$\boxed{1}$

$$\frac{3s+1}{(s+2)(s-1)} = \frac{A}{(s+2)} + \frac{B}{(s-1)}$$

$$\frac{3s+1}{s-1} = A + \frac{s+2}{s-1}B$$

$$s = -2 \Rightarrow A = \frac{3(-2)+1}{-2-1} = \frac{-6+1}{-3} =$$

$$\ddot{y} + \dot{y} - 2y = 3\dot{u} + u$$

$$x_1 = y$$
$$x_2 = \dot{y}$$

$$\dot{x}_2 = -\dot{y} + 2y + 3\dot{u} + u$$
$$= -x_2 + 2x_1 + 3\dot{u} + u$$

$$\dot{x}_2 - 3\dot{u} = -x_2 + 2x_1 + u$$

let $z = x_2 - 3u$; $-x_2 = -3u - z$

$$\dot{z} = \dot{x}_2 - 3\dot{u} = -x_2 + 2x_1 + u$$
$$= -z - 3u + 2x_1 + u$$

$$\dot{x}_1 = x_2 = z + 3u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ z \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} u$$

$$\cancel{z}\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ z \end{bmatrix}$$

$$\dot{x}_1 = z + 3u \qquad\qquad z = \dot{x}_1 - 3u$$

$$\dot{z} = 2x_1 - z - 2u \qquad\qquad \dot{z} = \ddot{x}_1 - 3\dot{u}$$

$$\ddot{x}_1 - 3\dot{u} = 2x_1 - \dot{x}_1 + 3u - 2u$$

$$\ddot{x}_1 + \dot{x}_1 - 2x_1 = 3\dot{u} + u$$

$$x_1 = \frac{3s+1}{s^2+1s-2}\,u \quad \checkmark$$

<u>Another SS system:</u>

let $\quad \underline{w} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} x \\ z \end{pmatrix} = T\underline{x}$

then $\quad \dot{\underline{w}} = T\dot{\underline{x}} = TAT^{-1}w + TBu$

$$TAT^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$TB = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$CT = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \checkmark$$

$\boxed{3}$ (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} \underline{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$ (4.1)

(b) $\dfrac{\cancel{6}s + 6}{s^3 + 3s^2 + 5s + 1}$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$x_3 \ \ddot{x} =$

$$y = [6 \ 6 \ 0] \ \underline{x}$$

$\boxed{4}$ (a)

$$x_1 = y$$
$$\dot{x}_1 = x_2 = \dot{y}$$
$$3\dot{x}_2 = -5x_2 - 4x_1 + 4\dot{u} + 3u$$

$$3\dot{x}_2 - 4\dot{u} = -5x_2 - 4x_1 + 3u$$

$$z = 3x_2 - 4u$$
$$\dot{z} = -5x_2 - 4x_1 + 3u$$
$$= -\frac{5}{3}(z + 4u) - 4x_1 + 3u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4/3 & -5/3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \frac{4/3 s + 1}{s^2 + 5/3 + 4/3}$$

$$y = [1 \quad 4/3] \ \underline{x}$$

4.2

$$y = 2\,\frac{s+3}{s+10}\,u$$

$$\dot{y} + 10y = 2\dot{u} + 6u$$

$$(\dot{y} - 2\dot{u}) = -10y + 6u$$

$$x = y - 2u$$

$$\dot{x} = -10(x+2u) + 6u$$

$$\boxed{\begin{aligned} \dot{x} &= -10x - 14u \\ y &= x + 2u \end{aligned}}$$

$$\frac{2s+6}{s+10} = a + \frac{b}{s+10}$$

$$b = -14$$

$$\frac{2s+6}{s+10} = \frac{2s+20}{s+10} - \frac{14}{s+10}$$

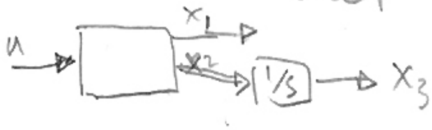
$$a = \frac{(2s+6)+14}{s+10} = \frac{2s+20}{s+10} = 2$$

$$\boxed{\begin{aligned} \dot{x} &= -10x - 14 \\ y &= x + 2u \end{aligned}}$$

$\boxed{5}$

$$\mathcal{C}(A,B) = \begin{bmatrix} 1 & 1.5 & -2.75 \\ 2 & -2 & -0.5 \\ 0 & 2 & -2 \end{bmatrix} \begin{matrix} 1 & 1.5 \\ 2 & -2 \\ 0 & 2 \end{matrix}$$

$$\det \mathcal{C} = 4 - 11 + 1 + 6 = 0$$



$$= [1]\{4+1\} - 2\{-3+5.5\} = 5-5=0$$

$\boxed{6}$

$$e^{At} = F(A) = \alpha_0 + \alpha_1 A \qquad \lambda = \sigma \pm j\omega$$

$$e^{\lambda_1 t} = \alpha_0 + \alpha_1 \lambda_1 = \alpha_0 + \alpha_1(\sigma + j\omega)$$

$$e^{\lambda_2 t} = \alpha_0 + \alpha_1 \lambda_2 = \alpha_0 + \alpha_1(\sigma - j\omega)$$

$$e^{\sigma + j\omega t} + e^{\sigma - j\omega t} = 2(\alpha_0 + \sigma\alpha_1)$$

$$e^{\sigma + j\omega t} - e^{\sigma - j\omega t} = 2\alpha_1 j\omega$$

$$\sin x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\frac{1}{\omega} e^{\sigma t} \sin \omega t = \alpha_1 \qquad A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

$$e^{\sigma t} \cos\omega t - \frac{\sigma}{\omega} e^{\sigma t} \sin\omega t = \alpha_0$$

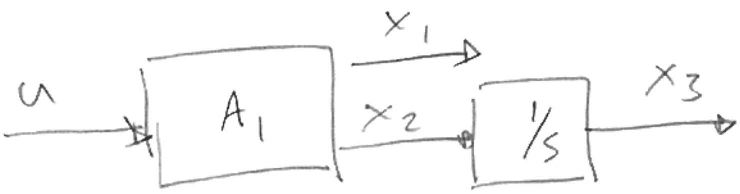
$$e^{At} = \begin{bmatrix} e^{\sigma t}\cos\omega t - \frac{\sigma}{\omega} e^{\sigma t}\sin\omega t & 0 \\ 0 & \text{same} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\sigma}{\omega} e^{\sigma t}\sin\omega t & \frac{\omega}{\omega} e^{\sigma t}\sin\omega t \\ -\frac{\omega}{\omega} e^{\sigma t}\sin\omega t & \frac{\sigma}{\omega} e^{\sigma t}\sin\omega t \end{bmatrix}$$

$\boxed{5}$

$W^T B = 0 \qquad W = V^T = \begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \end{bmatrix}$

for $w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$\dot{x}_3 = x_2 \qquad s x_3 = x_2$

$x_3 = \frac{1}{s} x_2$

eigenvalue at $s = 0$

since it is clearly the only mode that is not *directly* *effected* by B, it must be uncontrollable

~~A~~ ~~B~~

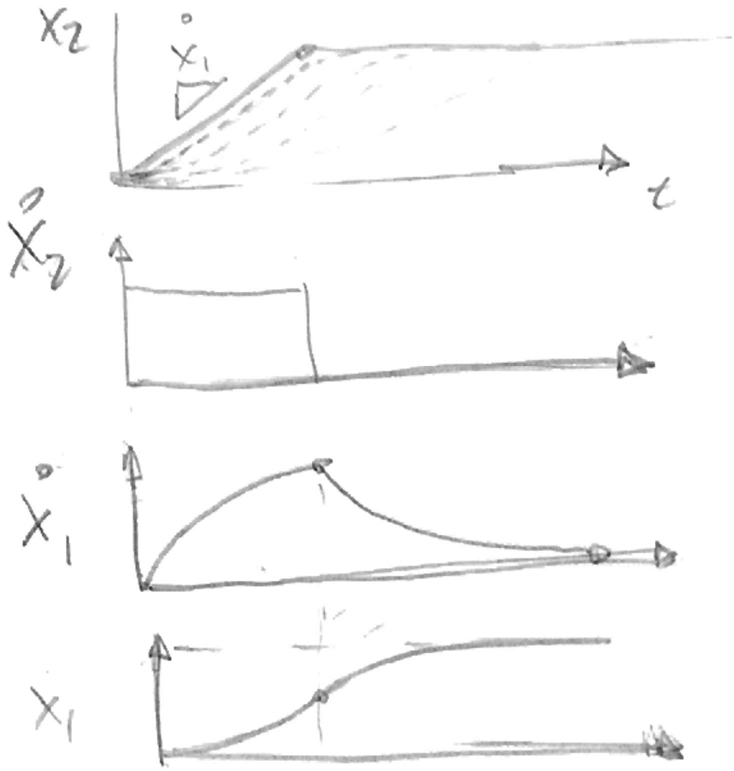
$\mathcal{C}(A_1, B_1) = \begin{bmatrix} 1 & 1.5 \\ 2 & -2 \end{bmatrix}$ ✓

$\boxed{7}$

$$\begin{bmatrix} \dot x_1 \\ \ddot x_1 \\ \dot x_2 \\ \ddot x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\mu & 0 & \mu \\ 0 & 0 & 0 & 1 \\ 0 & \mu & 0 & -\mu \end{bmatrix} \begin{bmatrix} x_1 \\ \dot x_1 \\ x_2 \\ \dot x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} 0 & 0 & \mu & -2\mu^2 \\ 0 & \mu & -2\mu^2 & +4\mu^3 \\ 0 & 1 & -\mu & +2\mu^2 \\ 1 & -\mu & 2\mu^2 & -4\mu^3 \end{bmatrix}$$

Un Controllable. $\mu = 0$ loses controllability. (4th Column = $-4 *$ 3rd Column). Rank goes from 3 to 2 if $\mu = 0$



$$\dot V_1 = \mu V_2 - \mu V_1$$

$$V_1 = \frac{\mu}{s+\mu} V_2$$