Topic #3

16.31 Feedback Control

Frequency response methods

- Analysis
- Synthesis
- Performance
- Stability

Introduction

- Root locus methods have:
  - Advantages:
    * Good indicator if transient response;
    * Explicity shows location of all closed-loop poles;
    * Trade-offs in the design are fairly clear.
  - Disadvantages:
    * Requires a transfer function model (poles and zeros);
    * Difficult to infer all performance metrics;
    * Hard to determine response to steady-state (sinusoids)

- Frequency response methods are a good complement to the root locus techniques:
  - Can infer performance and stability from the same plot
  - Can use measured data rather than a transfer function model
  - The design process can be independent of the system order
  - Time delays are handled correctly
  - Graphical techniques (analysis and synthesis) are quite simple.

Frequency response Function

- Given a system with a transfer function $G(s)$, we call the $G(\mathrm{j}\omega)$, $\omega \in [0, \infty)$ the *frequency response function* (FRF)

$$G(\mathrm{j}\omega) = |G(\mathrm{j}\omega)| \arg G(\mathrm{j}\omega)$$

  – The FRF can be used to find the **steady-state** response of a system to a sinusoidal input. If

$$e(t) \rightarrow \boxed{\mathrm{G(s)}} \rightarrow y(t)$$

  and $e(t) = \sin 2t$, $|G(2\mathrm{j})| = 0.3$, $\arg G(2\mathrm{j}) = 80^\circ$, then the steady-state output is

$$y(t) = 0.3 \sin(2t - 80^\circ)$$

  $\Rightarrow$ The FRF clearly shows the magnitude (and phase) of the response of a system to sinusoidal input

- A variety of ways to display this:
  1. Polar (**Nyquist**) plot – Re vs. Im of $G(\mathrm{j}\omega)$ in complex plane.
     – Hard to visualize, not useful for synthesis, but gives definitive tests for stability and is the basis of the robustness analysis.
  2. Nichols Plot – $|G(\mathrm{j}\omega)|$ vs. $\arg G(\mathrm{j}\omega)$, which is very handy for systems with lightly damped poles.
  3. **Bode** Plot – Log $|G(\mathrm{j}\omega)|$ and $\arg G(\mathrm{j}\omega)$ vs. Log frequency.
     – Simplest tool for visualization and synthesis
     – Typically plot $20\log|G|$ which is given the symbol dB

- Use logarithmic since if

$$\begin{aligned} \log|G(s)| &= \left|\frac{(s+1)(s+2)}{(s+3)(s+4)}\right| \\ &= \log|s+1| + \log|s+2| - \log|s+3| - \log|s+4| \end{aligned}$$

  and each of these factors can be calculated separately and then added to get the total FRF.

- Can also split the phase plot since

$$\begin{aligned} \arg\frac{(s+1)(s+2)}{(s+3)(s+4)} = &\quad \arg(s+1) + \arg(s+2) \\ &- \arg(s+3) - \arg(s+4) \end{aligned}$$

- The keypoint in the sketching of the plots is that good straightline approximations exist and can be used to obtain a good prediction of the system response.

Example

- Draw Bode for

$$G(s) = \frac{s+1}{s/10+1}$$

$$|G(j\omega)| = \frac{|j\omega+1|}{|j\omega/10+1|}$$

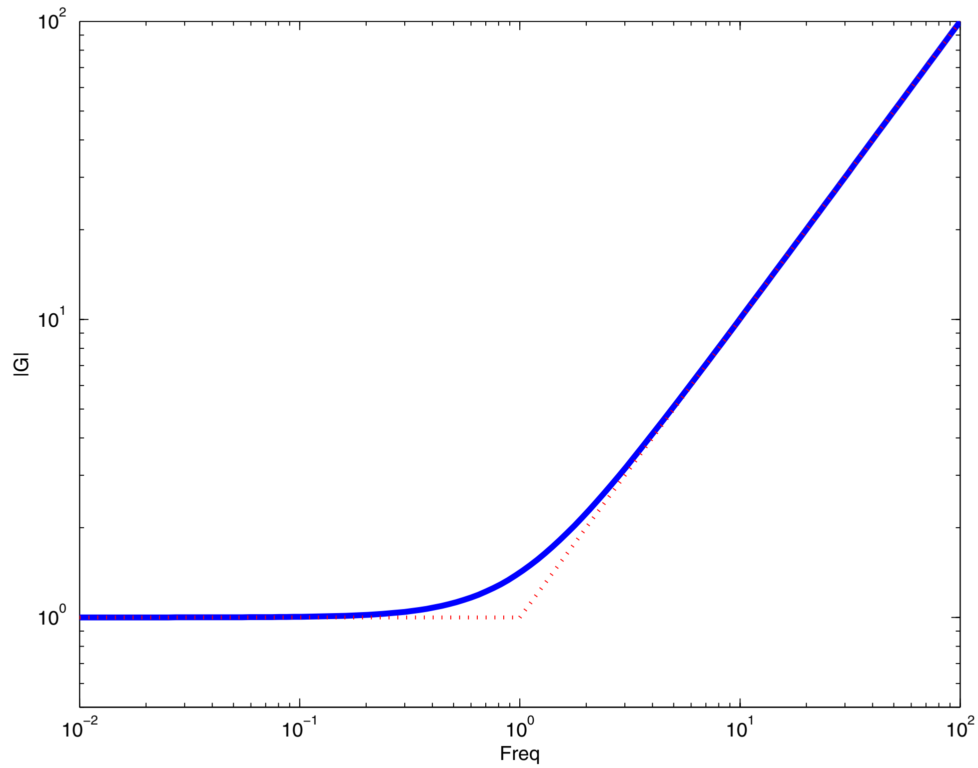
$$\log|G(j\omega)| = \log[1+(\omega/1)^2]^{1/2} - \log[1+(\omega/10)^2]^{1/2}$$

- Approximation

$$\log[1+(\omega/\omega_i)^2]^{1/2} \approx \begin{cases} 0 & \omega \ll \omega_i \\ \log[\omega/\omega_i] & \omega \gg \omega_i \end{cases}$$

Two straightline approximations that intersect at $\omega \equiv \omega_i$

- Error at ω_i obvious, but not huge and the straightline approximations are very easy to work with.



To form the composite sketch,

– Arrange representation of transfer function so that DC gain of each element is unity (except for parts that have poles or zeros at the origin) – absorb the gain into the overall plant gain.

– Draw all component sketches

– Start at low frequency (DC) with the component that has the lowest frequency pole or zero (i.e. s=0)

– Use this component to draw the sketch up to the frequency of the next pole/zero.

– Change the slope of the sketch at this point to account for the new dynamics: -1 for pole, +1 for zero, -2 for double poles, . . .

– Scale by overall DC gain

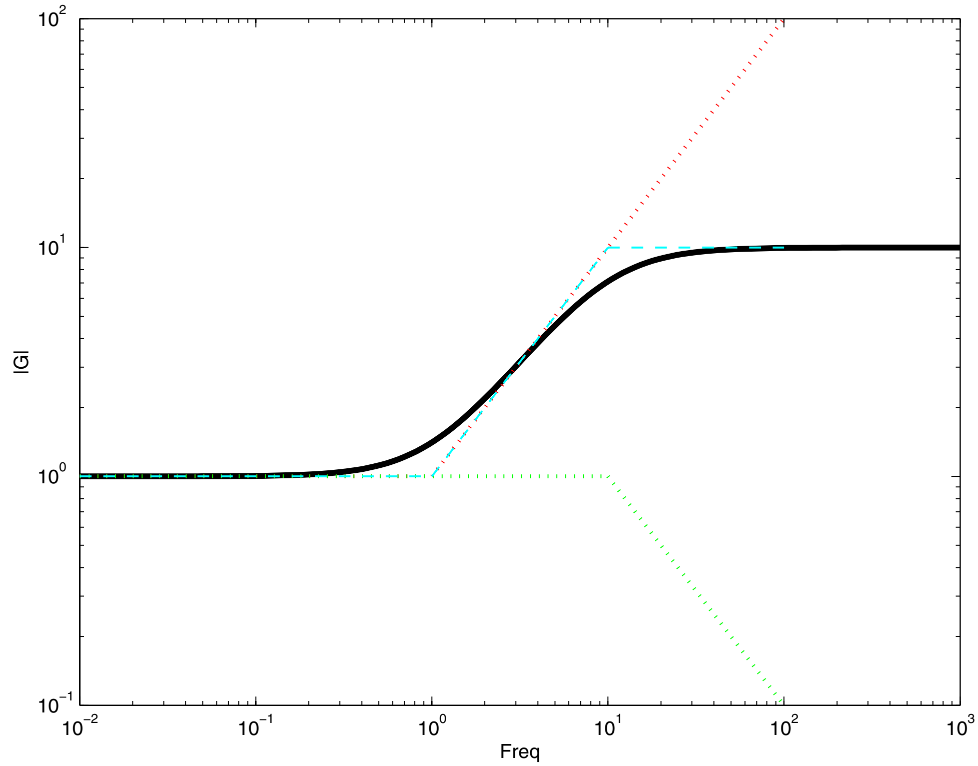


Figure 1: $G(s) = 10(s + 1)/(s + 10)$ which is a "lead".

- Since $\arg G(j\omega) = \arg(1 + j\omega) - \arg(1 + j\omega/10)$, we can construct phase plot for complete system in a similar fashion
  - Know that $\arg(1 + j\omega/\omega_i) = \tan^{-1}(\omega/\omega_i)$
- Can use straightline approximations

$$\arg(1 + j\omega/\omega_i) \approx \begin{cases} 0 & \omega/\omega_i \leq 0.1 \\ 90^\circ & \omega/\omega_i \geq 10 \\ 45^\circ & \omega/\omega_i = 1 \end{cases}$$

- Draw the components using breakpoints that are at $\omega_i/10$ and $10\omega_i$

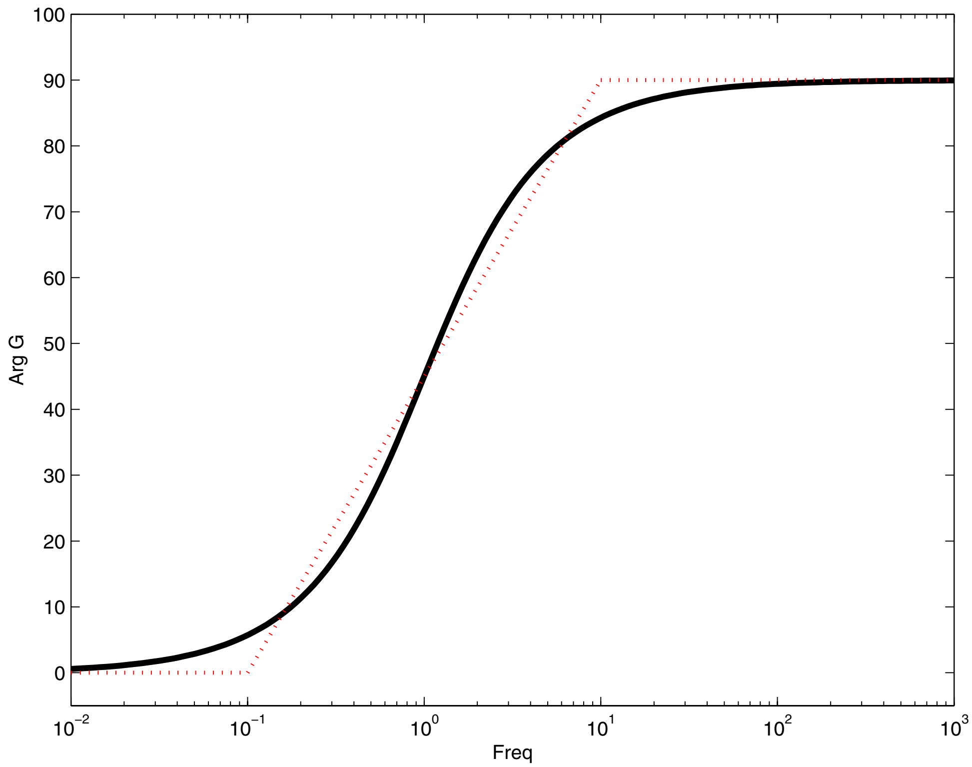


Figure 2: Phase plot for $(s + 1)$

- Then add them up starting from zero frequency and changing the slope as $\omega \to \infty$

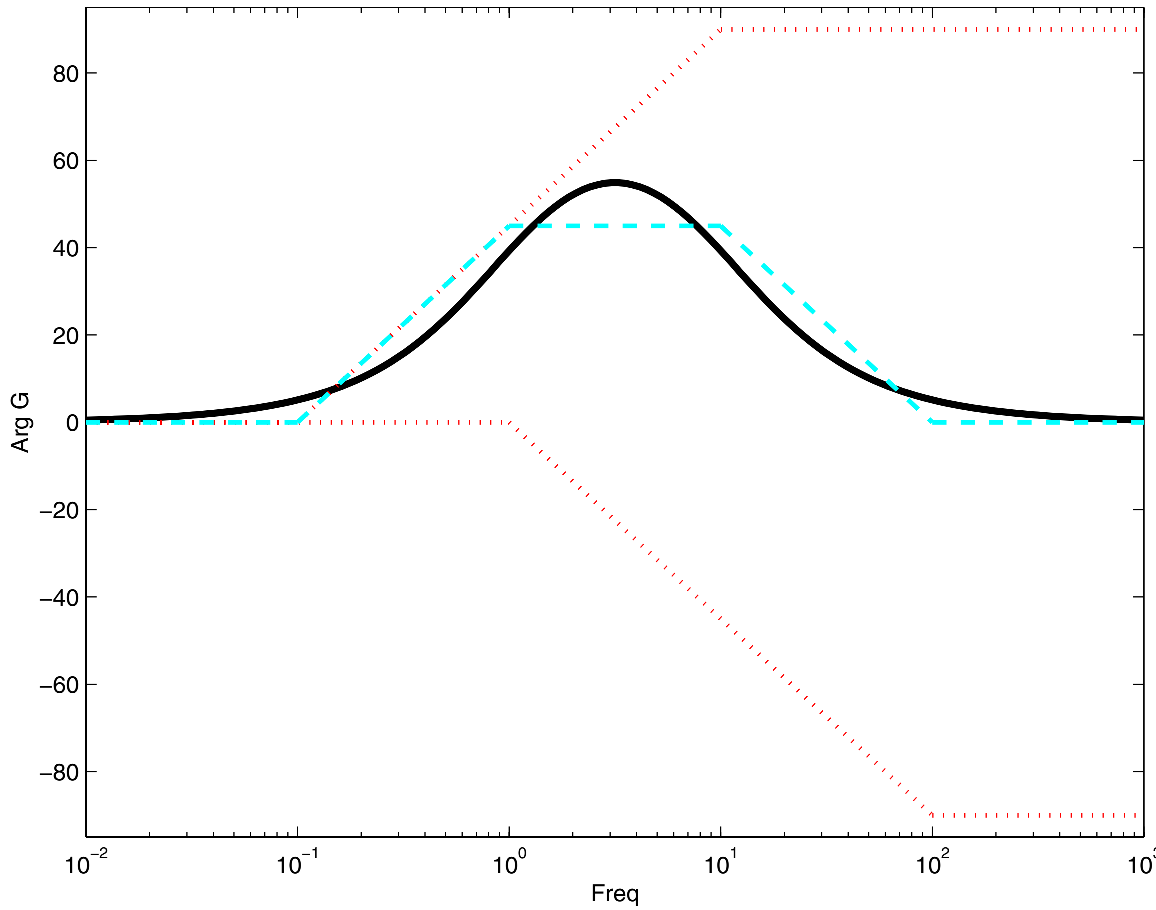
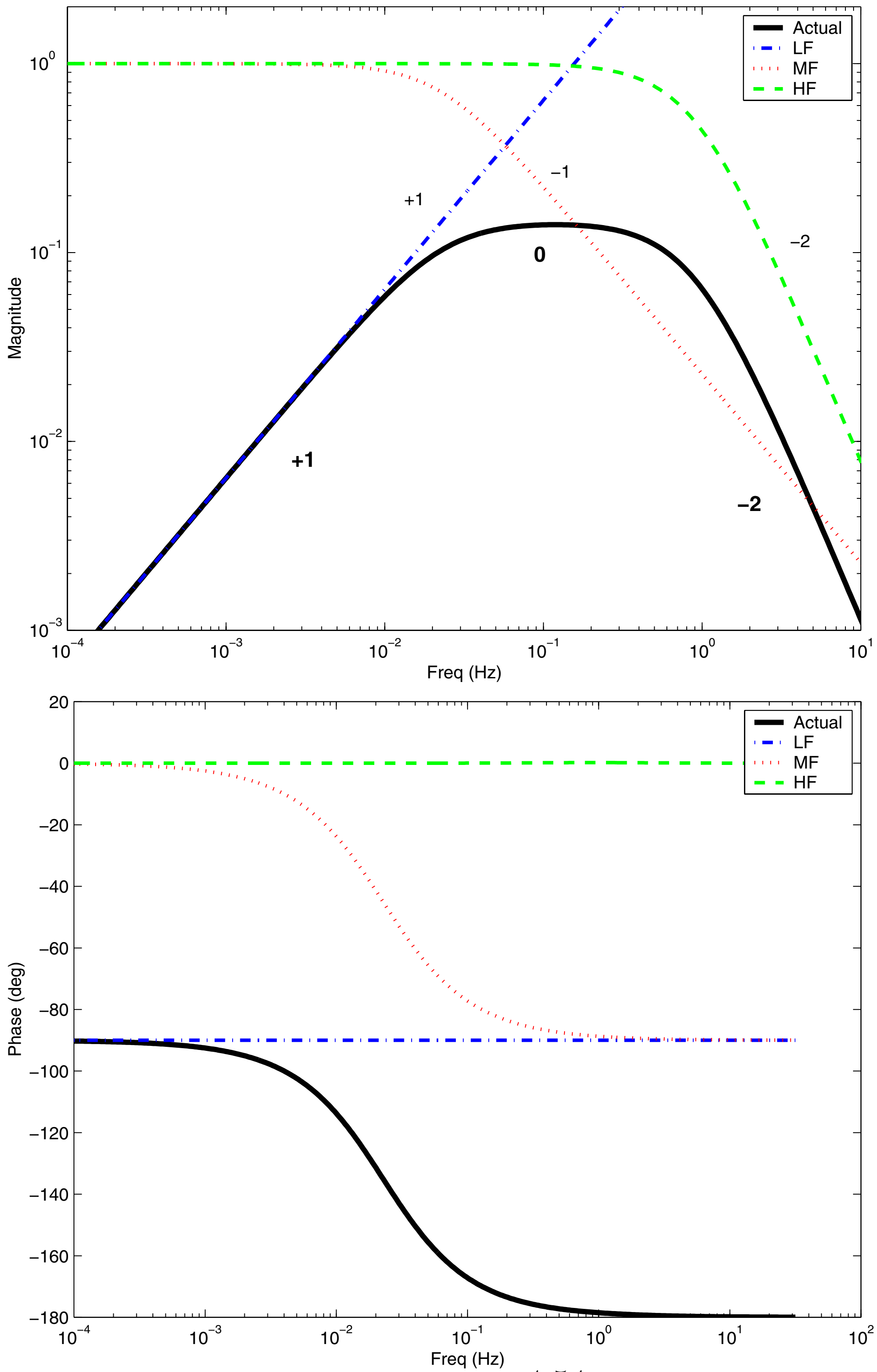


Figure 3: Phase plot $G(s) = 10(s + 1)/(s + 10)$ which is a "lead".



Bode for $G(s) = \dfrac{4.54s}{s^3 + 0.1818s^2 - 31.1818s - 4.4545}$.
The poles are at (-0.892, 0.886, -0.0227)

Non-minimum Phase Systems

- Bode plots are particularly complicated when we have non-minimum phase systems
  - A system that has a pole/zero in the RHP is called non-minimum phase.
  - The reason is clearer once you have studied the Bode *Gain-Phase relationship*
  - **Key point:** We can construct two (and many more) systems that have identical magnitude plots, but very different phase diagrams.
- Consider $G_1(s) = \frac{s+1}{s+2}$ and $G_2(s) = \frac{s-1}{s+2}$

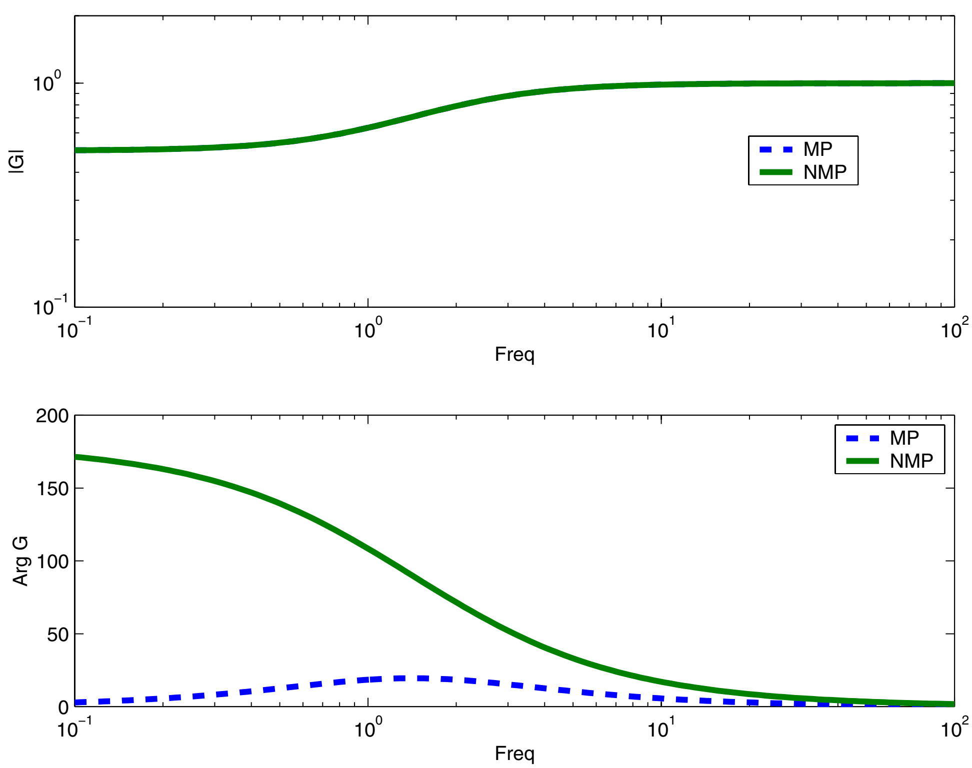


Figure 4: Magnitude plots are identical, but the phase plots are dramatically different. NMP has a 180 deg phase loss over this frequency range.