16.31 Fall 2005
Quiz 1 Sample Questions Fri 7-Oct-05 ver 1.0

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Comments

Here are a set of possible problems that could appear on Quiz 1

Problem 1

Consider the function

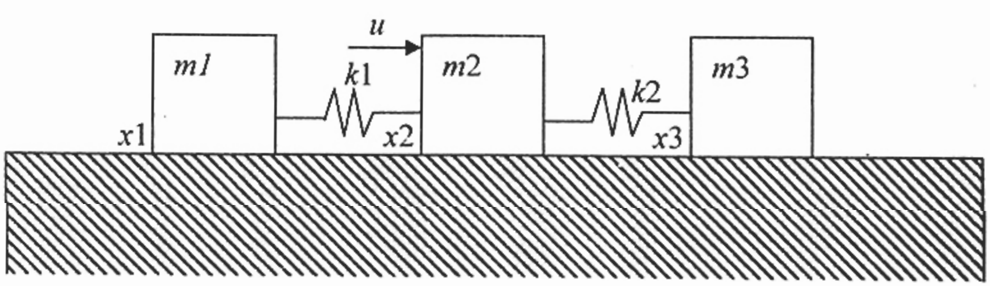
$$\frac{3s+1}{s^2+s-2}$$

- Find two different state sapce representations of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

Problem 2

The following mechanical system is given to you:



Three masses slide with no friction on a flat surface. Their masses are m_1, m_2, and m_3 respectively, and they are connected together by two springs with stiffness constants k_1 and k_2. A force u is applied to the system.

- Write down the equations of motion for this system in form of a linear time-invariant system like

$$\dot{x} = Ax + Bu$$

- Write the condition that must hold on spring stiffnesses k_1 and k_2 and the masses m_1, m_2, and m_3 for the system to be controllable.
- Assume all masses are 1 kg, and all spring stiffnesses are 1 kg/m. Write the transfer function from input force u to output u_2.
- Draw an equivalent mechanical system with the same transfer function.

Problem 3

Derive state-space models for sytesm described by the following transfer functions

$$G(s) = \frac{6}{s^3 + 3s^2 + 5s + 1}$$
$$G(s) = \frac{6(s+1)}{s^3 + 3s^2 + 5s + 1}$$

Problem 4

Derive state-space models for sytesm described by the following transfer functions

$$G(s) = \frac{4s+3}{3s^2 + 5s + 4}$$
$$G(s) = 2\frac{s+3}{s+10}$$
$$G(s) = \frac{s^2 + 3s + 4}{s^2 + 7s + 9}$$

1 Problem 5

Consider the system (A, B)

$$A = \begin{pmatrix} -0.5 & 1 & 0 \\ -1 & -0.5 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

- Is this system controllable?
- If not completely controllable, what quantities are uncontrollable?

2 Problem 6

If

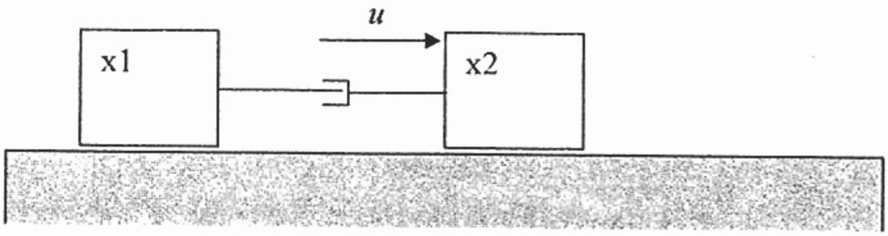
$$A = \begin{pmatrix} \sigma & \omega \\ -\omega & \sigma \end{pmatrix}$$

Show that

$$e^{At} = \begin{pmatrix} e^{\sigma t}\cos\omega t & e^{\sigma t}\sin\omega t \\ -e^{\sigma t}\sin\omega t & e^{\sigma t}\cos\omega t \end{pmatrix}$$

3 Problem 7

Consider the following problem



Two masses are connected by a single dashpot. The second mass can be moved by force acutation u.

The equations of motion are

$$\ddot{x}_1 = \mu(\dot{x}_2 - \dot{x}_1) \quad (1)$$
$$\ddot{x}_2 = \mu(\dot{x}_1 - \dot{x}_2) + u \quad (2)$$

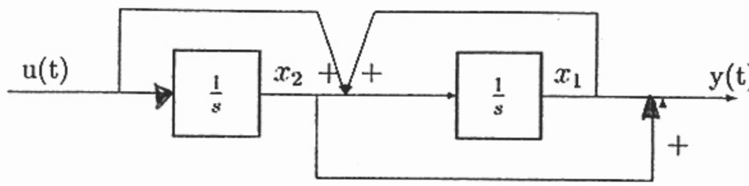
where $\mu \geq 0$.

- Write a state-space model for this system.
- Is this system controllable?
- How does controllabily change with the value of μ?
- Asusme the second mass moves forward by one unit of length, starting at rest and stopping at rest after travelling one unit forward. How far does the first mass move it it was also initially at rest? Does it depend on the specific path followed by the second mass? Does it depend on μ? (Don't forget to considerthe case $\mu = 0$)

  You may want to consider first what happens if the second mass moves forward at constant speed, then stops when it reaches one unit.

Problem 8

Consider the linear time-invariant system shown below:



- Write down the state space equations for this system using states given in the figure
- What is the transfer function $G(s) = Y(s)/U(s)$ for this system?
- Is this system controllable? Show all calculations
- Find e^{At} explicitly

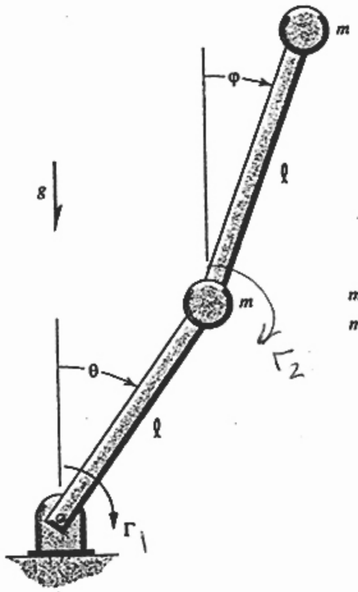
Problem 9

The longitudinal flight characteristics of the Boeing 747 flying near sea level at 221 ft/sec are given in standard notation by

$$A = \begin{pmatrix} -0.021 & 0.122 & 0 & -0.322 & 1 \\ -0.209 & -0.530 & 2.21 & 0 & -0.044 \\ 0.017 & -0.164 & -0.412 & 0 & 0.544 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.25 \end{pmatrix} \quad B = \begin{pmatrix} 0.010 \\ -0.064 \\ -0.378 \\ 0 \\ 0 \end{pmatrix}$$

Is (A, B) Controllable?

Problem 10



$$m\ell^2(2\ddot{\theta}+\ddot{\varphi}) - 2mg\ell\theta = \Gamma_1$$
$$m\ell^2(\ddot{\theta}+\ddot{\varphi}) - mg\ell\varphi = 0 \quad \Gamma_2$$

Figure 2.5-1 Double-pinned inverted pendulum system for Exercise 2.5-9.

The linearized dynamics are given above

The states are $x = [\theta, \dot{\theta}, \varphi, \dot{\varphi}]^T$

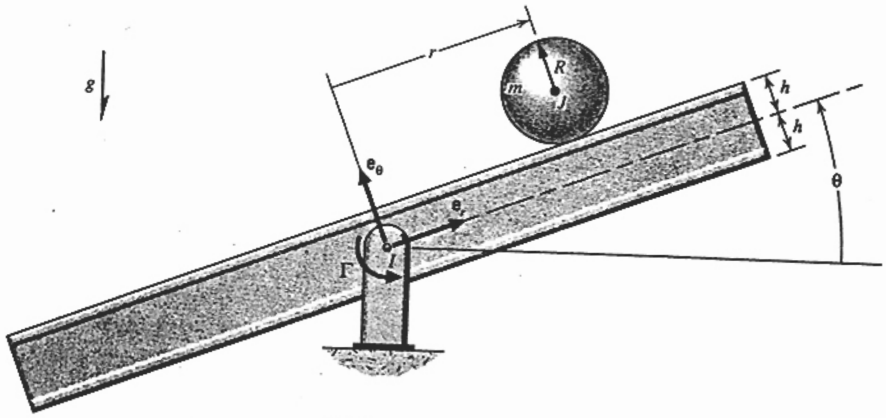
The control is $u_{\Gamma_1} = \Gamma_1 / m\ell^2$ or $u_{\Gamma_2} = \Gamma_2 / m\ell^2$

$$\omega^2 = g/\ell$$

- Write the system in state space form
$$\dot{x} = Ax + Bu$$

- Is the system controllable with u_{Γ_1}?
- Is the system controllable with u_{Γ_2}?

Problem 11



(a) Ball and beam system

the equations of motion (

$$\left[m + \frac{J}{R^2}\right]\ddot{r} - \left[\frac{J}{R} + m(R+h)\right]\ddot{\theta} - mr\dot{\theta}^2 + mg\sin\theta = 0$$

$$\frac{hJ}{R^2}\ddot{r} + \left[I + mr^2 - \frac{hJ}{R}\right]\ddot{\theta} + 2mr\dot{r}\dot{\theta} - mr(R+h)\,\dot{\theta}^2 + mgr\cos\theta = \Gamma.$$

Assuming that $(r,\dot{r},\theta,\dot{\theta})$ are small quantities,

- Identify an equilibrium point
- Linearize the equations about the eq point
- Write the system in the form

$$\dot{x} = Ax + Bu$$

- Is the system controllable with torque Γ?