16.31 Fall 2005 - Homework 7

Date Out: Monday 21 November 2005
Date Due: Wednesday 30 November 2005 2pm

Problem 1: LQR hand calculation

Bélanger Problem 7.20. In addition to doing the problem as stated, plot integrated control effort, $J_u = \int_0^\infty u^2 dt$, and integrated output deviation, $J_x = \int_0^\infty y^2 dt$, as functions of r, as well as the over all cost $J = J_u + rJ_x$ and comment.

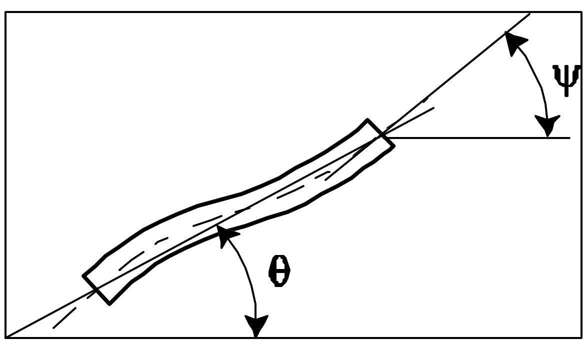
Problem 2: Incorporation of integrators into LQR designs

Repeat Problem 5(b) from Homework 6, but incorporate integrators which for the error between commanded and actual n_z and γ. Your goal should be to improve the time it takes these variables to reach there commanded values, and to keep overshoot and surface deflections as small as possible.

Problem 3: LQR for a satellite

(from Bryson, A. E., Control of Spacecraft and Aircraft, Princeton University Press, 1994)

The Orbiting Solar Observatory-8 (OSO-8) was launched in 1975. The attitude control system for this spacecraft had to keep its despun section (the one supporting the telescope) pointed at the sun. The despun section was sufficiently flexible that two vibration modes, one at 93.2 rad/sec and the other at 255 rad/sec, had to be considered in designing the control system. A diagram of the telescope is shown in the figure below.



The rigid-body attitude is given by q, but the angle that the satellite's optical axis is pointing is corrupted by the flexible modes, and is given by y.
The equations of motion for this vehicle are given by $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, $\mathbf{y} = \mathbf{Cx}$, where $\mathbf{A}$ is block diagonal with 2 by 2 blocks

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -0.8686 & 0 \end{bmatrix}, \text{ and } \mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ -6.5025 & 0 \end{bmatrix}.$$

The states are $\mathbf{x} = [q \quad \dot{q} \quad c \quad \dot{c} \quad h \quad \dot{h}]^T$ (the last 4 states are associated with the vibration modes). The output that we are interested in is $\mathbf{y} = y$, the attitude of the optical axis. The remaining matrices needed to define the dynamics are:

$$\mathbf{B} = [0 \quad 1 \quad 0 \quad 0.1493 \quad 0 \quad -1.1493]^T \text{, and}$$

$$\mathbf{C} = [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0].$$

(a) Plot a locus of the closed-loop roots vs. the parameter Q, where the roots are placed (using full-state feedback) to minimize the performance index

$$J = \int_0^\infty \left(Qq^2 + \mathbf{u}^2\right)dt.$$

(b) Determine the state-feedback gains for Q=1 and plot the outputs y for the initial condition $\mathbf{x} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$. This initial condition corresponds to the telescope being initially offset by one degree and no flexible dynamics are excited.

(We will incorporate a state estimator to implement this controller with limited measurements in the next Homework.)