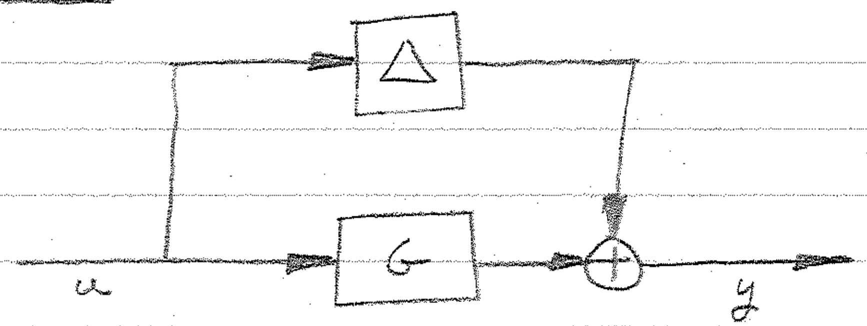
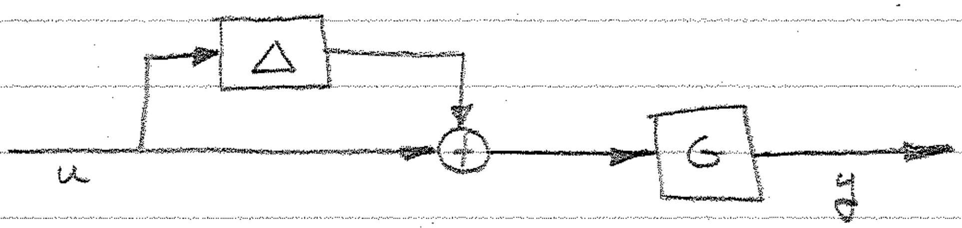
Modeling Uncertainties

There are several ways to model uncertainties:

Additive



Multiplicative

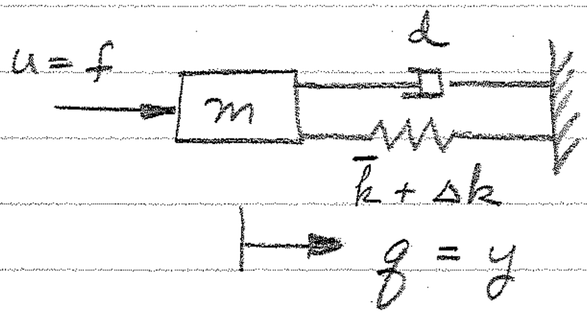


In both cases, the nominal plant occurs when $\Delta = 0$.

The multiplicative error describes the percentage or fractional error; the additive model describe the absolute error.

There are other, more physical models

<u>Example</u> Consider spring mass system with stiffness uncertainty:



States: $x_1 = q$
$x_2 = \dot{q}$

State equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k+\Delta k}{m} & -d/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -k/m & -d/m \end{bmatrix} x + \begin{bmatrix} 0 \\ -1/m \end{bmatrix} \Delta k \begin{bmatrix} 1 & 0 \end{bmatrix} x$$
$$+ \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

This looks like feedback!

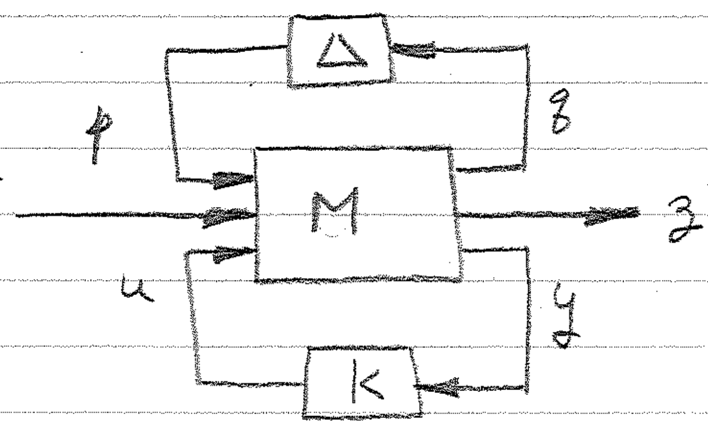
$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -k/m & -d/m \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ -1/m \end{bmatrix}}_{B_p} p + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

$$q = \underbrace{[1 \quad 0]}_{C_q} x$$

$$y = [1 \quad 0]\, x$$

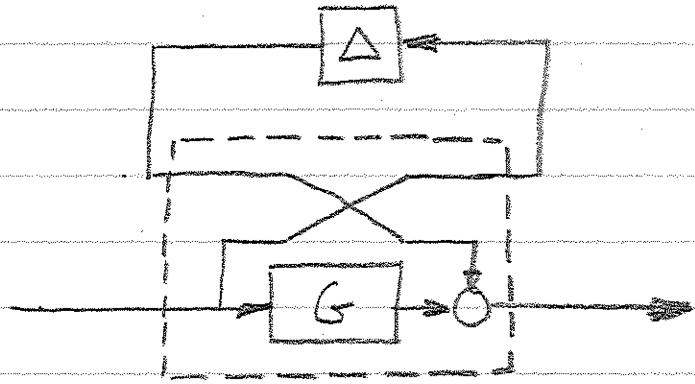
$$p = \Delta k\, q$$

Very often, uncertainty in a control system looks like feedback — plant is a "linear fractional transformation"



[This is almost the "standard model"]

Additive model:



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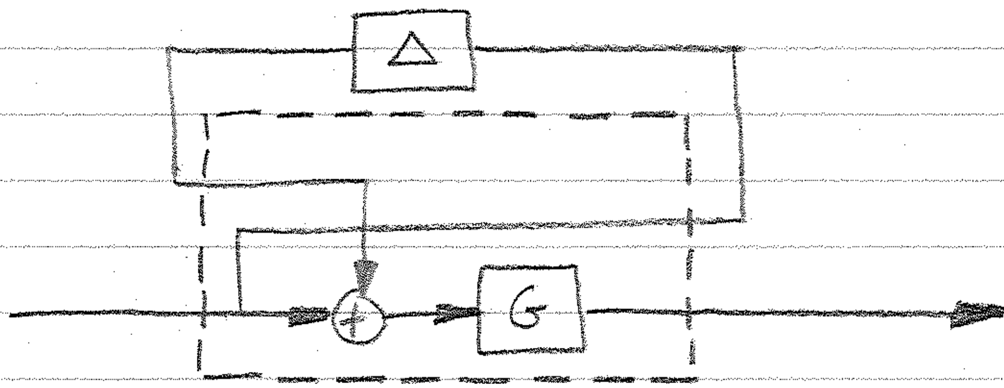
Multiplicative model:



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So almost every uncertainty model is a LFT!

Stability robustness Is system stable for all $\Delta \in \mathbf{\Delta}$?

Performance Robustness Does system meet requirements for all $\Delta \in \mathbf{\Delta}$?

To check for stability, break the loop at the Δ.

Example: Spring mass system.

$\bar{k} = 1 \qquad d = 0$

$m = 1$

feedback: $u = -0.2\,\dot{y} = -0.2\,x_2$

$$\Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix} x + \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix} \Delta k \, \underbrace{(1 \quad 0)\,x}_{q}}_{p}$$

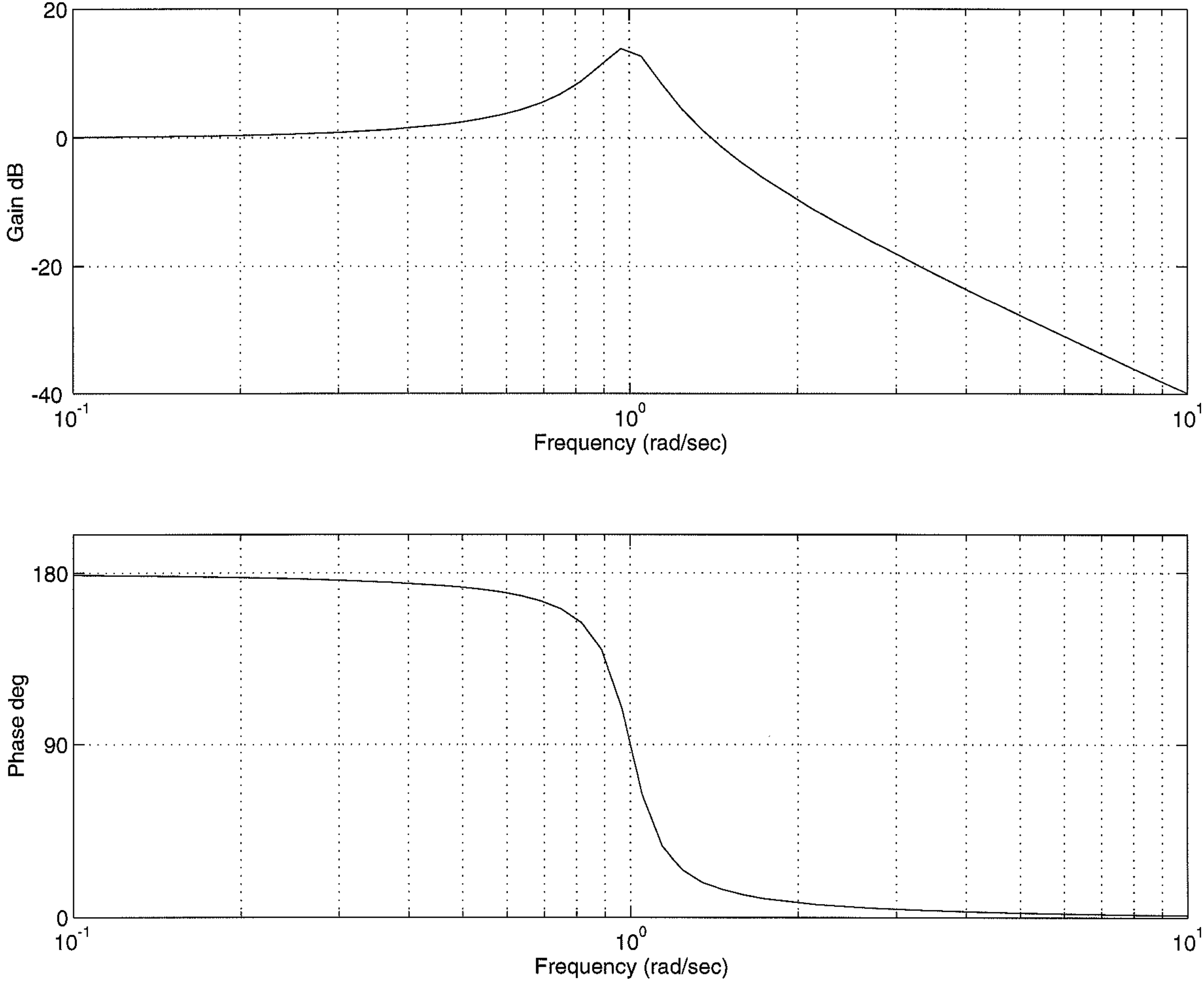
Transfer function

$$\frac{q}{p}(s) = \frac{-1}{s^2 + 0.2s + 1}$$

Bode plot next page —

System is stable for <u>all</u> Δ such that

$$|\Delta k(j\omega)| < 0.2042$$



20
0
-20
-40
Gain dB
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)
180
90
0
Phase deg
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

But, is stable for all real Δk such that

$$-1 < \Delta k < \infty$$

So treating Δk as complex is conservative

- It is much easier to treat complex uncertainties
- Remember that this often leads to conservatism.