

16.31 Homework 6

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Problem 1

In class, we will need the result that

$$\frac{d}{dK} \text{Tr}[XKY] = X^T Y^T$$

Prove this result. To do so, you will need to use the indicial form for matrices. For example, the matrix product XY really means that the ij element of the product is given by

$$[XY]_{ij} = \sum_k X_{ik} Y_{kj}$$

and the trace of a matrix X is given by

$$\text{Tr}[X] = \sum_k X_{kk}$$

Problem 2

In class, we will argue that the choice

$$K = \Sigma C^T V^{-1}$$

is the best choice of observer gain, because it minimizes the rate of change of the covariance, given by

$$\dot{\Sigma} = (A - KC)\Sigma + \Sigma(A - KC)^T + GWG^T + KVK^T$$

In fact, we will only prove that this choice makes $\dot{\Sigma}$ stationary — it might actually maximize $\dot{\Sigma}$. Show that this choice for K actually minimizes $\dot{\Sigma}$ in the following sense: Call $K^* = \Sigma C^T V^{-1}$, and the resulting rate of change of the covariance $(\dot{\Sigma})^*$. Then prove that every other choice of K , say, $K = K^* + \delta K$, produces a rate of change of the covariance, $\dot{\Sigma}$, that is larger than $(\dot{\Sigma})^*$. Hint: Subtract $(\dot{\Sigma})^*$ from $\dot{\Sigma}$, and show that the difference is a positive definite matrix.

Problem 3

Using duality and what you know about the LQR problem, describe an algorithm for solving the algebraic Riccati equation

$$0 = A\Sigma + \Sigma A^T + GWG^T - \Sigma C^T V^{-1} C \Sigma$$

for the matrix Σ .