16.31 Fall 2005 Sample Quiz 2 ver 1.1

Charles P. Coleman

Monday 31 October 2005

Comments

This is a one (1) hour exam.
This is a closed book/closed notes exam.
You are allowed two (2) single pages of notes (both sides).
No calculators or electronic devices may be used to complete the quiz.
To maximize your score, show all work and make sure your overall approach is clear.
The maximum score on this quiz is 50 points.

Problem 1

Consider the system (A, B)

$$A = \begin{pmatrix} -0.5 & 1 & 0 \\ -1 & -0.5 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad C^T = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Is this system observable?
- If not completely controllable, what quantities are observable?
- What quantities are unobservable?

Problem 2

- If $(A, b$ is given and it is NOT controllable, can we always choose c so that (c, A) is observable? A proof or counterexample will suffice.
- If $(A, b$ is given and controllable, can we always choose c so that (c, A) is observable?

Problem 3

For the plant

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u$$

design a state feedback control to place the closed-loop eigenvalues at $-2 \pm 2j$.

Problem 4

Design the observer matrix L to estimate the state of the system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u, \quad y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

from the output y. Place the observer eigenvalues at $-10 \pm 10j$.

Problem 4

Brian and Jennie are given the following transfer function

$$G(s) = \frac{s^2 - s}{s^3 + s^2 - s - 1}$$

- Jennie claims she can find a controllable realization for this system of dimension 4. Is she right? If she is, is her realization observable?
- Brian claims he can find an observable realization of this system of dimension 5. Is he right? If he is not, what dimension would be appropriate to build an observable realization?
- Jennie now says she can find a controllable AND observable realization of this system of dimension 2. Is she right?

Problem 5

For the system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

- Is it possible to arbitrarily locate the regulator poles for this system? What about the estimator poles?
- Design a full state feedback regulator with closed-loop poles at $-2 \pm 2j$.
- Use Ackermann's formula to design an estimator with poles at $-4, -5$.

Problem 6

Consider the transfer function

$$G(s) = \frac{s+1}{s^4 + 4s^3 + 3s^2 + 5s + 5} + 1$$

- Write a controllable state-space realization for this system
- Write an observable state-space realization for this system
- Write a controllable and observable state-space realization for this system

Problem 7

Consider the system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -\epsilon \end{pmatrix} x$$

where ϵ is a small positive quantity

- Show this system is always stable
- Compute a quadratic Lyapunov function for this system when $\epsilon = 0.1$
- In the limit when $\epsilon \to 0$ but remains positive, describe all quadratic Lyapunov functions that prove stability of this system

Problem 8

Consider the transfer function

$$G(s) = \frac{3s+1}{s^2 + s - 2}$$

- Find a state-space representation that is not controllable
- Find a state-space representation that is not observable
- Find a state-space representation that is neither controllable nor observable

Problem 9

Show that feedback does not destroy complete controllability. Specifically show that if (A, B) is completely controllable, then $(A - BK, B)$ is completely controllable.

Problem 10

Given the partitioned system

$$\begin{pmatrix} \dot{w} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} w \\ y \end{pmatrix}$$

with output y is completely observable. Show that the combination of A_{11} as system and A_{21} as output matrix is completely observable - (A_{11}, A_{21}) is c.o. (It may be simpler to prove the complete controllability of the transposed combination.)