16.31

LECTURE # 2

- DOMINANT POLES
- ROOT LOCUS BASICS
- PERFORMANCE ISSUES
- DYNAMIC COMPENSATION P. I. D.
- SYNTHESIS I, II

REFERENCES TO FRANKLIN + POWELL

HIGHER-ORDER SYSTEMS

- OUR RELATIONSHIPS BETWEEN TIME RESPONSE TO A STEP AND THE POLE LOCATIONS WERE CALCULATED FOR A SECOND-ORDER SYSTEM.
  - GIVES GOOD INSIGHTS
  - ACTUALLY GOOD APPROXIMATIONS FOR MANY HIGHER-ORDER SYSTEMS BECAUSE THEIR TRANSIENT RESPONSE IS DOMINATED BY A PAIR OF COMPLEX-CONJUGATE POLES.

- RELATIVE DOMINANCE OF CLOSED-LOOP POLES DETERMINED BY
  1. RATIO OF REAL PARTS OF THE CLOSED-LOOP POLES $\hookrightarrow$ σ OR $\zeta\omega_n$
  2. RELATIVE MAGNITUDES OF THE RESIDUES EVALUATED AT THE CLOSED-LOOP POLES.

(1) SLOWER POLES TEND TO DOMINATE MORE THAN FAST ONES (WHICH DECAY QUICKLY), ALL ELSE BEING EQUAL. (FACTOR OF 5)

(2) A ZERO NEAR A POLE WILL TEND TO REDUCE ITS EFFECT ON THE SYSTEM RESPONSE (AND RESULT IN A SMALLER RESIDUE).

EXAMPLE: ① $\frac{Y}{U} = G_1 = \frac{5}{(s+1)(s+5)}$ STEP INPUT $U = 1/s$

- PARTIAL FRACTION EXPANSION

RESIDUES

$$Y = G_1 U = \frac{5}{s(s+1)(s+5)} = \frac{1}{s} + \frac{(-5/4)}{s+1} + \frac{(1/4)}{s+5}$$

↓

$$y(t) = 1 - 1.25e^{-t} + 0.25e^{-5t}$$

→ 1.25 LARGER THAN 0.25
→ e^{-t} SLOWER THAN e^{-5t}
∴ EXPECT THIS TERM TO DOMINATE RESPONSE
⇒ SLOW

②

$$G_2 = \frac{5.5(s+0.91)}{(s+1)(s+5)}$$

$$Y = G_2 U = \frac{5.5(s+0.91)}{s(s+1)(s+5)} = \frac{1}{s} + \frac{0.124}{s+1} + \frac{(-1.125)}{s+5}$$

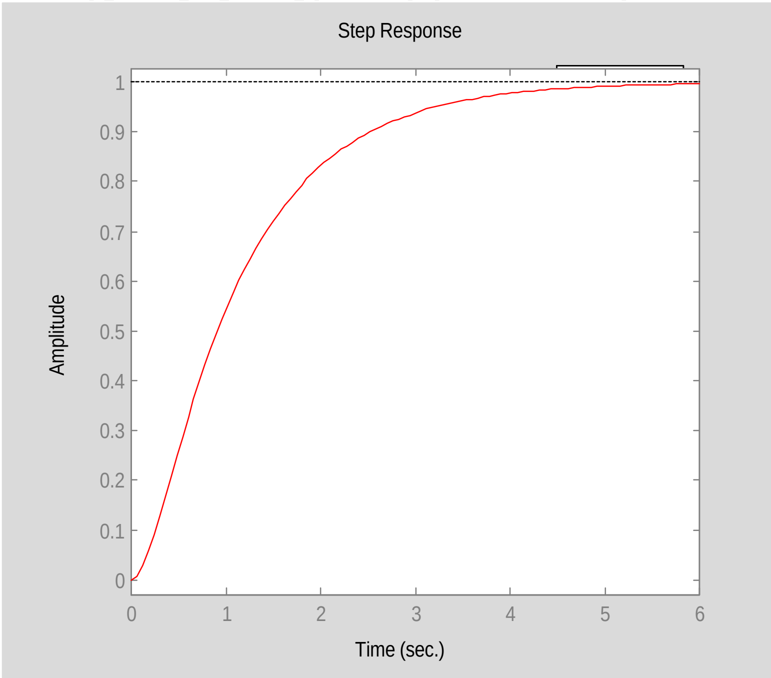
$$\Rightarrow y(t) = 1 + \underbrace{0.124\,e^{-t}} - 1.125\,e^{-5t}$$

- MUCH SMALLER CONTRIBUTION THIS TIME

⇒ EXPECT e^{-5t} (FAST) TO DOMINATE THE INITIAL RESPONSE → SHOULD THEN SEE THE CONTRIBUTION OF THE SLOW MODE.

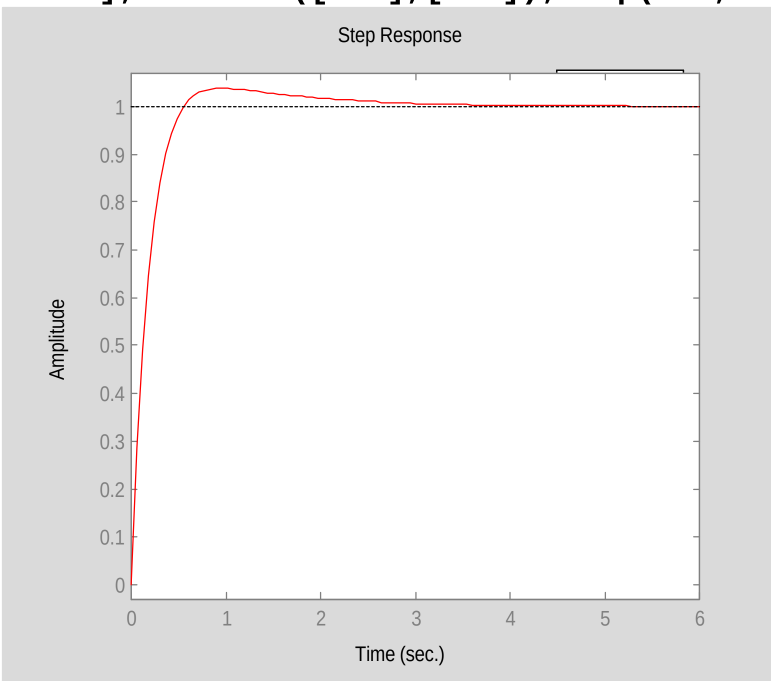
Slow dominant pole

```
num=5;den=conv([1 1],[1 5]);step(num,den,6)
```



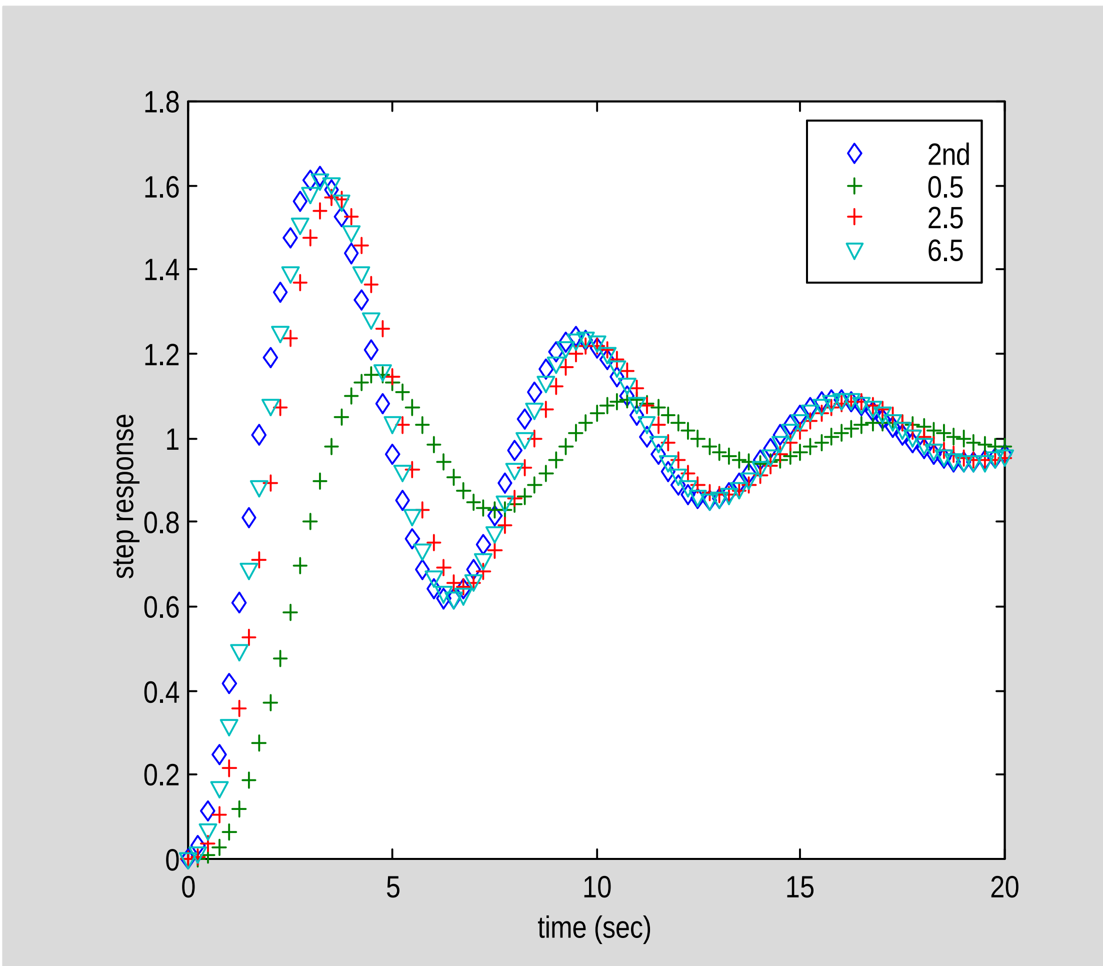
Fast dominant pole

```
num=5.5*[1 0.91];den=conv([1 1],[1 5]);step(num,den,6)
```



Similar example, but with second order dynamics combined with a simple real pole.

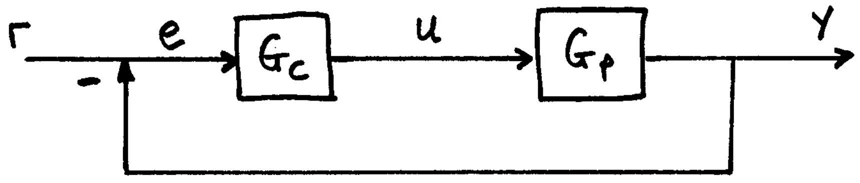
```
z=.15;wn=1;plist=[wn/2:1:10*wn];
nd=wn^2;dd=[1 2*z*wn wn^2];t=[0:.25:20]';
sys=tf(nd,dd);[y]=step(sys,t);
for p=plist;
  num=nd;den=conv([1/p 1],dd);
  sys=tf(num,den);[ytemp]=step(sys,t);
  y=[y ytemp];
end

plot(t,y(:,1),'d',t,y(:,2),'+',t,y(:,4),'+',t,y(:,8),'v');
ylabel('step response');xlabel('time (sec)')
legend('2nd',num2str(plist(1)),num2str(plist(3)),num2str(plist(7)))
```



For values of p=2.5 and 6.5, the response is very similar to the second order system. The response with p=0.5 is clearly no longer dominated by the second-order dynamics

ROOT LOCUS BASICS

BASIC FEEDBACK SYSTEM



G_p : PLANT TRANSFER FUNCTION $= \frac{N_p}{D_p}$

G_c : CONTROLLER T.F. $= K \frac{N_c}{D_c}$

{ N_c, D_c MONIC. POLYNOMIALS IN "S" }

- u : CONTROL COMMANDS
- y : SYSTEM OUTPUT
- r : REFERENCE INPUT (OFTEN = 0)
- e : RESPONSE ERROR

- WE WILL DISCUSS THE PERFORMANCE GOALS IN MORE DETAIL LATER. FOR NOW JUST CONCENTRATE ON LOCATION OF SYSTEM POLES.

- THIS IS THE "UNITY FEEDBACK" FORM. WE COULD PUT THE CONTROL G_c IN THE FEEDBACK PATH WITHOUT CHANGING THE POLE LOCATIONS.

- DISTURBANCES ADDED LATER.

BASIC QUESTIONS:

- ANALYSIS – GIVEN N_c, D_c, WHERE DO THE CLOSED-LOOP POLES GO AS A FUNCTION OF THE GAIN K. I.E. WHAT K TO PICK?

- SYNTHESIS – GIVEN N_p, D_p, HOW SHOULD WE PICK K, N_c, D_c TO GET THE CLOSED-LOOP POLES WHERE WE WANT THEM.

- NOTE: BOTH PRESUME THAT WE KNOW WHERE THE CLOSED-LOOP POLES SHOULD BE LOCATED. THIS THEME WILL RUN THROUGHOUT THE ENTIRE COURSE + WE WILL SPEND A LOT OF TIME LOOKING AT IT.

BLOCK DIAGRAM ANALYSIS

EASY TO SHOW THAT $\dfrac{Y}{r} = \dfrac{G_c G_p}{1 + G_c G_p} = G_{CL}$

$$G_{CL} = \frac{K N_c N_p}{D_c D_p + K N_c N_p}$$

CLOSED-LOOP TRANSFER FUNCTION

- THE DENOMINATOR IS CALLED THE (ϕ_c) CHARACTERISTIC EQUATION, + THE ROOTS OF $\phi_c = 0$ ARE CALLED THE CLOSED-LOOP POLES.

- POLES ARE CLEARLY A FUNCTION OF K (FOR GIVEN N_c, N_p, D_c, D_p) $\rightarrow$ A "LOCUS OF ROOTS"

OBSERVATIONS:

- ROOT LOCUS IS CONSISTENT WITH FIXING THE COMPENSATOR DYNAMICS N_c, D_c AND THEN CHANGING THE GAIN K

- ROOT LOCUS ENABLES US TO DETERMINE KEY FEATURES OF THE CLOSED-LOOP SYSTEM (TRANSIENT) RESPONSE (FROM THE POLE LOCATIONS) GIVEN THE OPEN-LOOP INFORMATION - N_p, D_p, N_c, D_c, K.

    $\Rightarrow$ WILL SEE THAT IT IS HARD TO INFER SOME PERFORMANCE PROPERTIES FROM THIS LOCUS $\rightarrow$ USE BODE TECHNIQUES TOO.

- APPROACH SUGGESTED BY W.R. EVANS

Evans, W. R. Graphical analysis of control systems, *AIEE Transactions* , vol. 67, , 1948, pp 547-551; and Control system synthesis by root locus method, *AIEE Transactions* , vol. 69, 1950, pp. 66-69.

ROOT LOCUS ANALYSIS

- IN GENERAL, THE FULL ROOT LOCUS IS VERY COMPLEX AND REQUIRES MATLAB® TOOLS LIKE "RLOCUS(NUM, DEN)"
  - EVANS' ORIGINAL PAPER USES A SOURCES + SINKS ANALOGY.
  - FULL PLOTTING RULES ON FPE PAGE 260.

⇒ WE NEED TO DEVELOP SOME BASIC DRAWING SKILLS SO WE CAN DO "BACK OF THE ENVELOPE" DESIGNS.

- BASIC POINTS: ASSUME N_c, D_c KNOWN.

LET $L_d = \frac{N_c}{D_c} \cdot \frac{N_p}{D_p}$

THEN $\phi_c = 1 + K L_d = 0$?

$\Rightarrow \quad L_d = -\frac{1}{K}$ $\qquad$ K REAL, POSITIVE

⇒ CAN USE THIS TO ANSWER BASIC QUESTION: IS S_0 ON THE ROOT LOCUS?

ANS: ONLY IF $\angle L_d(s_0) = 180° \pm 360° \cdot L$
$L = 0, 1, 2, \ldots$

- K POSITIVE → 180° LOCUS
  NEGATIVE → 0° LOCUS.

BASIC QUESTIONS

① WHERE DOES THE LOCUS START ?

- POLES

② WHERE DOES THE LOCUS END ?

- ZEROS
- ASYMPTOTES

③ WHEN / WHERE IS THE LOCUS ON THE REAL LINE ?

- LOCUS POINTS ON THE REAL LINE ARE TO THE LEFT OF AN ODD NUMBER OF REAL-AXIS POLES AND ZEROS.

④ GIVEN THAT s_0 IS ON THE LOCUS, WHAT GAIN IS NEEDED TO GET THIS CLOSED-LOOP POLE ?

$$K_R = \frac{1}{|L_d(s_0)|}$$

$$L_d(s) = \frac{N_c}{D_c} \cdot \frac{N}{D}$$

$$K(s) = K_R \frac{N_c}{D_c}$$

BASIC QUESTIONS

1) WHERE DOES THE LOCUS START?

IF $K \to 0$, THEN, WITH $L_d = N/D$

$$\phi_c = 0 = 1 + K\frac{N}{D} \Rightarrow D + KN = 0$$

$$\therefore \text{ AS } K \to 0 \Rightarrow \underset{\text{LOCUS STARTS AT}}{D = 0}$$

- ROOTS OF D ARE THE POLES OF THE PLANT AND COMPENSATOR.

2) WHERE DOES THE LOCUS END?

ALREADY SHOWED THAT FOR s_0 TO BE ON THE LOCUS, WE MUST HAVE

$$L_d(s_0) = -\frac{1}{K}$$

$\therefore$ AS $K \to \infty$, POLES MUST SOLVE:

$$L_d = \frac{N}{D} = \frac{N_c N_p}{D_c D_p} = 0$$

- SEVERAL POSSIBILITIES
  - A) POLES LOCATED AT VALUES OF s FOR WHICH $N \equiv 0$ (ZEROS OF PLANT / COMPENSATOR)
  - B) LOOP ($L_d(s)$) HAS MORE POLES THAN ZEROS $\to$ AS $|s| \to \infty$, $|L_d(s)| \to 0$. WE MUST ALSO ENSURE THAT THE PHASE CONDITION IS SATISFIED AS WELL.

- MORE DETAIL ON CASE $K \to \infty$

- ASSUME THERE ARE n ZEROES AND p POLES. FOR LARGE $|s|$, $L_d \simeq \dfrac{1}{(s-\alpha)^{p-n}}$
  $(p \geq n)$

- COMPLEX ANALYSIS OF EQUATION

$$1 + \frac{K}{(s-\alpha)^{p-n}} = 0$$

ROOT LOCUS AS $K \to \infty$

- TELLS US THAT:

(1) n POLES HEAD TO THE ZEROES OF L

(2) THE REMAINING $p-n$ POLES HEAD TO $|s| = \infty$ ALONG ASYMPTOTES DEFINED BY THE RADIAL LINES

$$\phi_L = \frac{180° + 360°(L-1)}{p-n} \qquad L = 1, 2, \ldots$$

$\Rightarrow$ # ASYMPTOTES GOVERNED BY THE # POLES COMPARED TO # ZEROS (RELATIVE DEGREE).

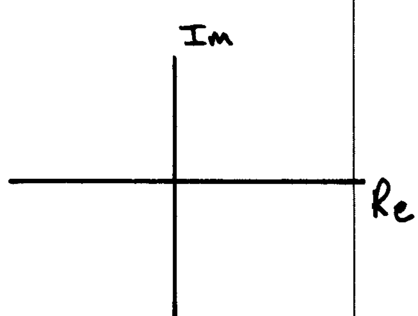
IF $N(z_i) = 0$, $D(p_j) = 0$, THEN THE CENTROID OF THE ASYMPTOTES GIVEN BY

$$\alpha = \frac{\sum p_i - \sum z_i}{p-n}$$

- EXAMPLE: $G(s) = s^{-4}$, $G_c = 1$

$p - n = 4$

$\alpha = 0$



3) S_0 IS ON THE REAL LINE, IS IT PART OF THE LOCUS? (ASSUME $K \geq 0$)

- SPECIAL CASE OF EARLIER QUESTION #1 BECAUSE THE ANSWER IS MORE CONCRETE.

⇒ WRITE $L_d(s) = \dfrac{\prod^{n} (s - z_i)}{\prod^{p} (s - p_j)}$

$$\angle L_d(s_0) = \angle \left[\frac{\prod^{n} (s_0 - z_i)}{\prod^{p} (s_0 - p_j)} \right] = \sum_i^n \angle (s_0 - z_i) - \sum_j^p \angle (s_0 - p_j)$$

"PHASE OF"

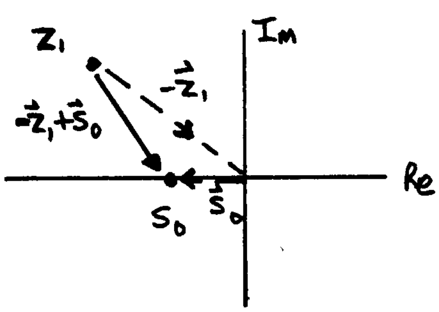
$$= 180° \pm 360° \cdot l \qquad l = 0, 1, \ldots$$

⇒ IF S_0 REAL, THEN COMPLEX CONJUGATE PAIRS OF POLES + ZEROS DO NOT CONTRIBUTE TO THIS CALCULATION. CONSIDER ZERO PAIR z_1, z_2 WITH $z_2 = z_1^*$.

GET A TERM LIKE $\angle (s_0 - z_1) + \angle (s_0 - z_2)$

$$= \angle (s_0 - z_1) + \angle (s_0 - z_1^*)$$

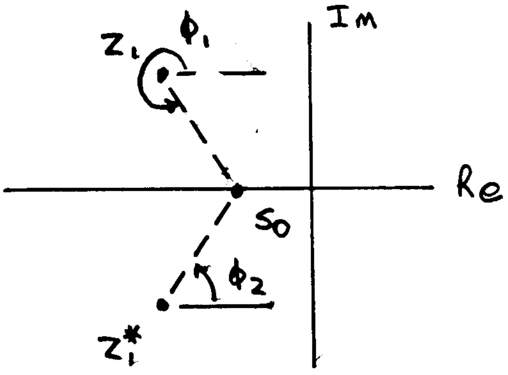
- FIRST, WHAT IS $s_0 - z_1$?



- THINK OF THESE AS VECTORS IN THE S-PLANE.
  - START AT z_1, GO $-\vec{z}_1$ TO ORIGIN, THEN GO $\vec{s}_0$ TO S_0

⇒ $S_0 - z_1$ IS THE VECTOR THAT CONNECTS z_1 TO S_0.

- SECOND, WHAT IS $\angle(s_0 - z_1)$?



- THE PHASE IS MEASURED FROM THE REAL LINE TO THE VECTOR.
- BY GEOMETRY, $\phi_1 + \phi_2 = 360°$

$\Rightarrow$ $\angle(s_0 - z_1) + \angle(s_0 - z_1^*) = \phi_1 + \phi_2 = 360°$

$\Rightarrow$ COMPLEX CONJUGATE ZEROS/POLES ADD MULTIPLES OF 360° TO THE PHASE, BUT THIS AMOUNT OF PHASE IS NOT IMPORTANT IN THE CALCULATION WE ARE DOING!

$\rightarrow$ COMPLEX POLES/ZEROS CAN BE IGNORED.

- THUS ONLY THE <u>REAL</u> POLES AND ZEROS OF $L(s)$ DETERMINE IF A PORTION OF THE REAL LINE IS PART OF THE LOCUS.

  $\rightarrow$ LOCUS POINTS ON THE REAL LINE ARE TO THE <u>LEFT</u> OF AN <u>ODD</u> NUMBER OF REAL-AXIS POLES AND ZEROS. (180° LOCUS)

  WHY?

4) ASSUMING s_0 IS ON THE LOCUS, WHAT GAIN DO WE NEED TO GET THIS CLOSED-LOOP POLE ?

$$K_p = \frac{1}{|L_d(s_0)|}$$

- EXAMPLES ON NEXT PAGE :
  - NOTE SEQUENCES 2 → 3

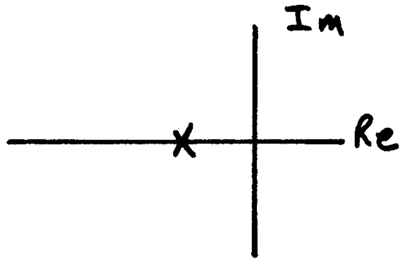
    2 → 4

    2 → 5

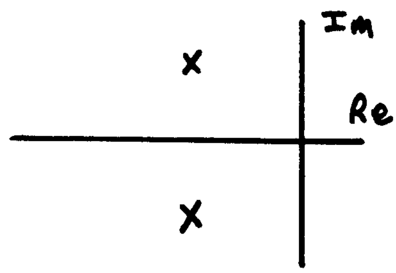
  ARE SIMILAR TO CONTROL DESIGN FOR #2 SINCE WE ARE ADDING "COMPENSATOR" DYNAMICS TO MODIFY THE LOOP , $L_d(s)$

EXAMPLES $K \geq 0$

1

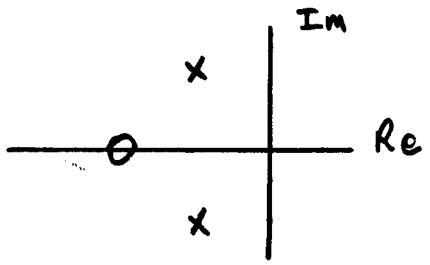


2



ADD ZERO

3



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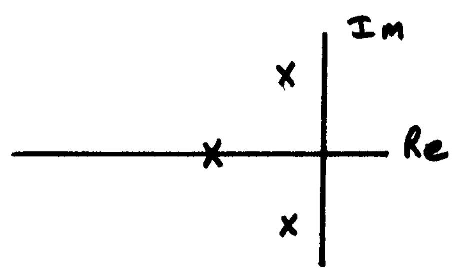
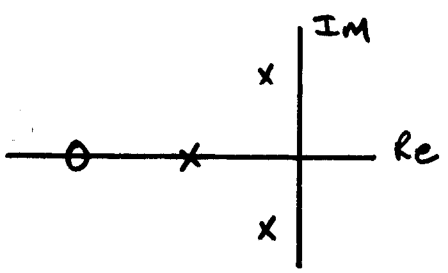


ADD ZERO

5



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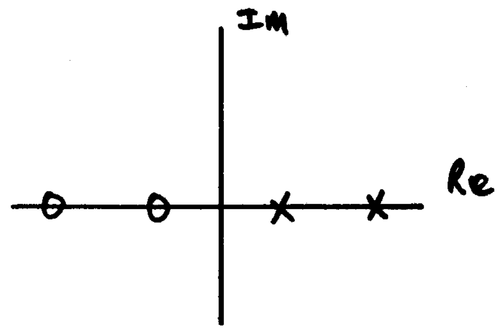
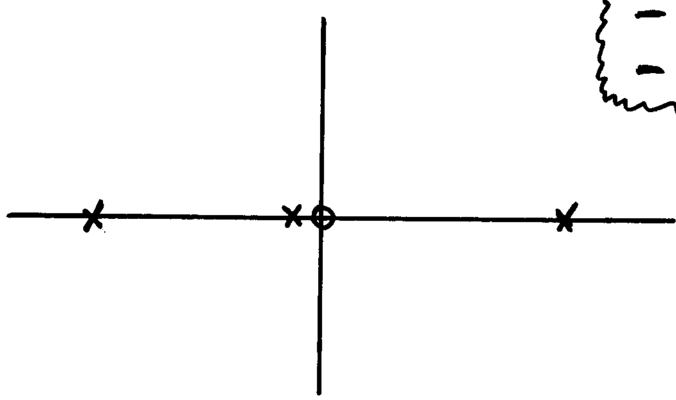




- # ASYMPTOTES
- REAL LINE

KEYS TO A GOOD SKETCH

ROOT LOCUS - ADD PLANT GAIN

- WE ASSUMED ON PAGE 2-13 THAT BOTH THE PLANT AND COMPENSATOR POLYNOMIALS WERE MONIC

- MEANT WE COULD WRITE $L_d(s) = \dfrac{\prod_i (s-z_i)}{\prod_j (s-p_j)}$ GAIN ~ 1

⇒ USED THE COMPENSATOR GAIN K_c TO PLOT THE ROOT LOCUS

⇒ MOST OFTEN FIND THAT THE PLANT NOT MONIC AND MUST ALSO ACCOUNT FOR THE PLANT GAIN ⇒ $L_d(s) = K_p \dfrac{\prod_i (s-z_i)}{\prod_j (s-p_j)}$

PLANT GAIN

⇒ VERY EASY TO ACCOUNT FOR THE MAGNITUDE OF K_p. ⇒ $\tilde{K}_c = K_c \cdot |K_p|$

⇒ MUST ALSO ACCOUNT FOR THE SIGN OF K_p

→ IF $K_p < 0$, THEN MUST MODIFY OUR PREVIOUS STATEMENTS ON 2-13

→ NEED $\angle L_d(s) = -180° \pm 360° L$ STILL !

ASSUME $K_c \geq 0$ (STILL), BUT $K_p < 0$

→ WE NEED $\angle \left[\dfrac{\prod_i (s-z_i)}{\prod_j (s-p_j)} \right] = 0° \pm 360° L$ $\Big\{$ COMPARE WITH 2-13

- OUR ULTIMATE GOAL IS CONTROLLER SYNTHESIS → MUCH MORE THAN FIDDLING WITH THE GAIN KNOB.

$$\phi_c = 1 + G_c G_p = 1 + \frac{K_c N_c}{D_c} \cdot \frac{N_p}{D_p} = 0$$

- WE CAN MAKE $N_c \neq 1$ AND $D_c \neq 1$

  ⇒ DYNAMIC COMPENSATION.

- WE THEN PLOT THE POLES AND ZEROES OF THE PRODUCT $\frac{N_c N_p}{D_c D_p}$ AND PLOT VERSUS K_c.

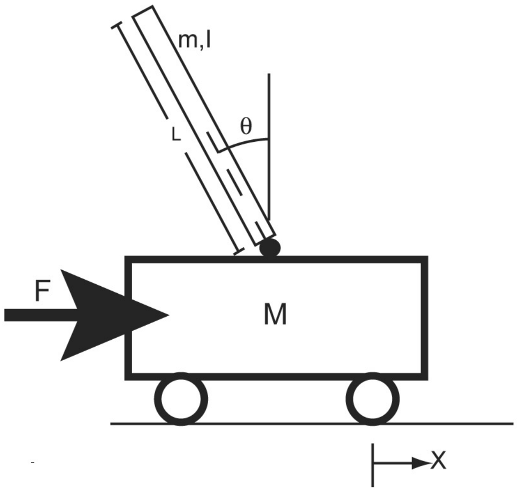
- IF OUR SYSTEM $G_p = \frac{1}{(s+a)^2+b^2}$ (IE #2)

  THEN ADDING A ZERO IN THE COMPENSATOR $N_c = (s+c)$, $D_c = 1$ RESULTS IN A SYSTEM ROOT LOCUS LIKE #3

[PURE ZERO FEASIBLE?]

- SIMPLE PLOTS LIKE THESE LEAD US STRAIGHT INTO CONTROL SYNTHESIS, BUT HOW GET THE POLES TO WHERE WE WANT THEM?

EXAMPLE: CART WITH AN INVERTED PENDULUM.



- NONLINEAR EQUATIONS OF MOTION CAN BE DEVELOPED FOR LARGE ANGLE MOTION (SEE 30-32)

- FORCE ACTUATOR, θ SENSOR

- LINEARIZE FOR SMALL θ

$$(I + mL^2)\ddot{\theta} - mgL\theta = mL\ddot{x}$$

$$(M+m)\ddot{x} + b\dot{x} - mL\ddot{\theta} = F$$

$$\begin{bmatrix} (I+mL^2)s^2 - mgL & -mLs^2 \\ -mLs^2 & (M+m)s^2 + bs \end{bmatrix} \begin{bmatrix} \theta(s) \\ x(s) \end{bmatrix} = \begin{bmatrix} 0 \\ F(s) \end{bmatrix}$$

$$\frac{\theta}{F} = \frac{mLs^2}{\left[(I+mL^2)s^2 - mgL\right]\left[(M+m)s^2 + bs\right] - (mLs^2)^2}$$

- CANNOT SAY TOO MUCH MORE
- LET $M = 0.5$, $m = 0.2$, $b = 0.1$, $I = 0.006$, $L = 0.3$

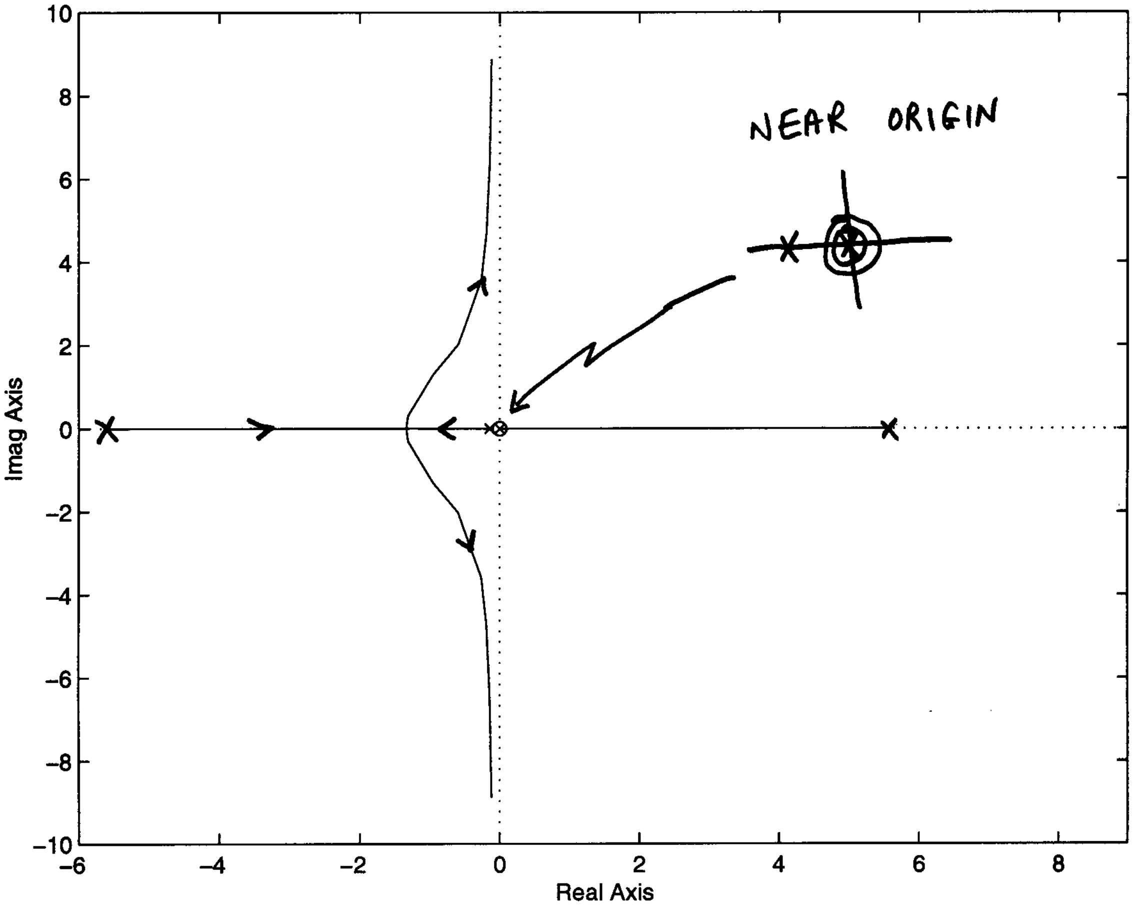
$\Rightarrow$ GIVES $$\frac{\theta}{F} = \frac{4.54\, s^2}{s^4 + 0.1818\, s^3 - 31.18\, s^2 - 4.45\, s}$$

$\therefore$ HAS AN UNSTABLE POLE (AS EXPECTED)

$s = \pm 5.6, -0.14, 0$

E205: Inverted pendulum example

```
num = 4.54*[1 0 0];
den = [1.0000    0.1818  -31.1818   -4.4545        0]
rlocus(num,den)
```



PERFORMANCE ISSUES (SECTION 4.3)

- INTERESTED IN KNOWING HOW WELL OUR CLOSED-LOOP SYSTEM CAN TRACK VARIOUS INPUTS
  - STEPS / RAMPS / PARABOLAS ...
  - BOTH TRANSIENT AND STEADY STATE

- FOR PERFECT STEADY STATE TRACKING, WE WOULD LIKE $\lim_{t \to \infty} e(t) = 0$

  $\Rightarrow$ CAN DETERMINE THIS USING THE CLP TRANSFER FUNCTION AND THE FINAL VALUE THEOREM $\lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,e(s)$

- STEP INPUT $r(t) = 1(t) \rightarrow R(s) = \frac{1}{s}$

$$\frac{y}{r} = \frac{G_c G_p}{1 + G_c G_p}, \qquad \frac{y}{e} = G_c G_p$$

$$\Rightarrow \frac{e}{r} = \frac{1}{1 + G_c G_p}$$

$$\therefore \quad e = \frac{r(s)}{1 + G_c G_p} = \frac{1/s}{1 + G_c G_p}$$

$$\Rightarrow \lim_{s \to 0} s\,e(s) = \lim_{s \to 0} \frac{1}{1 + G_c G_p} = \frac{1}{1 + G_c(0) G_p(0)} = e(\infty)$$

- <u>BOTTOM LINE</u>: STEADY STATE ERROR TO A STEP GIVEN BY

$$e_{ss} = \frac{1}{1 + G_c(0)\,G_p(0)}$$

$\Rightarrow$ TO MAKE e_{ss} SMALL, WE NEED TO MAKE (BOTH OR) ONE OF $G_c(0)$, $G_p(0)$ VERY LARGE.

- CLEARLY, IF $G_p(s)$ HAS A "FREE INTEGRATOR" SO THAT IT LOOKS LIKE $G_p(s) = \dfrac{1}{s(s+\alpha)}$

THEN $G_p(0) \to \infty$

$\Rightarrow e_{ss} \to 0$

- CAN CONTINUE THE DISCUSSION BY LOOKING AT VARIOUS INPUT TYPES (STEP, RAMP, PARABOLA) WITH SYSTEMS THAT HAVE DIFFERENT #'S OF FREE INTEGRATORS (<u>TYPE</u>)

	STEP	RAMP	PARABOLA
TYPE 0	$\frac{1}{1+K_p}$	∞	∞
TYPE 1	0	$\frac{1}{K_v}$	∞
TYPE 2	0	0	$\frac{1}{K_a}$

- DEFINITIONS:

$$K_p = \lim_{s \to 0} G_c(s) G_p(s)$$ POSITION ERROR CONSTANT

$$K_v = \lim_{s \to 0} s G_c(s) G_p(s)$$ VELOCITY ERROR CONSTANT

$$K_a = \lim_{s \to 0} s^2 G_c(s) G_p(s)$$ ACCELERATION ERROR CONSTANT

- A GOOD WAY TO KEEP TRACK OF HOW WELL YOUR SYSTEM IS DOING IN TERMS OF STEADY STATE PERFORMANCE.

DYNAMIC COMPENSATION

- LAST TIME WE STUDIED HOW TO DRAW A ROOT LOCUS FOR THE GIVEN PLANT DYNAMICS.

  ⇒ WHAT IF OUR DESIRED POLE LOCATIONS ARE NOT ON THIS LOCUS?

- WE NEED TO MODIFY THE LOCUS ITSELF BY ADDING EXTRA DYNAMICS IN N_c, D_c

  ⇒ DYNAMIC COMPENSATION.

  ⇒ THEN REDRAW THE LOCUS + FIND THE GAIN TO PUT THE CLOSED LOOP POLES WHERE WE WANT THEM.

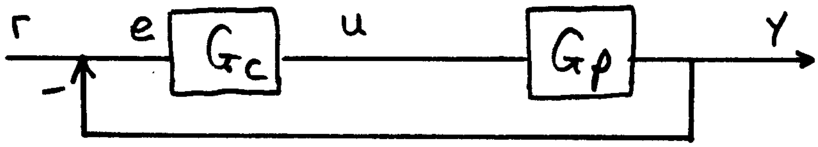
- NEW QUESTIONS:

  - WHAT TYPE OF COMPENSATION SHOULD WE USE?

  - HOW DO WE FIGURE OUT WHERE TO PUT THE ADDITIONAL DYNAMICS?

TYPES OF CONTROL DYNAMICS

- THERE ARE 3 CLASSIC TYPES OF CONTROLLERS :



CONTROLLER : $\quad u = G_c(s)\, e$

$\Rightarrow$ WHAT IS G_c ?

1) PROPORTIONAL FEEDBACK : $\quad G_c(s) = K$ (A CONSTANT)

I.E. $N_c = D_c = 1$

$u = K e$

$\Rightarrow$ SAME CASE WE JUST LOOKED AT. CONTROLLER ONLY CONSISTS OF A "GAIN KNOB", WE HAVE TO TAKE THE LOCUS "AS GIVEN" SINCE WE HAVE NO EXTRA DYNAMICS TO MODIFY IT.

$\Rightarrow$ USUALLY VERY LIMITED APPROACH, BUT A GOOD PLACE TO START.

2) INTEGRAL FEEDBACK

$$u(t) = \left[\int_0^t e(\tau)\, d\tau\right] K_I$$

$$\Rightarrow \quad G_c(s) = \frac{K_I}{s}$$

- USED TO REDUCE / ELIMINATE STEADY-STATE ERRORS
  - IF $e(\tau) \simeq$ CONSTANT, $u(t)$ WILL BECOME VERY LARGE + HOPEFULLY CORRECT THE ERROR

• EXAMPLE: $G_p = \dfrac{1}{(s+a)(s+b)}$ $\quad a > b > 0$

- WITH PROPORTIONAL FEEDBACK $e_{ss} = \dfrac{1}{1 + \frac{K}{ab}}$

  $\Rightarrow$ CAN MAKE e_{ss} SMALL, BUT NEED K LARGE.

- WITH INTEGRAL CONTROL $e_{ss} = 0$

  SINCE $G_c(s)\big|_{s\to 0} = \infty$

• INTEGRAL FEEDBACK IMPROVES THE STEADY STATE RESPONSE, BUT THIS IS OFTEN AT THE EXPENSE OF THE TRANSIENT RESPONSE (THIS GETS WORSE $\rightarrow$ NOT AS WELL DAMPED)

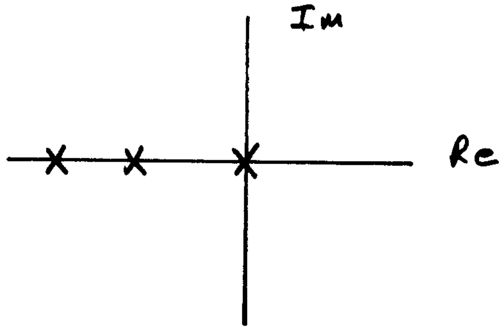
WITH INTEGRAL FEEDBACK

$$G_c = \frac{K_I}{s}$$

$$G_p(0) = \frac{1}{ab}$$

$\Rightarrow e_{ss} = 0$

ROOT LOCUS

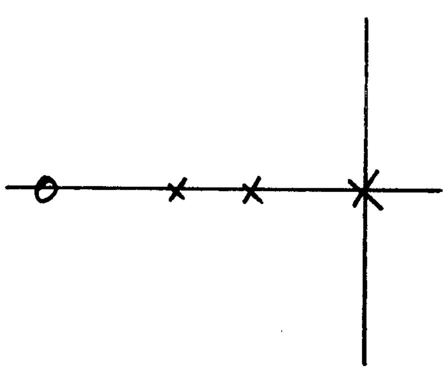


- INCREASING K_I TO INCREASE THE SPEED OF THE RESPONSE PUSHES THE POLES TOWARDS THE IMAGINARY AXIS → OSCILLATORY.

PROPORTIONAL - INTEGRAL

NOW $G_c = K_1 + \dfrac{K_2}{s} = \dfrac{K_1 s + K_2}{s}$

∴ BOTH A POLE AND A ZERO.



COMBINATION OF PROPORTIONAL - INTEGRAL (PI) SOLVES MANY OF THE PROBLEMS WITH JUST INTEGRAL (I).

- \# ASYMPTOTES ?
- CENTROID

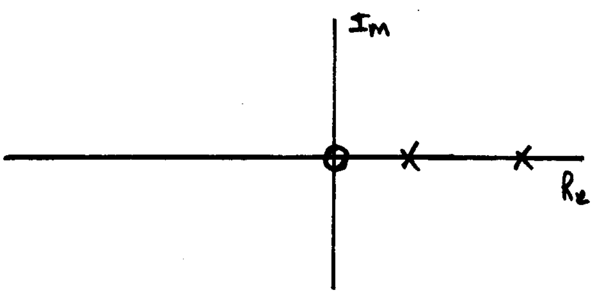
3) <u>DERIVATIVE FEEDBACK</u> $u = K_D \dot{e}$ (RATE)

$$\Rightarrow \quad G_c(s) = s \cdot K_D$$

- DOES NOT DO MUCH (ANYTHING?) TO HELP THE STEADY STATE ERROR
- DERIVATIVE CONTROL PROVIDES FEEDBACK THAT IS PROPORTIONAL TO THE RATE OF CHANGE OF $e(t)$ $\Rightarrow$ CONTROL RESPONSE <u>ANTICIPATES</u> FUTURE ERRORS.
  - VERY BENEFICIAL
  - USED A LOT!

<u>EXAMPLE:</u> $G(s) = \dfrac{1}{(s-a)(s-b)}$ $G_c(s) = s \cdot K_D$

$a > b > 0$



DERIVATIVE FEEDBACK IS VERY USEFUL FOR DRAWING THE ROOT LOCUS INTO THE LHP

$\Rightarrow$ INCREASED DAMPING

$\Rightarrow$ MORE STABLE RESPONSE.

ALSO TYPICALLY USE COMBINATION OF <u>PROPORTIONAL + DERIVATIVE</u> $\Rightarrow$ $G_c(s) = K_1 + K_2 s$

(PD)

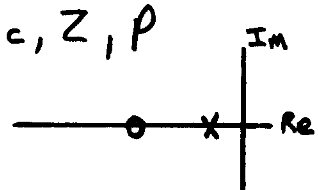
SYNTHESIS

- FIRST LOOK AT WHERE WE WANT THE DOMINANT POLES TO BE LOCATED.
- WILL PROPORTIONAL FEEDBACK DO THE JOB?
- WHAT KIND OF DYNAMICS SHOULD WE ADD?

⇒ TYPICALLY USE THE FOLLOWING BUILDING BLOCK

$$G_B = \frac{K_c\,(s+z)}{(s+p)} \qquad \begin{array}{l} z>0 \\ p>0 \end{array}$$

→ CAN LOOK LIKE MANY TYPES OF COMPENSATORS DEPENDING ON HOW WE PICK K_c, z, p



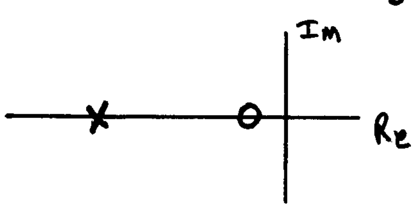
(A) PICK $z > p$, p SMALL

LAG THEN G_B IS SIMILAR TO $\dfrac{K\,(s+\alpha)}{s}$

(B) PICK $p \gg z$. AT LOW FREQUENCY

LEAD G_B IS SIMILAR TO $K\,(s+z)$

SINCE IMPACT OF $\dfrac{p}{s+p}$ IS SMALL.



- TO DETERMINE HOW TO PICK K, P, Z, WE MUST USE THE PHASE CONDITION OF THE ROOT LOCUS
  + MAGNITUDE!

– CONSIDER $G_p(s) = \frac{1}{s^2}$

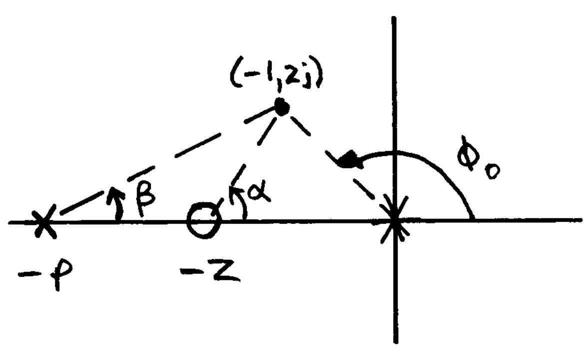
⇒ WE WANT THE CLP POLES AT $-1 \pm j2$

- WILL PROPORTIONAL DO?
- SO WHAT DYNAMICS DO WE NEED TO ADD?

$$G_c(s) = \frac{K_c (s+z)}{(s+p)}$$

⇒ LOOP $L_d(s) = \dfrac{s+z}{(s+p)\, s^2}$

- EVALUATE PHASE OF $L_d(s)$ AT $s_0 = -1 \pm j2$. SINCE WE WANT s_0 TO BE ON THE NEW LOCUS: $\angle L_d(s_0) = 180° \pm 360° l$



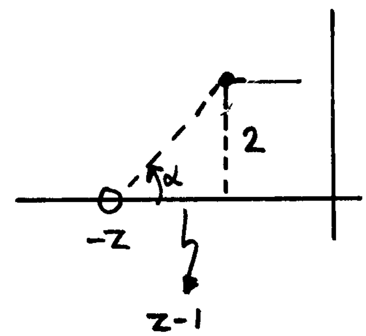
FOUR TERMS IN $\angle L_d(s_0)$

- POLES AT ORIGIN BOTH CONTRIBUTE 117°

⇒ CONTRIBUTION OF POLE/ZERO CLEAR FROM GEOMETRY.

- GEOMETRY FOR THE ZERO

$$\tan \alpha = \frac{2}{z-1}$$



$$\phi_z = \alpha$$

- FOR THE POLE $\quad \tan \beta = \dfrac{2}{p-1}$

$$\phi_p = \beta$$

- WE KNOW THAT A ZERO IS REQUIRED NEAR THE ORIGIN TO PULL THE 2 POLES AT $s=0$ INTO THE LHP.
  - $\Rightarrow$ PUT ZERO FIRST
  - $\Rightarrow$ DESIGN RULE, SET $p = 10z$

- <u>PHASE CONDITION</u> $\quad -2(117^\circ) + \alpha - \beta = 180^\circ$

$$\phi_z - \sum \phi_{POLES} = 180^\circ$$

$$\Rightarrow -234^\circ + \alpha - \beta = 180^\circ$$

$$\tan^{-1}\left(\frac{2}{z-1}\right) - \tan^{-1}\left(\frac{2}{10z-1}\right) = 54^\circ$$

RECALL: $\tan(A-B) = \dfrac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$

$$\Rightarrow \frac{\frac{2}{z-1} - \frac{2}{10z-1}}{1 + \frac{2}{z-1} \cdot \frac{2}{10z-1}} = 1.38 \quad \Rightarrow \left.\begin{matrix} z = 2.23 \\ p = 22.3 \end{matrix}\right\} \text{FIND } K$$

- <u>ALTERNATIVE APPROACH</u>

- IF ONE SET OF POLES IS AT $-1 \pm 2j$ AND WE KNOW THERE ARE 3 IN TOTAL, THEN THE CHARACTERISTIC EQUATION MUST LOOK LIKE
$$(s^2+2s+5)\cdot(s+\alpha)=0$$

- WITH $G_c(s) = \dfrac{K(s+z)}{s+p}$ $\qquad p = 10z$

THEN $\phi_c(s) = 1 + G_p(s)G_c(s) = 0$

$$\Rightarrow \quad s^2(s+10z) + K(s+z) = 0$$
$$\Rightarrow \quad s^3 + s^2\,10z + s(K) + zK = 0$$

- NOW COMPARE THE CHARACTERISTIC EQUATION WE GET FROM THE CONTROLLER WITH THE ONE WE EXPECT TO SEE:

i) $s^3 + s^2 10z + sK + zK = 0$

ii) $s^3 + s^2(\alpha+2) + s(2\alpha+5) + 5\alpha = 0$

$$\Rightarrow \left.\begin{aligned} \alpha+2 &= 10z \\ K &= 2\alpha+5 \\ zK &= 5\alpha \end{aligned}\right\}$$

SOLVE FOR α, z, K

$K = \dfrac{25}{5-2z}$, $\alpha = \dfrac{5z}{5-2z}$

$\Rightarrow$ $z = 2.23$
$\alpha = 20.65$
$K = 46.3$

Example: G(s)=1/2^2
Design Gc(s) to put the clp poles at –1 + 2j

```
z=roots([-20 49 -10]);z=max(z),k=25/(5-2*z),alpha=5*z/(5-2*z),
num=1;den=[1 0 0];
knum=k*[1 z];kden=[1 10*z];
rlocus(conv(num,knum),conv(den,kden));
hold;plot(-alpha+eps*j,'d');plot([-1+2*j,-1-2*j],'d');hold off
r=rlocus(conv(num,knum),conv(den,kden),1)'
```

```
z =      2.2253
k =     45.5062
alpha = 20.2531
```

These are the actual roots that I found from the locus using a gain of 1 (recall that the K gain is already in the compensator)

```
r =
 -20.2531
  -1.0000 - 2.0000i
  -1.0000 + 2.0000i
```