Uncertainty and Visual Exploration

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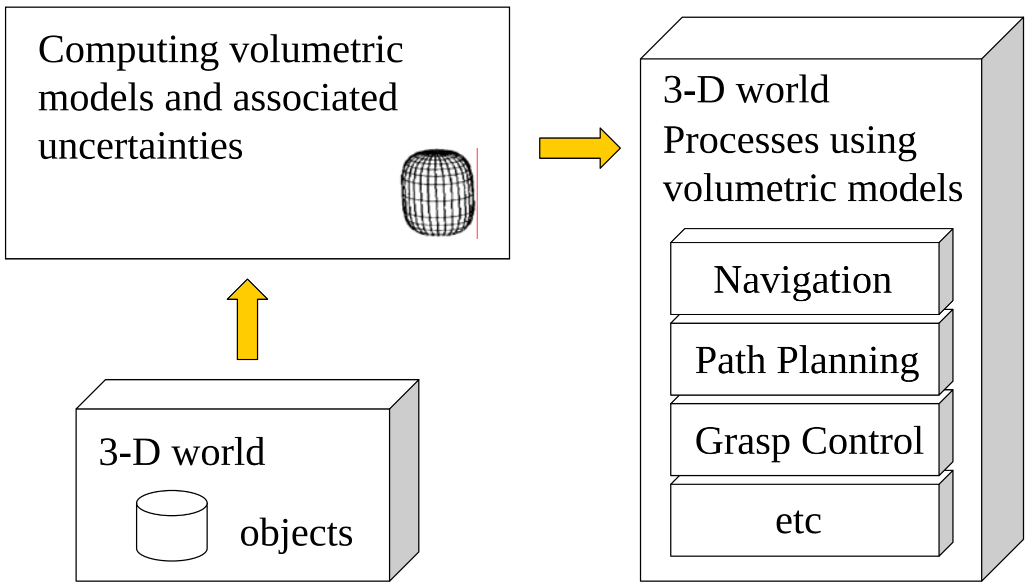
P. Whaite, F. P. Ferrie, *From Uncertainty to Visual Exploration*, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 13, NO. 10, October 1991

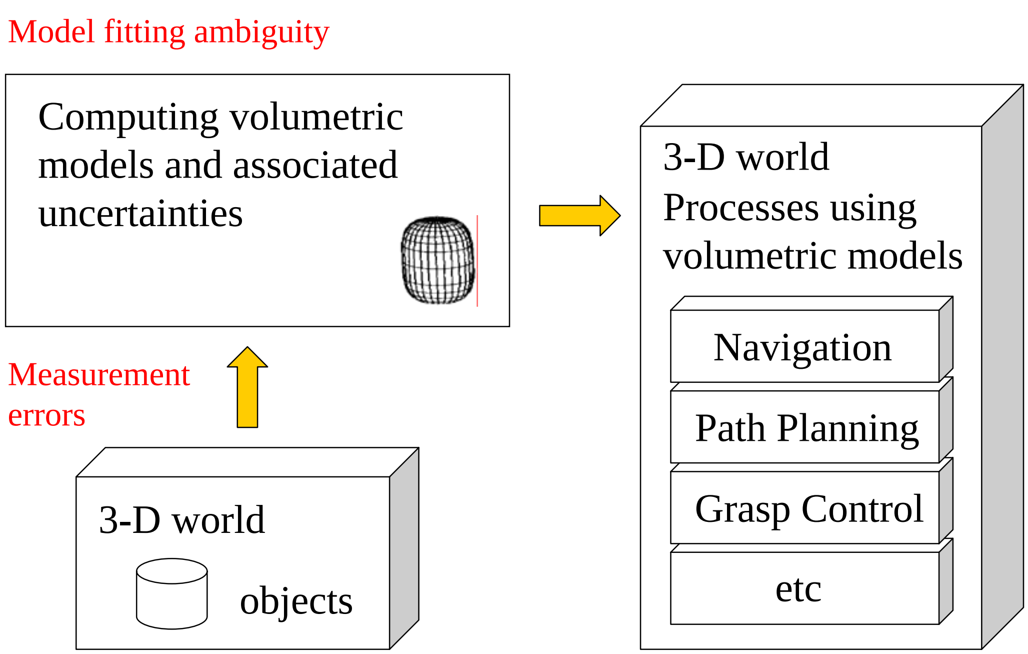
Outline

- Using uncertainty for visual exploration: strategy
- Volumetric representation of real-world objects: superellipsoids
- Metrics
- Nonuniqueness
- Communicating nonuniqueness with confidence
- Uncertainty in 3-D space
- Gaze planning

Using Uncertainty for Visual Exploration: Strategy



Sources of Uncertainty

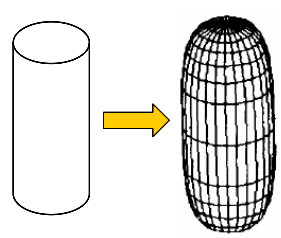


Using Uncertainty for Visual Exploration: Strategy

Instead of trying to resolve uncertainty, communicate uncertainty together with computed volumetric models to 3-D world processes using these models to try to make use of uncertainty for world exploration

Volumetric Representation of Real-World Objects: Superellipsoids

$$f(x,a) = \left(\left| \frac{x}{a_x} \right|^{2/\varepsilon_2} + \left| \frac{y}{a_y} \right|^{2/\varepsilon_2} \right)^{\varepsilon_2/\varepsilon_1} + \left| \frac{z}{a_z} \right|^{2/\varepsilon_1}$$



$\mathbf{x} = (x, y, z)$ - data point

$\mathbf{a} = (a_x, a_y, a_z, \varepsilon_2, \varepsilon_1)$ - superellipsoid parameters

$a_x > 0,\ a_y > 0,\ a_z > 0,\ 0 \le \varepsilon_2 \le 2,\ 0 \le \varepsilon_1 \le 2$

f - *inside-outside function*:

$f(\mathbf{x},\mathbf{a}) = 1$ then **x** is on the surface

$f(\mathbf{x},\mathbf{a}) > 1$ then **x** is outside the surface

$f(\mathbf{x},\mathbf{a}) < 1$ then **x** is inside the surface

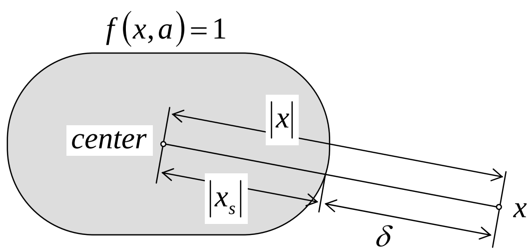
Levenburg-Marquardt least squares minimization method

- Minimize in steepest descent fashion the squared sum:

$$X^2(a) = \sum_{i=1}^{N} \frac{D^2(x_i, a)}{\sigma_i^2}$$

  - $D(x_i,\mathbf{a})$ **-** metric: measure of distance from data point x_i to superellipsoid surface described by parameters **a**
  - σ_i **-** distance error

Metrics



- Metric 1: example of bad metric

$$D_1 = f(x,a) - 1 = \left(1 + \frac{\delta}{|x_s|}\right)^{2/\varepsilon_1} - 1$$

  - biased toward large ε_1

Metrics

- Metric 2: example of bad metric

$$D_2 = f^{\varepsilon_1}(x,a) - 1 = \frac{\delta}{|x_s|}\left(2 + \frac{\delta}{|x_s|}\right)$$

  – biased toward large $|\,x_s\,|$

- Metric 3: example of bad metric

$$D_3 = (a_x a_y a_z)^{\frac{1}{2}}\left(f^{\varepsilon_1}(x,a) - 1\right)$$

"<u>minimum volume metric</u>"

  – model shape depends on volume exponent, metric is over-committed to object classes

Metrics

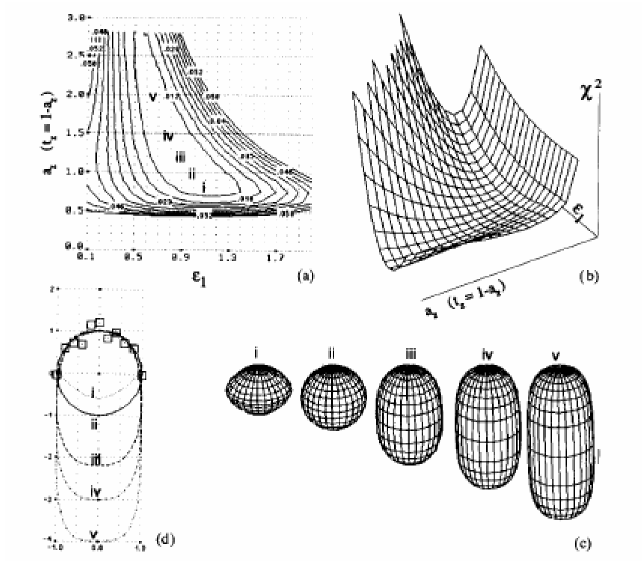
- Metric 4: good metric

$$\left.\begin{matrix} D_1 \\ |x| = \delta + |x_s| \end{matrix}\right\} \Rightarrow \quad D_4 = \delta = |x|\left(1 - \frac{1}{f^{\varepsilon_1/2}(x,a)}\right)$$

  – designed to expose weaknesses in the interpretation (i.e. uncertainty)
  – makes minimum assumptions about object shape

Nonuniqueness

- Ambiguity is inherent in the fitting process.



Communicating Nonuniqueness with Confidence

- Nonuniqueness region in the parameter space in the absence of measurement noise:

$$\{a \in A \mid X^2(a) = 0\} \quad \text{(valley floor)}$$

- Noise distorts the region, "swamps" the smallest misfit errors => impossible to decide if the error is due to misfit or random perturbations in the data

Communicating Nonuniqueness with Confidence

- Confidence level γ
- Acceptable error $X_{\gamma}^{2}(a)$ for parameters **a**
- Chance variations in the data will cause the error to exceed $X_{\gamma}^{2}(a)$ (100 - γ) % of time
- Complication:
  - There is no analytic solution for the probability distribution of the error at arbitrary parameters **a**
- Simplifying assumption: choose constant value of X_{γ}^{2} independent of parameters

Communicating Nonuniqueness with Confidence

- Region of non-unique (equivalent) models in the parameter space is enclosed by the surface:

$$X^2(a) < X_{\gamma}^{2} = \text{constant} \qquad (*)$$

- Complex shape of $X^2(a)$ is non-linear and not easily described mathematically => use Taylor expansion:

$$X^2(\hat{a} + \delta a) = X^2(\hat{a}) + \frac{\partial X^2}{\partial a}\delta a + \frac{1}{2}\delta a^T H \delta a + o\left(|\delta a|^2\right)$$

Communicating Nonuniqueness with Confidence

- $\delta a = a - \hat{a}$ is the displacement from the minimum position
- H is Hessian matrix
- At a minimum position $\frac{\partial X^2(\hat{a})}{\partial a} = 0$ Therefore:

$$\Delta X^2(\delta a) = X^2(\hat{a} + \delta a) - X^2(\hat{a}) = \frac{1}{2}\delta a^T H \delta a$$

$$\Rightarrow \quad \frac{1}{2}\delta a^T H \delta a < X_\gamma^2 \qquad (*)$$

Communicating Nonuniqueness with Confidence

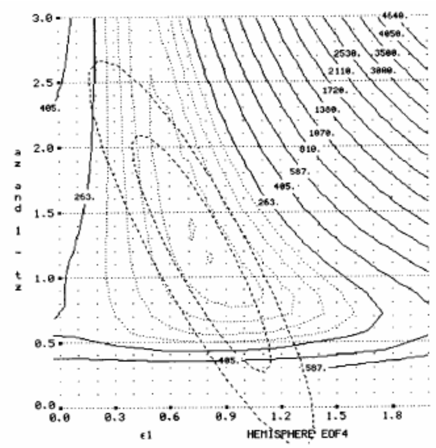
- Assumptions:
  - Local linearity of X^2
  - Normal distribution of noise
- Covariance matrix: $C = 2H^{-1}$
- Then:

$$(*) \qquad \delta a^T C^{-1} \delta a < X_\gamma^2$$

<u>ellipsoid of confidence</u>

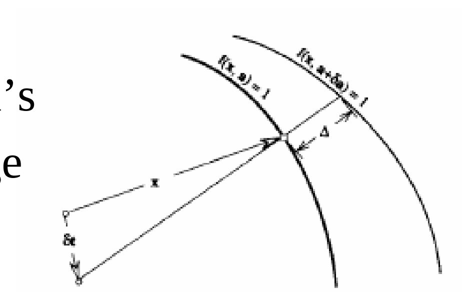
- Covariance matrix is used to communicate the nonuniqueness of the fitted model

Ellipsoid of Confidence



- Limitation of the ellipsoid of confidence: can only represent nonuniqueness at a single minimum in parameter space. Presence of multiple minima is a difficult problem.

Uncertainty in 3-D Space



- Δ is the change in the model's surface in response to change δa in model's parameter estimate
- Δ is not easily described analytically => use Taylor expansion:

$$\Delta = D_4\left(x, \hat{a} + \delta a\right) = \frac{\partial D_4}{\partial a}\delta a + o\left(\left|\delta a^2\right|\right) + \ldots$$

Uncertainty in 3-D Space

- Constrain parameters to lie within the ellipsoid of confidence => the surface perturbation is constrained as well
- Want to know maximum surface variation
- Constrained optimization problem => use Lagrange multipliers to maximize the quantity:

$$L = \Delta + \mu\left(\delta a^T C^{-1} \delta a - \Delta X_\gamma^2\right)$$

Uncertainty in 3-D Space

$$\begin{cases} \dfrac{\partial L}{\partial \delta a} = \dfrac{\partial D_4}{\partial a} + 2\mu C^{-1} \delta a = 0 \\ \dfrac{\partial L}{\partial \mu} = \delta a^T C^{-1} \delta a - \Delta X_\gamma^2 = 0 \end{cases} \qquad (*)$$

. . .

$$\text{Result:} \qquad \Delta_{\max} = \pm\sqrt{\Delta X_\gamma^2}\sqrt{\frac{\partial D_4^T}{\partial a} C \frac{\partial D_4}{\partial a}}$$

Uncertainty in 3-D Space

$$\Delta_{max} = \pm\sqrt{\Delta X_{\gamma}^{2}}\sqrt{\frac{\partial D_4^T}{\partial a} C \frac{\partial D_4}{\partial a}}$$

- Two solutions for Δ_{max}: (+) and (-) => each point **x** on the fitted superellipsoid is inside one of the models from the ellipsoid of confidence and outside another
- Ellipsoid of confidence in the parameter space maps into a shell around the fitted surface in 3-D space => shell of uncertainty
- The thicker the shell of uncertainty => the more uncertainty about the true position on the surface

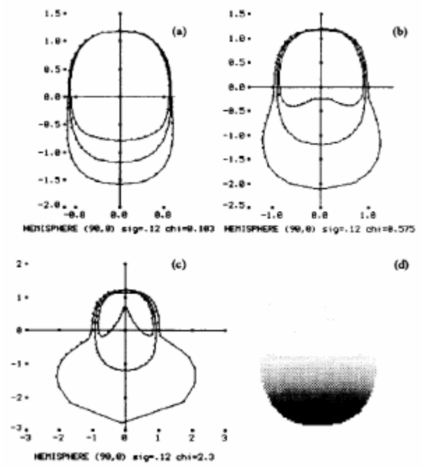
Uncertainty in 3-D Space

- Two types of processes in 3-D world that use the obtained volumetric representations:
  - Type 1: ones that need to minimize the obtained uncertainty: path planning, grasp control etc.
  - Type 2: ones that can use the obtained uncertainty: gaze planning
- Useful function for both types of processes:

$$U(x,\hat{a}) = \left|\frac{\Delta_{max}}{\sqrt{\Delta X_{\gamma}^{2}}}\right| = \sqrt{\frac{\partial D_4^T}{\partial a} C \frac{\partial D_4}{\partial a}}$$

surface uncertainty measure

Uncertainty in 3-D Space



Plots for three levels of confidence:
(a) γ = 5 %
(b) γ = 25 %
(c) γ = 68 %

uncertainty image: surface of the fitted model colored with levels of gray proportional to uncertainty measure U

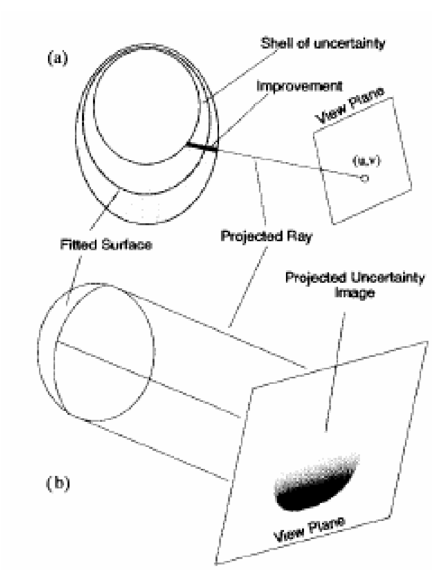
- Breakdown of local linearity assumption for high levels of confidence

Gaze Planning

- Measure of improvement

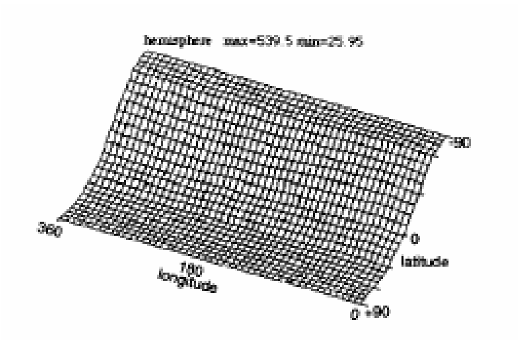
$$I(\theta,\phi) = \sum_{u,v\in P} U\left(x\left(P\left(u,v,\theta,\phi\right)\right),\hat{a}\right)$$

P - projection
U - surface uncertainty measure
(θ,ϕ) - view position
(u,v) - viewplane coordinates
x(P(u,v, θ,ϕ)) - point of intersection of the projected ray with the surface of the model



Gaze Planning

- Measure of improvement as a function of view position

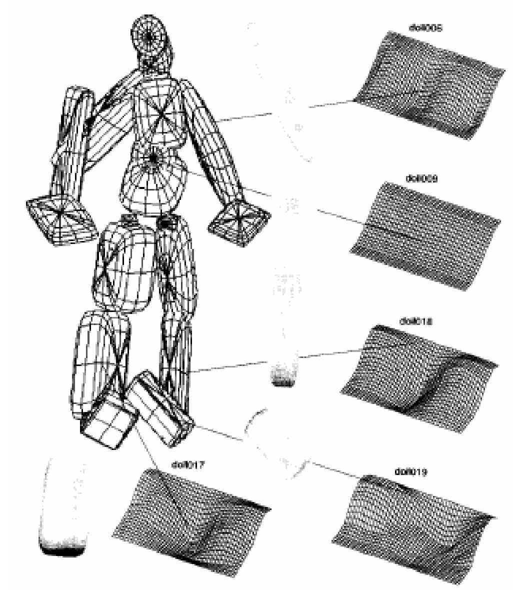


example of range scanning of a noisy hemisphere

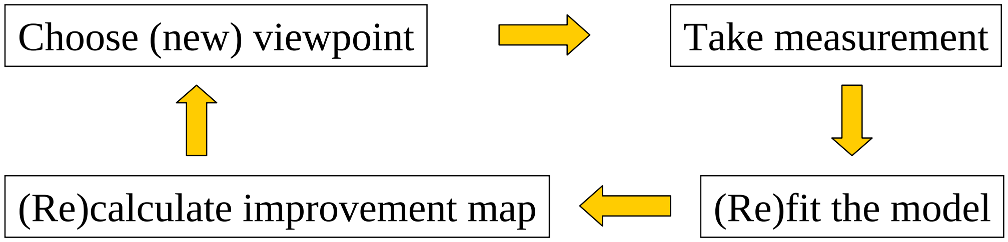
- +90 deg latitude (north pole): zero improvement
- -90 deg latitude (south pole): maximum improvement

Gaze Planning: Real Example

- Wooden mannequin
- Gaze planning strategies:
  - choose a viewpoint corresponding to a maximum peak on the improvement map
  - choose a viewpoint corresponding to a steepest ascent on the improvement map



Gaze Planning Strategy



Continue until there is negligible decrease in the overall uncertainty

Conclusions

- Ambiguity can be used to plan new direction of view that minimizes the ambiguity of subsequent interpretation
- Unresolved problems:
  - Real world is not only composed of objects that can be represented by volumetric models
  - Metrics are either suitable for good model fit or for good uncertainty measure but not for both