16.31 Homework 3

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Note: Although due October 11, please try to complete by October 6, as there will be another problem set issued then.

1. Given the plant $G(s) = 1/s^2$, design a lead compensator so that the dominant poles are located at $-2 \pm 2j$.

2. Determine the required compensation for the system

$$G(s) = \frac{K}{(s+8)(s+14)(s+20)}$$

   to meet the following specifications:

   (a) Overshoot $\leq 5\%$

   (b) 10-90% rise time $t_r \leq 150$ msec

   Simulate the response of this closed-loop system to a step response. Comment on the steady-state error. You should find that it is quite large.

   Determine what modifications you would need to make to this controller so that the system also has

   (c) $K_p > 6$

   thereby reducing the steady state error. Simulate the response of this new closed-loop system and confirm that all the specifications are met.

3. Develop a state space model for the transfer function (not in modal / diagonal form).

$$G_1(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)}$$

   Discuss what state vector you chose and why. Develop a "modal" state space model for this transfer function as well. Confirm that both models yield the same transfer function when you compute

$$\hat{G}(s) = C(sI - A)^{-1}B + D$$

4. A set of state-space equations is given by:

$$\dot{x}_1 = x_1 \left(u - \beta x_2 \right)$$
$$\dot{x}_2 = x_2 \left(-\alpha + \beta x_1 \right)$$

where u is the input and α and β are positive constants.

(a) Is this system linear or nonlinear, time-varying or time-invariant?

(b) Determine the equilibrium points for this system (constant operating points), assuming a constant input $u = 1$.

(c) Near the positive equilibrium point from (b), find a linearized state-space model of the system.