16.31 Homework 8

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Problem 1

In this problem, you will see that the LQG design technique can yield control systems with very low gain and phase margins. Thus, the LQG approach, while "optimal," does not necessarily yield controllers that are "good."

Consider the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s-1}{(s-2)(s+3)}$$

Find a compensator

$$K(s) = -\frac{U(s)}{Y(s)}$$

using LQG methods. To do this, do the following:

1. Assume that the process noise w enters the system in the same way as the control u. That is, assume that $B_w = B_u$.
2. Assume that the performance variable is the same as the measurement variable. That is, assume that $C_z = C_y$.
3. Use unit weights in the LQR problem. That is, use $Q_z = R = 1$.
4. Assume that the process noise w and measurement noise v has unit intensity. That is, assume that $W = V = 1$.

What is the resulting compensator?

Find the gain and phase margins for the closed-loop system. Are they acceptable? Can you find a compensator with better margins?

Problem 2

Show that the maximum singular value of the matrix T is the same as the maximum singular value of the matrix T^*, where $(\cdot)^*$ denotes the complex conjugate transpose.

Problem 3

Write a Matlab function, `infnorm(A,B,C)`, that returns the infinity norm of the transfer function of the system defined by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

using the Hamiltonian method described in class. Using this function, find the infinity norm for the system

$$
\begin{aligned}
A &= \begin{bmatrix} -0.1 & 1 \\ -2 & -0.3 \end{bmatrix} \\
B &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\end{aligned}
$$

Problem 4

Write a Matlab function, `infnorm2(A,B,C,D)`, that returns the infinity norm of the transfer function of the system defined by

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
$$

The presence of the D term complicates the problem significantly. Using the block diagram interpretation we discussed in class, determine the Hamiltonian that allows one to determine whether the infinity norm of the transfer function is less than γ. This result can then be used to define an iteration to find the infinity norm.

Using this function, find the infinity norm for the system

$$
\begin{aligned}
A &= \begin{bmatrix} -0.1 & 1 \\ -2 & -0.3 \end{bmatrix} \\
B &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\
D &= \begin{bmatrix} 20 & 3 \\ 4 & 26 \end{bmatrix}
\end{aligned}
$$