16.31 Fall 2005 - Homework 1

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Date Out: Friday 9 September 2005
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Problem 1

Bélanger, Problem 2.1
Comment: This problem, and Problems 2 and 3, may seem trivial, but they are very important exercises. Throughout this course you will be interconnecting systems either to implement a controller or observer, or to prove interconnected system properties (e.g. separation principle), or to implement a controller architecture. Hence, these problems are good practice! As you go through the course, try to identify where we are applying interconnection of systems for analysis and control!

Problem 2

Bélanger, Problem 2.2

Problem 3

Bélanger, Problem 2.3

Problem 4

Bélanger, Problem 2.21
Comment: The historical motivation for this problem is rocket control where the ballistic trajectory is known, but small course corrections around the trajectory are necessary in order to get the rocket to its target. Understanding the concept of linearization about a trajectory is as important to your control education as understanding the concept of linearization about a point. This is why this problem has been assigned.

Problem 5

The satellite shown in Figure 1 is in circular orbit around the earth and is subject to an inverse square field $-k/r$. The satellite has the ability to exert a thrust u_1 in the radial direction and a thrust u_2 in the tangential direction.

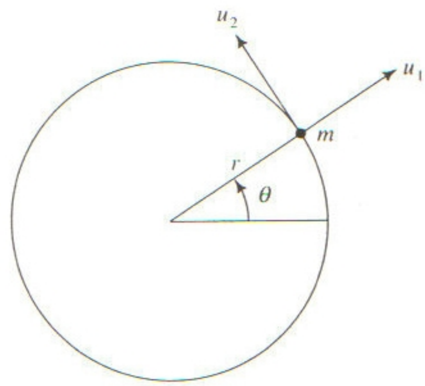

Figure 1: Satellite in circular orbit about the earth

Lagrangian analysis yields the following nonlinear dynamics:

$$m\ddot{r} - mr\dot{\theta} + \frac{k}{r^2} = u_1$$

$$2r\dot{r}\dot{\theta}m + r^2\ddot{\theta}m = ru_2$$

for the state variables r, the radial distance from the earth, and θ, the angular position of the satellite.

a) Let $m = 1$ and solve this system of equations for $\ddot{r}$ and $\ddot{\theta}$.

b) Let $r_1 = r$, $r_2 = \dot{r}$, $\theta_1 = \theta$, and $\theta_2 = \dot{\theta}$, and rewrite the equations for $\ddot{r}$ and $\ddot{\theta}$ as a system of first order nonlinear equations.

c) Assume $k \neq 0$. Does an equilibrium point exist with zero input $u_1(t) = u_2(t) = 0$ $(t \geq 0)$?
Let $\sigma = 1$, $k = \sigma^3\omega^2$, $u_1(t) = u_2(t) = 0$ $(t \geq 0)$ and let

$$r_1(t) = \sigma, r_2(t) = 0$$

$$\theta_1(t) = \omega t, \theta_2(t) = \omega$$

and define the following new state variables

$$\begin{aligned} x_1 &= r_1 - \sigma \\ x_2 &= r_2 \\ x_3 &= \theta_1 - \omega t \\ x_4 &= \theta_2 - \omega \end{aligned}$$

d) What differential equations are satisfied by these new state variables?

e) Linearize the resulting differential equations about the given trajectory and write the resulting system of equations in the form

$$\dot{x} = Ax + Bu$$

Comment: This problem is a classic and its solution can found in many control and satellite control/orbital mechanics books. However, you should try this classic problem on your own. This problem show that often we have to manipulate the resulting dynamical equations to put them in a form suitable for analysis in the framework that we use in this class. You do this in part a) and b).

In this problem we combine the concepts of linearization about a point and linearization about a trajectory. In part c) you are asked to determine if an equilibrium point exits. In parts d) and e) you are asked to linearize the system about a trajectory applying the theory that you learned in Problem 4. The application of this problem to satellite control and orbital mechanics should illustrate to you the importance and utility of linearization about a trajectory!