Fall 2001

Topic #6

16.31 Feedback Control

Control Design using Bode Plots

- Performance Issues
- Synthesis
- Lead/Lag examples

BODE'S GAIN-PHASE RELATIONSHIP

- SYNTHESIS WOULD BE HARD IF WE HAD TO DEAL WITH BOTH MAGNITUDE AND PHASE PLOTS, BUT ...

- **THEOREM**: FOR ANY STABLE, MINIMUM PHASE SYSTEM, $\angle G(j\omega)$ IS UNIQUELY RELATED TO $|G(j\omega)|$.

- **RELATIONSHIP**: ON LOG-LOG PLOT, IF SLOPE OF MAGNITUDE CURVE IS CONSTANT OVER A DECADE OF FREQUENCY, WITH SLOPE n, THEN

$$\angle G(j\omega) \simeq 90° \cdot n$$

VERY IMPORTANT

- SO, IN THE CROSS-OVER FREQUENCY RANGE ($|G_c G_p| \simeq 1$), IF THE SLOPE IS:

s^0	NO CROSS-OVER	
s^{-1}	$\simeq 90°$	
s^{-2}	$\simeq 180°$	TOO MUCH PHASE AS $PM \simeq 0$

$\Rightarrow$ SELECT $G_c(s)$ SO THAT LTF CROSSES OVER WITH A SLOPE OF -1

OTHER PERFORMANCE ISSUES

- STEP RESPONSE ERROR $e_{ss} = \dfrac{1}{1 + G_c(0)\,G_p(0)}$

GOOD NEWS: WE CAN FIND $G_cG_p(0)$ DIRECTLY FROM THE LOW FREQ BODE PLOT (TYPE 0)

- FOR TYPE I SYSTEMS, THE LOW FREQ ASYMPTOTE IS INFINITE (SLOPE −1)
  - ⇒ STEP ERROR OF ZERO
  - ⇒ VELOCITY ERROR (RAMP INPUT) $e_{ss} = \dfrac{1}{K_v}$

⇒ K_v IS THE GAIN OF THE CURVE AT LOW FREQ

$$K_v = \lim_{s \to 0} s\, G_c(s) G_p(s).$$

- FOR TYPE I SYSTEMS $G_cG_p(s) \simeq \dfrac{K_v}{s}$ AT LOW FREQUENCY
  - ⇒ CAN FIND K_v BY EXTENDING THE LOW FREQUENCY ASYMPTOTE (IF NECESSARY) TO $\omega = 1$ AND FINDING $|G_cG_p|\big|_{\omega=1}$

$$K_v \simeq \omega \cdot |G_cG_p|$$

PERFORMANCE

- **HOW MUCH PHASE MARGIN?**

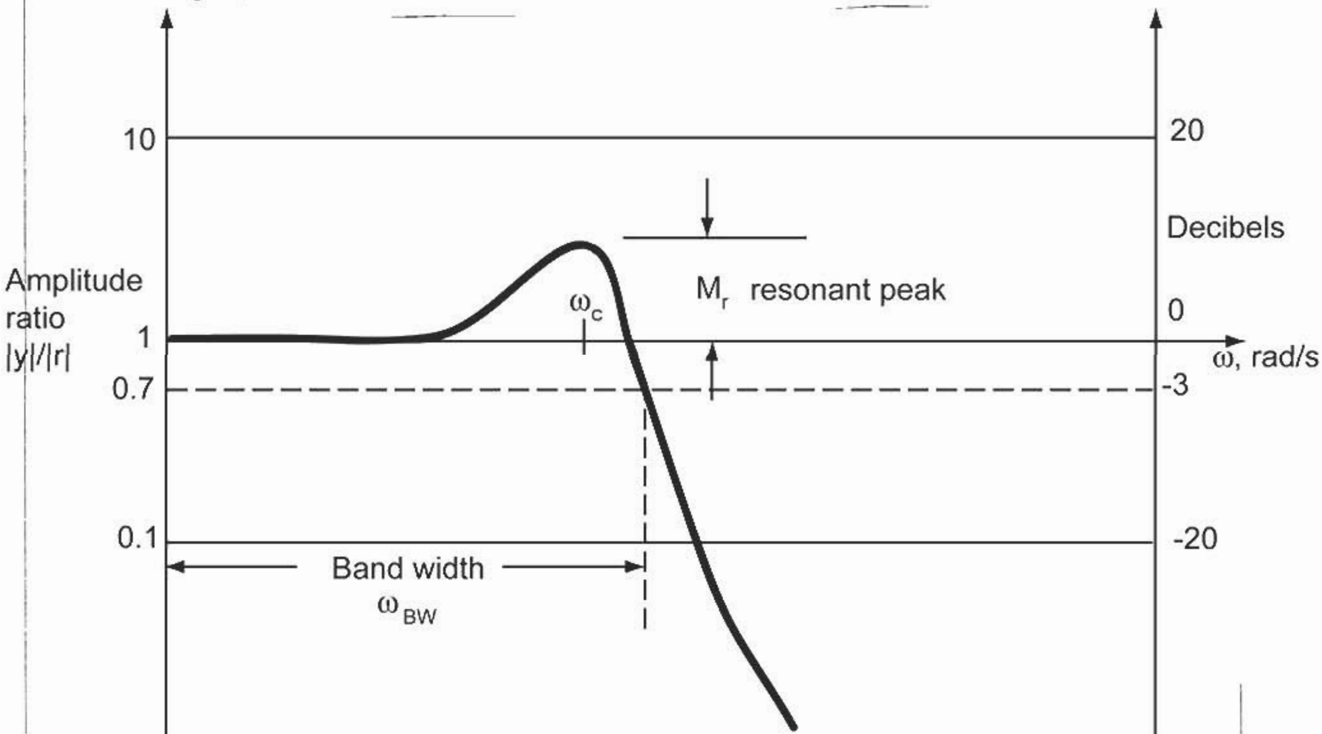
TIME RESPONSE OF 2^{nd} ORDER SYSTEM GIVES:

1) DAMPING RATIO OF CLP POLES ξ $\qquad \xi \simeq \frac{PM}{100}$, $PM < 70^\circ$

2) CLP RESONANT PEAK $\qquad M_r \simeq \frac{1}{2\sin\left(\frac{PM}{2}\right)}$

3) $\omega_{BW} \simeq 1.4\,\omega_c$

CLP BANDWIDTH $\approx$ 1.4 X CROSS OVER FREQUENCY. (ω_c)



- PEAK OF TF USUALLY CLOSE TO ω_c

$\Rightarrow$ SPECIFY (ULTIMATELY) ω_c AND PM

FREQUENCY RESPONSE DESIGN

- LOOKED AT BUILDING BLOCK $G_C(s) = \dfrac{K_c(s+z)}{(s+p)}$

  ⇒ ROOT LOCUS APPLICATIONS

- FREQUENCY RESPONSE CHARACTERISTICS?

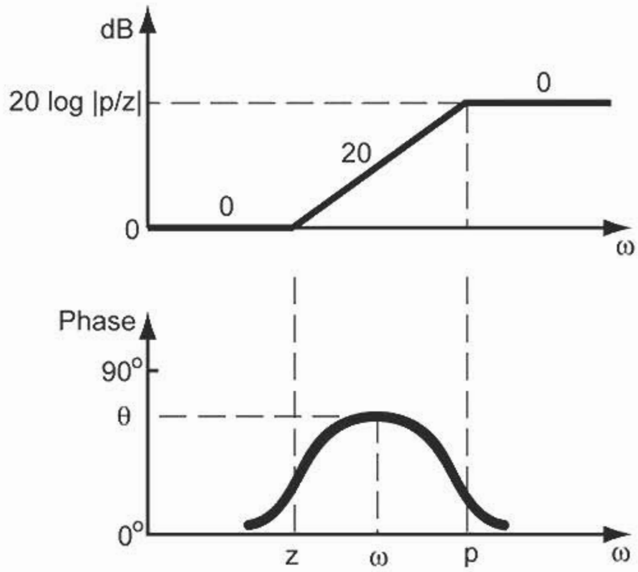
  ⇒ HOW USE $G_c(s)$ TO MODIFY THE LOOP TRANSFER FUNCTION (LTF) $G_c(s)G_p(s)$ TO GET DESIRED
  - BANDWIDTH
  - PHASE MARGIN.
  - ERROR CONSTANTS.

① **LEAD** $|z| < |p|$

- ZERO AT LOWER FREQUENCY THAN POLE

  ⇒ GAIN INCREASES WITH FREQUENCY

  ⇒ PHASE POSITIVE (I.E. ADDS PHASE LEAD)



PLOTTED WITH $K_c = \dfrac{p}{z}$

LEAD MECHANICS

$$\alpha = \frac{1 - \sin\phi_{MAX}}{1 + \sin\phi_{MAX}}$$

- MAXIMUM PHASE ADDED

$$\sin\phi_{MAX} = \frac{1-\alpha}{1+\alpha} \qquad \alpha = \frac{|z|}{|p|}$$

- FREQUENCY OF MAXIMUM PHASE ADDITION

$$\omega_{MAX} = \sqrt{|z|\cdot|p|}$$

- HIGH FREQUENCY GAIN INCREASE $\frac{1}{\alpha}$

$\Rightarrow$ COMPROMISE BETWEEN DESIRE FOR LARGE PHASE MARGIN (α SMALL) AND TENDANCY TO GENERATE LARGE GAINS AT HIGH FREQ.

$\Rightarrow$ KEEP $\frac{1}{\alpha} \leq 10$ $\quad (|p| \leq 10|z|)$

$\Rightarrow \phi_{MAX} \leq 60°$

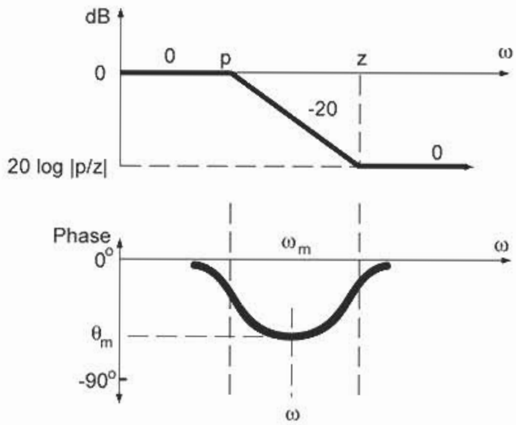
- USE MULTIPLE LEAD FILTERS TO GET MORE PHASE. $\quad G_c = K_c\left(\frac{s+z_1}{s+p_1}\right)\left(\frac{s+z_2}{s+p_2}\right)$
- SELECTION OF K_c IS PROBLEM SPECIFIC
  - COULD ARRANGE $|G_c(s)|_{s\to 0} = K_0$
  - OR SELECT K_c TO FORCE $\omega_c \equiv (\omega_c)_{DESIRED}$.
- USE LEAD TO ADD PHASE $\rightarrow$ INCREASE PHASE MARGIN $\rightarrow$ IMPROVE TRANSIENT RESPONSE.

② LAG $|p| < |z|$

POLE AT LOWER FREQUENCY THAN ZERO.

$\Rightarrow$ GAIN DECREASES AT HIGH FREQUENCY

$\Rightarrow$ PHASE NEGATIVE (I.E. ADDS LAG).



$$G_{LAG} = \frac{s/z + 1}{s/p + 1}$$

- LAG MECHANICS THE SAME AS FOR THE LEAD.

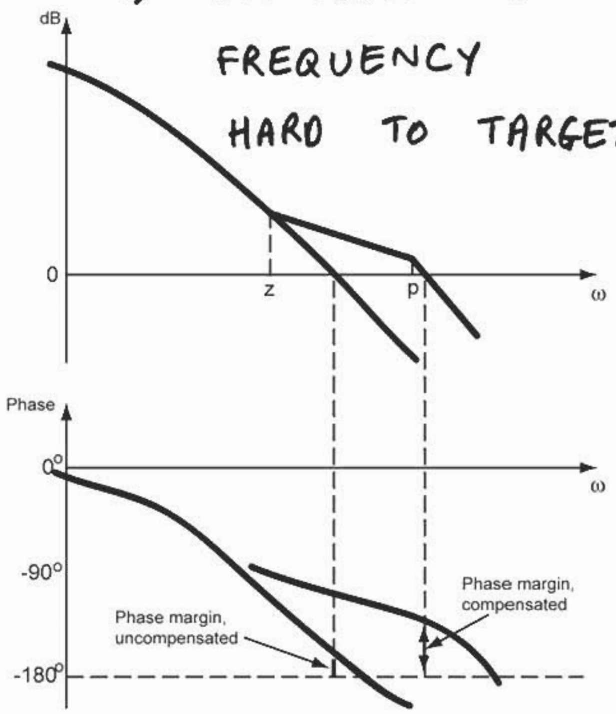
- USE LAG TO ADD $20 \log \alpha$ TO LOW FREQUENCY GAIN WITH (HOPEFULLY) A SMALL CHANGE TO THE PHASE MARGIN.

  ➡ PLOT SHOWS GAIN MUCH HIGHER AT LOW FREQUENCY.

$\Rightarrow$ KEEP LAG DYNAMICS TYPICALLY WELL BELOW THE LEAD.

LEAD COMPENSATION: TYPICAL PROCESS.

- ADDING CHANGES MAGNITUDE + PHASE
  - ⇒ DIFFICULT TO PREDICT NEW CROSSOVER FREQUENCY
  - HARD TO TARGET ϕ_M AT CORRECT LOCATION.



DESIGN APPROACHES DISCUSSED IN TEXT.

$$\phi_{REQ} = PM - \left(180 + \angle G(j\omega_c)\right)$$

- THE PROCESS IS SLIGHTLY SIMPLER IF WE TARGET THE LEAD COMPENSATOR DESIGN ONLY AT THE CROSS-OVER FREQUENCY RANGE.

  1) FIND ϕ_{MAX} REQUIRED → FIND $\alpha = \frac{|z|}{|p|}$

  2) PUT ϕ_{max} AT CROSS-OVER FREQUENCY
  $$\omega_c^2 = |p| \cdot |z|$$

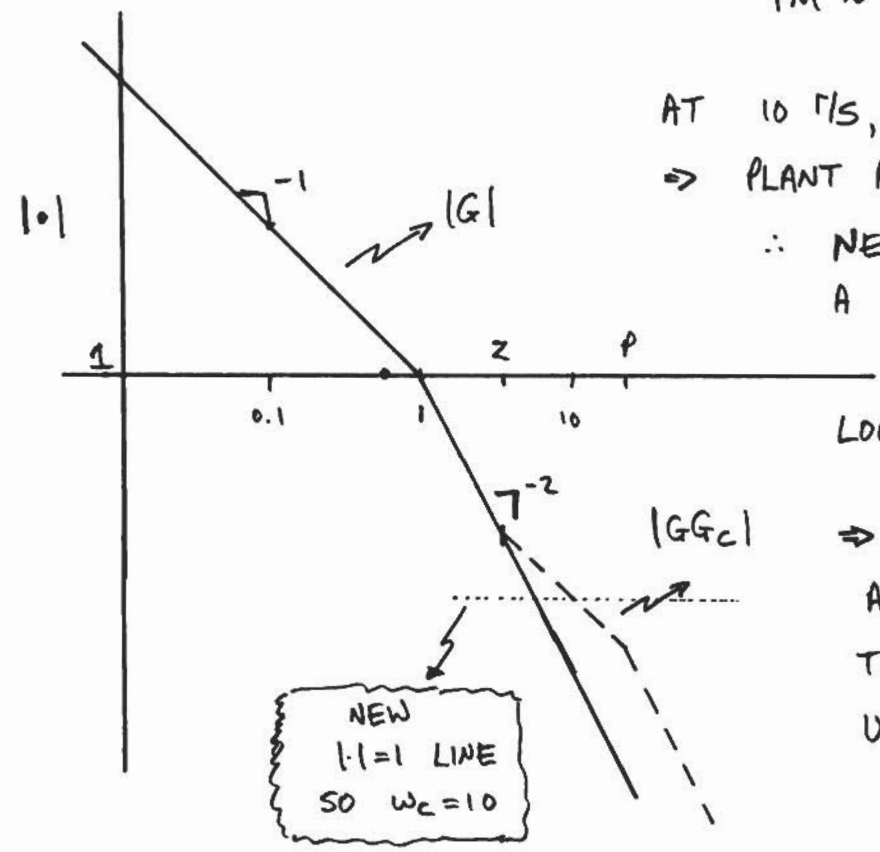
  3) SELECT K_c SO CROSSOVER <u>IS</u> <u>AT</u> ω_c

⇒ MEET BANDWIDTH / PHASE MARGIN SPECIFICATIONS

⇒ NO SPECIFIED CHANGE IN LOOP GAIN
→ ERROR CONSTANTS.

EXAMPLE: $G(s) = \dfrac{1}{s(s+1)}$

WOULD LIKE
$\omega_c \approx 10$ RAD/SEC
PM ~ 40°



AT 10 r/s, SLOPE ~ -2
⇒ PLANT PHASE ~ 180°
∴ NEED TO ADD A LEAD SO THAT SLOPE OF THE LOOP $G_cG(s) \sim -1$

⇒ ADDING A LEAD ALSO INCREASES THE PHASE, GIVING US OUR PM.

① $\dfrac{z}{p} = \dfrac{1-\sin\phi_m}{1+\sin\phi_m}$ $\left.\begin{array}{l}\angle G(j\omega_c) \doteq -180° \\ PM = 40°\end{array}\right\}$ $\phi_m = 40°$

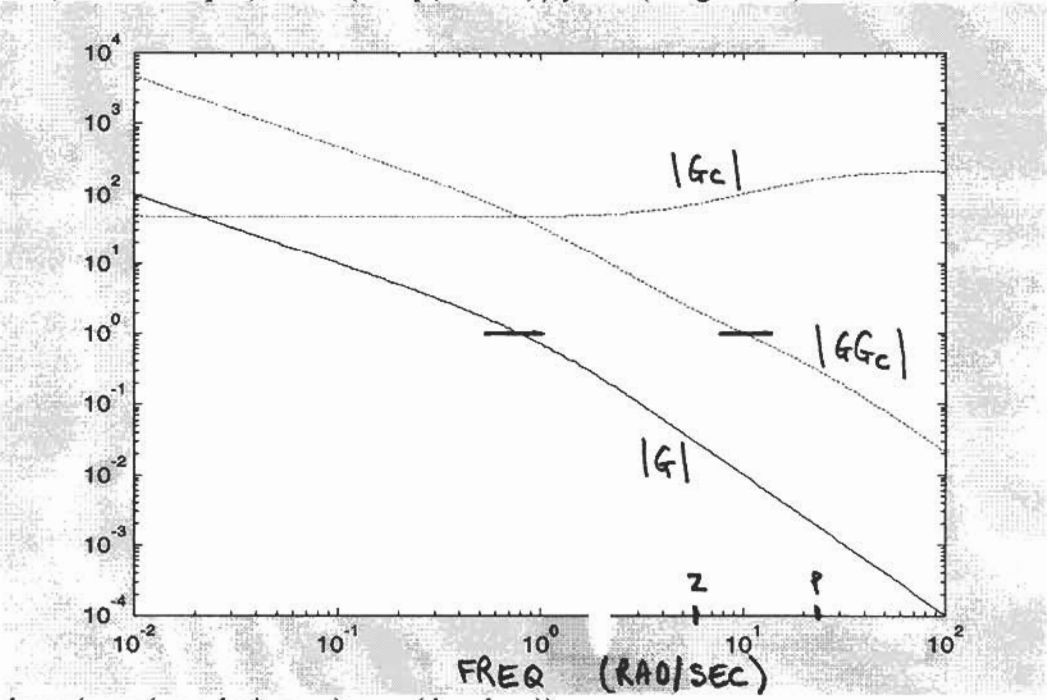
$\Rightarrow \dfrac{z}{p} = 0.22$

② $\omega_c^2 = z \cdot p = 10^2 \Rightarrow z = 4.7,\ p = 21.4$

③ PICK K_c SO $\left.\left|G_cG\right|\right|_{s=10j} = 1$

LEAD EXAMPLE

```
% g=1/s/(s+1)
wc=10;num=1;den=conv([1 0],[1 1]);Phim=40*pi/180;
zdp=(1-sin(Phim))/(1+sin(Phim));z=sqrt(100*zdp);p=z/zdp;
numc=[1 z];denc=[1 p];
kc=abs(polyval(conv(den,denc),j*wc)/polyval(conv(num,numc),j*wc));
f=logspace(-2,2,400);g=freqresp(num,den,f*sqrt(-1));
gc=freqresp(kc*numc,denc,f*sqrt(-1));
loglog(f,abs(g),f,abs(gc),'-.',f,abs(g.*gc),':');
title('Lead Example');xlabel('Freq (rad/sec)');ylabel('Magnitude')
```

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rlocus(conv(num,kc*numc),conv(den,denc))
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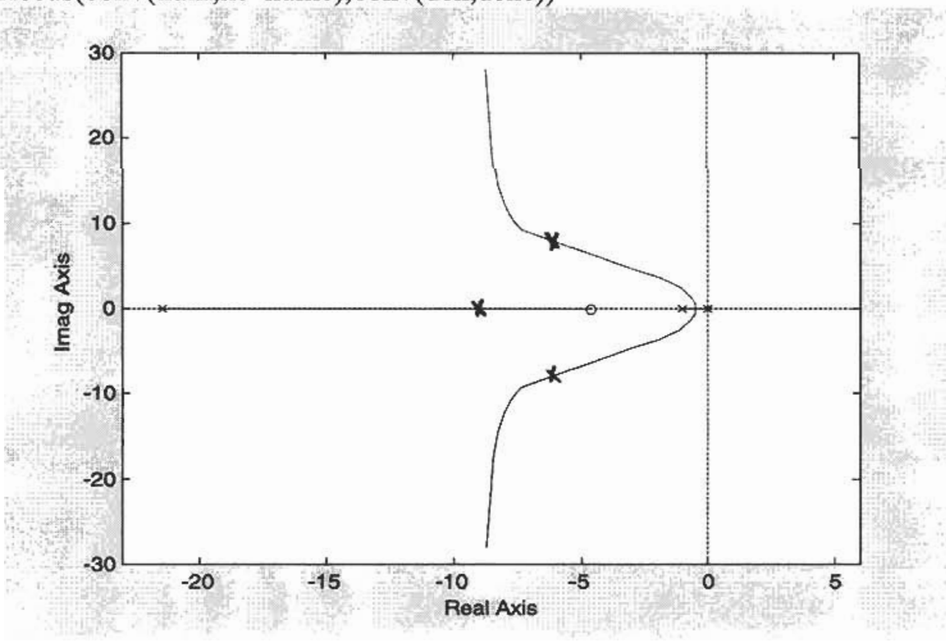


LAG COMPENSATION : TYPICAL PROCESS

- ASSUMPTION IS THAT WE NEED TO MODIFY (INCREASE) THE DC GAIN OF THE LOOP TRANSFER FUNCTION.

  ⇒ IF APPLY ONLY A GAIN, THEN ω_c TYPICALLY INCREASES, AND THE PHASE MARGIN DECREASES.

  THIS IS NOT GOOD → USE LAG COMP TO LOWER HIGH FREQ. GAIN (OR INCREASE LOW FREQ. GAIN)

- ANALYSIS SIMPLER WITH $G_{LAG} = \dfrac{K_c\,(s/z+1)}{(s/p+1)}$

  1) PICK $|G_{LAG}|_{s=0} \Rightarrow K_c$ TO GIVE THE DESIRED LOW FREQUENCY GAIN FOR $L = G_{LAG}(s)\,G_p(s)$

  2) PICK DESIRED GAIN REDUCTION AT HIGH FREQUENCY $20 \log\left(\frac{1}{\alpha}\right)$ $\left(\alpha = \dfrac{|z|}{|p|}\right)$

     ⇒ NORMALLY PICK α SO THAT ω_c NOT CHANGED.

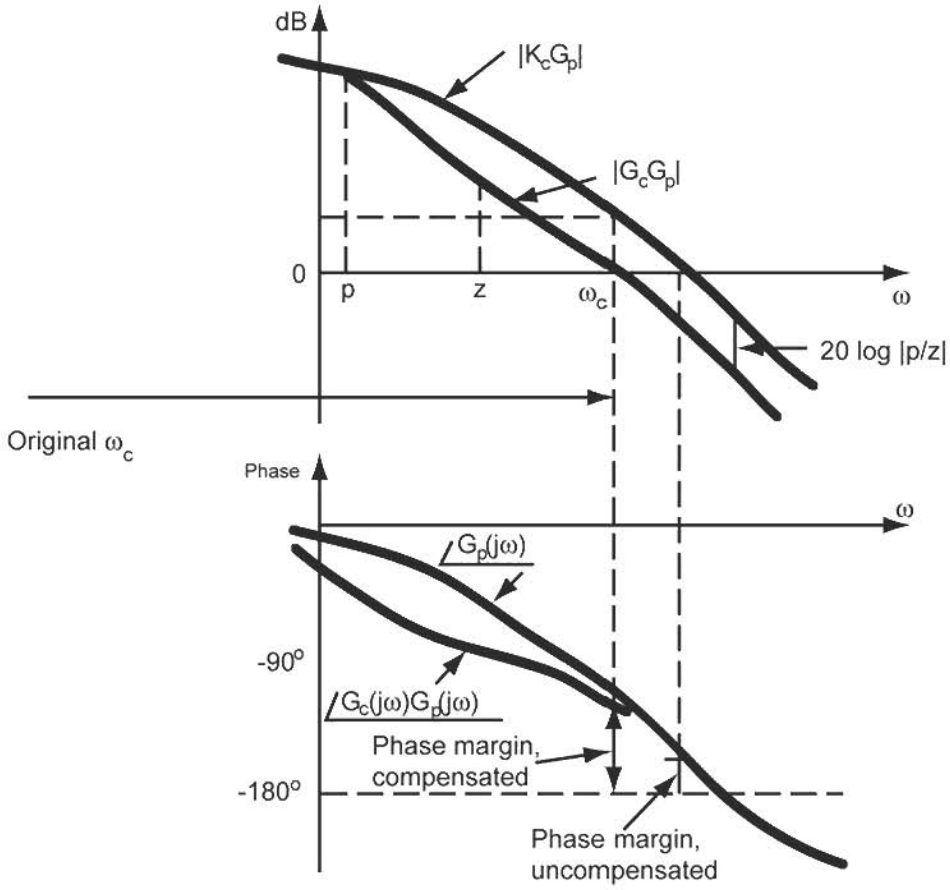
  3) HEURISTIC: WANT TO LIMIT FREQUENCY OF THE ZERO (+ POLE) SO THAT THERE IS A MINIMAL IMPACT OF THE PHASE LAG AT $\omega = \omega_c$

     $$\Rightarrow \boxed{\text{SET } z = \frac{\omega_c}{10}}$$

LAG DESIGN:

TWO WAYS TO GET DESIRED LOW FREQUENCY GAIN

① USING JUST K_c INCREASES ω_c AND LOWERS OUR PM → BAD



② ⇒ ADD LOW FREQ DYNAMICS (LAG) THAT INCREASE LOW FREQ GAIN, LEAVING GAIN NEAR + ABOVE ORIGINAL ω_c UNCHANGED

- MANY OTHER APPROACHES EXIST
  → TRY TO MEET ALL SPECS WITH ONLY ONE COMPENSATOR
  → APPROACH I OUTLINED SPLITS THE PROBLEM AND MAKES EACH STEP EASIER
    ⇒ TYPICALLY REQUIRES <u>BOTH</u> LEAD AND LAG TO MEET ALL SPECS.

- SEE BOOK / HANDOUT #9 FOR MORE EXAMPLES + <u>RECIPES</u> FOR OTHER DESIGN CASES.