16.31 Fall 2005
Lecture Fri 7-Oct-05 ver 1.2

Charles P. Coleman

October 9, 2005

TODAY

- TAKE AWAY:
  - Think about controllability and observability up front in the DESIGN phase.
  - Don't assume that control will save you!
- Design Examples
  - Satellite
  - Two Stick Cart
- References:
  - Grantham & Vincent (1993), Modern Control Systems Analysis and Design
  - Szidarovszky & Bahill (1997), Linear Systems Theory, 2nd Ed

TODAY
Design Issues
Satellite Design
Two Stick Cart

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?

Controllability and Observability - Real Life Designs

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?
  1. In general, a system contains real internal variables which are not accessible to either or both control or observation.

Controllability and Observability - Real Life Designs

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?
  1. In general, a system contains real internal variables which are not accessible to either or both control or observation.
  2. The relationship between states and either or both inputs and outputs may not be linearly independent.

Controllability and Observability - Real Life Designs

TODAY
Design Issues
Satellite Design
Two Stick Cart

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?

  1. In general, a system contains real internal variables which are not accessible to either or both control or observation.
  2. The relationship between states and either or both inputs and outputs may not be linearly independent.
  3. A system could have subsystems having identical dynamics.

Controllability and Observability - Real Life Designs

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?

  1. In general, a system contains real internal variables which are not accessible to either or both control or observation.
  2. The relationship between states and either or both inputs and outputs may not be linearly independent.
  3. A system could have subsystems having identical dynamics.
  4. Pole-zero cancellation in the transfer function of cascaded systems.

Controllability and Observability - Real Life Designs

- So far we've had a lot of theoretical definitions, but what can cause unobservability or uncontrollability in a mathematical model of a real system?

  1. In general, a system contains real internal variables which are not accessible to either or both control or observation.
  2. The relationship between states and either or both inputs and outputs may not be linearly independent.
  3. A system could have subsystems having identical dynamics.
  4. Pole-zero cancellation in the transfer function of cascaded systems.

- We should be concerned about these issues in the up front DESIGN of a controlled system!

Satellite Design

TODAY
Design Issues
Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions
Two Stick Cart

Geosynchronous Satellite

- States

$$\begin{aligned} r_1 &= \text{radial position} & \theta_1 &= \text{angular position} \\ r_2 &= \text{radial velocity} & \theta_2 &= \text{angular velocity} \end{aligned}$$

- Controls

$$u_1 = \text{radial thruster} \qquad u_2 = \text{tangential thruster}$$

- Outputs

$$y_1 = \text{radial position} \qquad y_2 = \text{angular position}$$

TODAY
Design Issues

Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions

Two Stick Cart

State Equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y_1 &= C_1 x \\ y_2 &= C_2 x \end{aligned}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$$

TODAY
Design Issues

Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions

Two Stick Cart

- DESIGN:
  - What actuators must be present for the system to be controllable?
  - What sensors must be present for the states to be observable from the measured output?
- Controls & Ouputs

$$
\begin{aligned}
u_1 &= \text{radial thruster} & u_2 &= \text{tangential thruster} \\
y_1 &= \text{radial position} & y_2 &= \text{angular position}
\end{aligned}
$$

Geosynchronous Satellite

- The satellite system is completely controllable with both thrusters

$$M_C = \begin{pmatrix} B & AB & A^2B & A^3B \end{pmatrix}$$

$$M_C = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 2\omega & -\omega^2 & 0 \\ 1 & 0 & 0 & 2\omega & -\omega^2 & 0 & 0 & 2\omega^3 \\ 0 & 0 & 0 & 1 & -2\omega & 0 & 0 & -4\omega^2 \\ 0 & 1 & -2\omega & 0 & 0 & -4\omega^2 & 2\omega^3 & 0 \end{pmatrix}$$

- M_C has rank 4
- What if the tangential thruster u_2 fails, and we are only left with the radial thruster u_1?

$$B_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

TODAY
Design Issues

Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions

Two Stick Cart

Geosynchronous Satellite Radial Thruster Control

- The satellite system is not controllable only using the radial thruster u_1.

$$M_C = \begin{pmatrix} B_1 & AB_1 & A^2B_1 & A^3B_1 \end{pmatrix}$$

$$M_C = \begin{pmatrix} 0 & 1 & 0 & -\omega^2 \\ 1 & 0 & -\omega^2 & 0 \\ 0 & 0 & -2\omega & 0 \\ 0 & -2\omega & 0 & 2\omega^3 \end{pmatrix}$$

- M_C has rank 3 since column 4 is a multiple of colum 2.
- We can't control the system with only the radial thruster u_1. Can we control the sytem only with the tangential thruster u_2?

$$B_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

TODAY
Design Issues

Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions

Two Stick Cart

- The satellite system is controllable only using the tangential thruster u_2.

$$M_C = \begin{pmatrix} B_2 & AB_2 & A^2B_2 & A^3B_2 \end{pmatrix}$$

$$M_C = \begin{pmatrix} 0 & 0 & 2\omega & 0 \\ 0 & 2\omega & 0 & 2\omega^3 \\ 0 & 1 & 0 & -4\omega^2 \\ 1 & 0 & -4\omega^2 & 0 \end{pmatrix}$$

- M_C has rank 4.
- Maybe we should have redundancy in tangential thruster control?

Geosynchronous Satellite Radial Measurement

- Are the states of the satellite observable if we only measure its radial distance?

$$C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} C_1 \\ C_1 A \\ C_1 A^2 \\ C_1 A^3 \end{pmatrix}$$

$$M_O = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & -\omega^2 & 0 & 0 \end{pmatrix}$$

- rank 3. 3rd column is zero!
- The system states are not observable soley from the measurement of the radial position of the satellite.

TODAY
Design Issues

Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions

Two Stick Cart

- Hopefully the system states are observable if we are only left with an angular position measurement.

$$C_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} C_2 \\ C_2 A \\ C_2 A^2 \\ C_2 A^3 \end{pmatrix}$$

$$M_O = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 0 \\ -6\omega^2 & 0 & 0 & -4\omega^2 \end{pmatrix}$$

- The system states are observable soley from the measurement of the tangential position of the satellite.

TODAY
Design Issues
Satellite Design
Satellite States
Satellite State Eqs
Satellite Design
Radial Control
Tangential Control
Radial Measurement
Tangential Measure
Conclusions
Two Stick Cart

- DESIGN Conclusions:
  - We must at least have tangential thruster to control the satellite system.
  - We must at least measure the angular position of the satellite to estimate the system's states.
- Controls & Ouputs

$$u_2 = \text{ tangential thruster} \qquad y_2 = \text{ angular position}$$

TODAY
Design Issues
Satellite Design
Two Stick Cart
Cart Dynamics
Cart State Equations
Cart Design
NEXT WEEK

Two Stick Cart

Cart with two sticks

TODAY
Design Issues
Satellite Design
Two Stick Cart
Cart Dynamics
Cart State Equations
Cart Design
NEXT WEEK

- System Parameters

$$\begin{array}{rlrl} M: & \text{cart mass} & & \\ M_1: & \text{mass of stick 1} & L_1: & \text{length of stick 1} \\ M_2: & \text{mass of stick 2} & L_2: & \text{length of stick 2} \\ & M_1 = M_2 & & \end{array}$$

- Dynamics (linearized!)

Sum horizontal forces

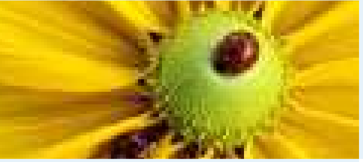
$$M\dot{v} = -M_1 g\theta_1 - M_2 g\theta_2 + u$$

Sum torques about each pivot point

$$M_1(\dot{v} + L_1\ddot{\theta}_1) = M_1 g\theta_1$$

$$M_2(\dot{v} + L_2\ddot{\theta}_2) = M_2 g\theta_2$$

Eliminate $\dot{v}$ from the equations



Cart with two sticks

- States

$$x_1 = \theta_1 \qquad x_3 = \dot{\theta}_1$$
$$x_2 = \theta_2 \qquad x_4 = \dot{\theta}_2$$

- State Equations

$$\dot{x} = Ax + Bu$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_1 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ -c \\ -d \end{pmatrix}$$

$$a_1 = \frac{(M+M_2)g}{ML_1} \qquad a_2 = \frac{M_2 g}{ML_1} \qquad c = \frac{1}{ML_1}$$
$$a_3 = \frac{M_2 g}{ML_2} \qquad a_4 = \frac{(M+M_2)g}{ML_2} \qquad d = \frac{1}{ML_2}$$

Cart System Design

TODAY
Design Issues
Satellite Design
Two Stick Cart
Cart Dynamics
Cart State Equations
Cart Design
NEXT WEEK

- DESIGN: What restrictions must we place on the system for it to be controllable?
- M_C must have rank 4.

$$M_C = \begin{pmatrix} B & AB & A^2B & A^3B \end{pmatrix}$$

$$M_C = \begin{pmatrix} 0 & -c & 0 & -(a_1c + a_2d) \\ 0 & -d & 0 & -(a_3c + a_4d) \\ -c & 0 & -(a_1c + a_2d) & 0 \\ -d & 0 & -(a_3c + a_4d) & 0 \end{pmatrix}$$

- M_C will have rank less than 4 if

$$\begin{aligned} \alpha \begin{pmatrix} c \\ d \end{pmatrix} &= \begin{pmatrix} (a_1c + a_2d) \\ (a_3c + a_4d) \end{pmatrix} \\ c(a_3c + a_4d) &= d(a_1c + a_2d) \end{aligned}$$

Cart System Design

- M_C will have rank less than 4 if

$$c(a_3 c + a_4 d) \quad = \quad d(a_1 c + a_2 d)$$

$$\frac{1}{ML_1}\left(\frac{M_2 g}{ML_2}\frac{1}{ML_1} + \frac{(M+M_2)g}{ML_2}\frac{1}{ML_2}\right) = \frac{1}{ML_2}\left(\frac{M_2 g}{ML_1}\frac{1}{ML_2} + \frac{(M+M_2)g}{ML_1}\frac{1}{ML_1}\right)$$

- If $L_1 = L_2$ the cart system is uncontrollable.
- This is an example of two subsystems having the same dynamics.
- Fix this in the DESIGN, 'cause control can't help you out! Unless?...

TODAY
Design Issues
Satellite Design
Two Stick Cart
Cart Dynamics
Cart State Equations
Cart Design
NEXT WEEK

NEXT WEEK

- Monday - Columbus Day
- Wednesday - Quiz Review with Dr Paduano
- Friday - Quiz 1