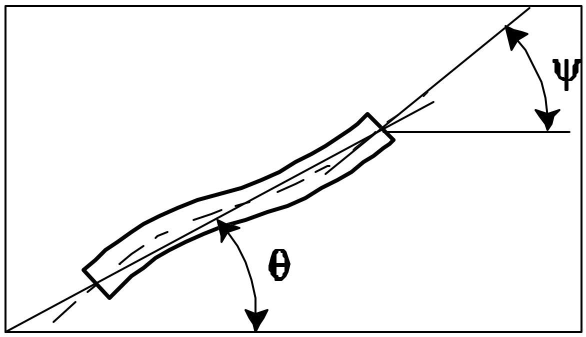
16.31 Fall 2005 - Homework 8

Date Out: Wednesday 30 November 2005
Date Due: Wednesday 7 December 2005 2pm

Problem 1: LQG for a satellite



<u>Further problem definition:</u>

The states of the satellite model given in Homework 7, Problem 3, are in radians and radians per second. The control input is given in normalized torque units, where a torque value of 10 is the maximum available torque.

The goal of this problem is to regulate the pointing angle, y, to zero, for an initial condition of:

$$\mathbf{x}_0 = \frac{p}{180} \cdot \begin{bmatrix} 5 & 0 & 0.5 & 0 & 0.5 & 0 \end{bmatrix}^T$$

(5 degrees pointing error, with 0.5 degree additional error introduced by each structural mode). Only one measurement is available to perform this task: an attitude sensor which is corrupted by vibration, such that the output equation is:

$$\mathbf{y} = \mathbf{Cx} = \begin{bmatrix} 1 & 0 & -0.75 & 0 & 0.25 & 0 \end{bmatrix} x .$$

In addition, the system is assumed to be excited by processes that impose unknown torques that drive all of the system modes, such that the forced system is of the form:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{B_w w} \text{ where } \mathbf{B_w} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Other than these changes, the system dynamics are identical to those used in the previous Homework, and have been distributed as a script to everyone in the class by email.

A script (m-file) into which you will insert your solutions, and I will subsequently run to view your solutions, has also been distributed. This script contains call statements that make the appropriate plots for me to judge the success of your design.

a) Design an LQG regulator that regulates the pointing angle y to zero, and damps all modes of the system. Damping of every system mode should be at least 4%, that is

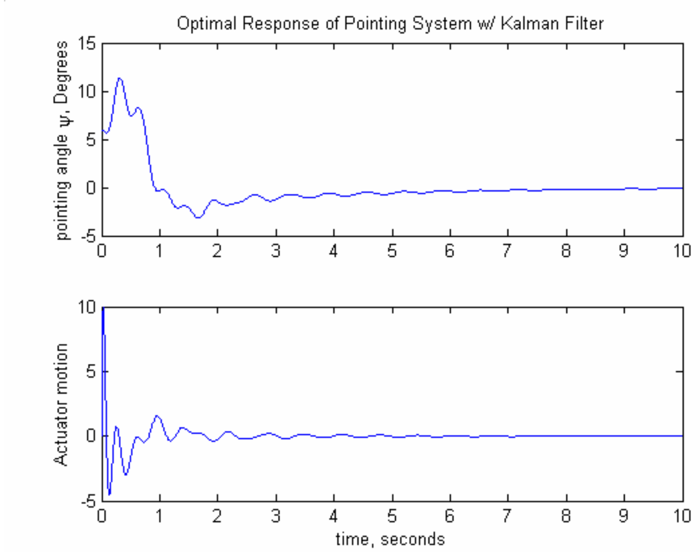
$$z \geq 0.04 \text{ for every eigenvalue.}$$

(you can use damp(eig(Acl)), where Acl is the closed-loop A matrix, to check this.)

Submit your LQR gain, K, and your Kalman filter gain G, in the script supplied, so that I can run the script and use my plotting programs to plot out your system's performance. My latest design is shown below, as an example of the scales I will use to judge performance. Overshoot, damping, settling time, and control activity required should all be as good as, or better, than my design.

b) In the script you submit, include code that plots the bode plot of the *LQG compensator* represented by K and G.

My compensated system response plots – See if you can do better!:



Kalman filter estimator response for the initial state defined above, but no feedback.