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MODELLING A FALLING FELINE

On The Implementation of Non-Holonomic Moment-of-Inertia Reorientation Control

Written Proposal
16.621
Fall 2001

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December 11, 2001

Table of Contents

1 Introduction

1.1 Overview

There is an aphorism that a dropped cat always lands on its feet. Though not always true, cats do exhibit a remarkable ability to reorient themselves while falling. An inverted cat dropped from a height with zero initial angular velocity and zero angular momentum is able to manipulate its body so as to rotate 180 degrees and land safely upright. This motion appears to violate the conservation of angular momentum since a rigid body initially at rest and under the influence of no external forces except for gravity should not be able to rotate without something to push against.

The mechanism behind this paradoxical activity is of some interest, and is hypothesized to be a form of non-holonomic motion control. The cat may first be treated as two distinct halves separated about the midpoint of the spine into a front and rear component. These two torso elements can actuate a torque between themselves (e.g. twist along the axis of the spine) and can each individually vary their moments of inertia via the extension or retraction of the cat's legs. A diagram of a falling cat reorienting is shown in Figure 1.

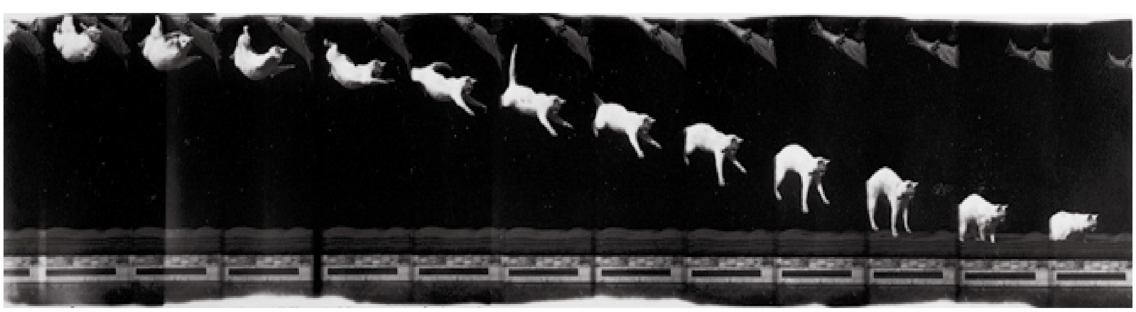


Figure 1 Chronophotograph of a cat falling, c 1893 by Marey.

1.2 Motivation

The incentive to understand non-holonomic orientation control as exhibited by a falling cat lies in its potential as an alternative to current methods of orientation control. Free objects requiring a specific orientation (such as communications satellites or interplanetary landing-craft or even every-day aircraft) currently tend to maintain and control this orientation via aerodynamic forces (wings and fins) or via momentum-exchange based thrusters. The cat's non-holonomic moment-of-inertia based control provides an intriguing alternative to these traditionally accepted means.

Of specific interest to the aerospace community, this orientation control scheme could be entirely electrically actuated. Installed on an orbiting satellite, such a system would draw all necessary power from the same source as onboard electronics (photovoltaic cells, nuclear plants, etc.) Such a change would eliminate the need for fuel-propellant control systems, which add unnecessary vibrations, exhaust particles, and mass to any space system.

Despite the potential for application of non-holonomic moment-of-inertia orientation control, the majority of research in this area has been largely theoretical. Actual implementation of an orientation control system based on this scheme will require an understanding of the mechanics of the motion, and of the relationships between component parameters and system performance.

1.3 Previous Work

The first formal research of the falling cat phenomenon can be traced to Etienne-Jules Marey of France who was interested in the movement of living creatures. In 1894 he developed a camera system that could take photos at a fast enough rate to visually capture the cat's motions during a fall. Research has continued into the modern day, with various tools of analytical dynamics finding applications in such a system. In 2001, DeSapio and Layon of Stanford proposed three abstracted models of a cat that could execute similar motions, with simplified internal actuations[1]. Non-holonomic reorientation is an active area of study, with research finding application in fields ranging from motion planning for coupled rigid bodies[2] to springboard diving technique[3].

2 Objective

The objective of this project is to experimentally assess the open loop behavior of a proposed non-holonomic reorientation scheme. The relationship between variability of component moments of inertia and performance parameters based on deflection accuracy and speed of response will be measured. Analysis of the system dynamic performance will help answer the questions: "Does the reorientation algorithm work? Does it work quickly enough? And can the algorithm be made to perform better by varying the ratio of maximum to minimum component moments of inertia?"

3 Existing Knowledge

3.1 Model Abstraction

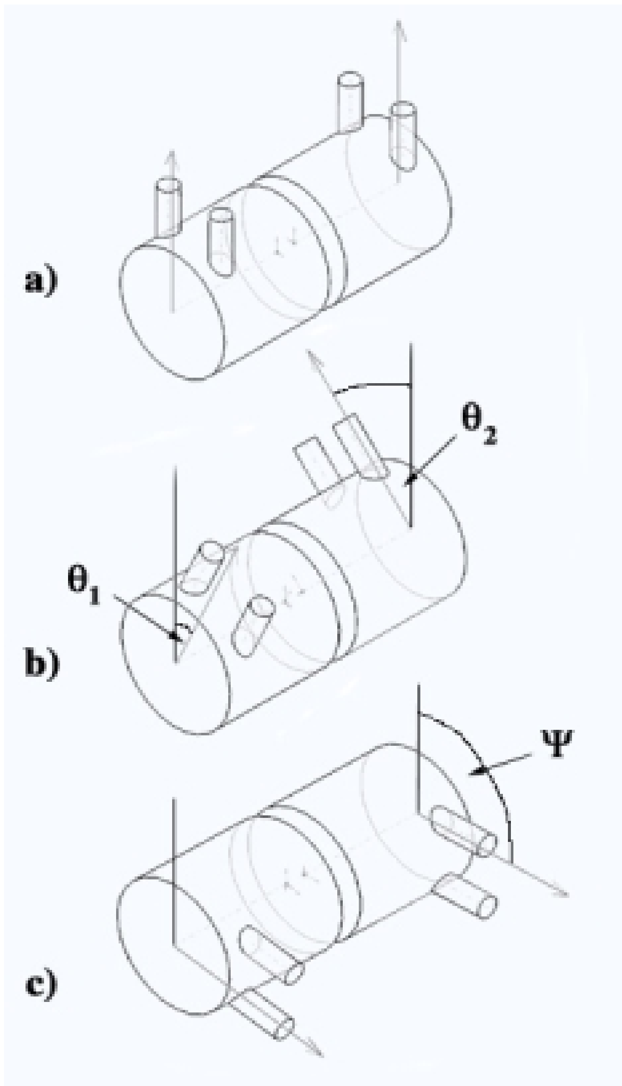


Figure 2 Simplified cat abstraction. (a) Initial resting position. (b) With partially rotated torso elements. (c) Final resting position.

Terms:

Final Net Deflection: $\boldsymbol{\psi}$
Angle of Deflection from neutral position of A torso element: $\boldsymbol{\theta_1}$
Angle of Deflection from neutral position of B torso element: $\boldsymbol{\theta_2}$
Angle of Deflection of primary servo: $\boldsymbol{\Theta = |\theta_1| + |\theta_2|}$
Moment of Inertia of torso element with legs retracted: $\mathbf{I_R}$
Moment of Inertia of torso element with legs extended: $\mathbf{I_E}$

3.2 Reorientation Abstraction

Suppose two identical torso elements with identical moments of inertia as per figure 2. The application of torque along the twisting axis between the two halves results in a rotation of one torso element, and an equal and opposite rotation of the other torso element. The application of an equal counter-torque then realigns the two halves (returns them to a "resting position"). However, no net deflection has occurred. The torso elements are in precisely the same position they were when they started. To accomplish a net change in system orientation (a change in ψ) the moments of inertia of each torso element need to be varied as well. This is illustrated below in the context of a falling cat:

When the cat is released, it immediately retracts one set of legs (either the front or the hind), and extends the other. With regards to the twisting axis (parallel to the spine), one half of the animal now has a larger moment of inertia than the other half.

The cat's muscles apply a torque about the twisting axis and a rotation of each torso element occurs. The half with the retracted legs and smaller moment of inertia will rotate through a greater angle than the half with the extended legs and larger moment of inertia. The ratio of the magnitude of these rotations is proportional to the ratio of the different torso element moments of inertia. The direction of rotation of the two elements is in opposite directions, as expected by the conservation of angular momentum.

The relative moments of inertia of each torso element are switched by extending the retracted legs, and retracting the extended legs.

A counter-torque equal in magnitude but in the opposite direction of the original torque is applied and the torso elements undergo another rotation.

When the two torso elements have rotated until the net twist angle between them (**Θ**) has returned to zero, the cat is once again in a "relaxed state." Measurements of the net deflection (**ψ**) only occur when the system has returned to a relaxed state.

Due to the varying moments of inertia, the final relaxed system will have undergone a net deflection about the axis of twisting and no longer be oriented as it was initially. If this net rotation is 180 degrees, the cat will have completely inverted, and then be able to land on its own four feet.

3.3 Theory

The constitutive relationship between angular momentum and moment of inertia is of the form

$$L = I\omega \tag{1}$$

The system starts at rest, with no initial angular momentum

$$L_{system} = 0 \tag{2}$$

Therefore changes in angular momentum of the two halves must be equal and opposite

$$L_A = L_B \tag{3}$$

$$I_A \omega_A = I_B \omega_B \tag{4}$$

$$\omega_A = \left(\frac{I_B}{I_A} \right) \omega_B \tag{5}$$

Multiplying by time so as to work with units of deflection instead of velocity yields

$$\theta_A = \left(\frac{I_B}{I_A} \right) \theta_B \tag{6}$$

Recall that each torso element shares the same value for maximum and minimum moment of inertia. The first rotation turns element A θ_A degrees in the positive direction, and element B θ_B degrees in the negative direction. Component moments of inertia are switched and the counter-torque applied. The second rotation then turns element A θ_B degrees in the negative direction and element B θ_A degrees in the positive direction. In both cases, the final net deflection angle is

$$\psi = (\theta_A - \theta_B) \tag{7}$$

From (6)

$$\psi = \left(\frac{I_B}{I_A} - 1\right)\theta_B = \left(1 - \frac{I_A}{I_B}\right)\theta_A \tag{8}$$

The required amount of rotation of the primary servo is

$$\Theta = |\theta_A| + |\theta_B| \tag{9}$$

Using (7) to convert into terms of final net deflection angle

$$\Theta = \frac{\psi}{\left(1 - \frac{I_A}{I_B}\right)} + \frac{\psi}{\left(\frac{I_B}{I_A} - 1\right)} \tag{10}$$

$$\Theta = \frac{\psi\left(\frac{I_B}{I_A} - 1\right) + \psi\left(1 - \frac{I_A}{I_B}\right)}{\frac{I_B}{I_A} + \frac{I_A}{I_B} - 2} \tag{11}$$

$$\Theta = \frac{\left(\frac{I_B}{I_A} - \frac{I_A}{I_B}\right)\psi}{\left(\frac{I_B}{I_A} + \frac{I_A}{I_B} - 2\right)} \tag{12}$$

Let IR = (I_E/I_R) = (I_A/I_B) = Variability of moment of inertia. Then the angle of deflection, Θ of the primary servo required at each of the two rotations to attain a final net deflection of ψ is:

$$\Theta = \frac{\left(\frac{1}{IR} - IR\right)}{\left(IR + \frac{1}{IR} - 2\right)}\psi \tag{13}$$

Which reduces to a linear relationship between Θ and ψ for any given IR. This equation will be utilized by the system controller software to determine the appropriate level of primary servo deflection required to attain any specific final net deflection.

3.4 Non-Holonomic Motion

The process of reorientation described in 3.3 can be described as non-holonomic. This implies that the route to reorientation is path-dependent, or that the various internal actuations (twisting between the torso elements, extension and retraction of the legs) must occur in a specific order to attain a specific end result. If the two applications of central torque and the varying of moments of inertia occur in a different order, then the cat will not achieve the same orientation. Any final position is possible, but getting to each position requires a specific series of motions and cannot be directly attained.

4 Experimental Approach

The project objective will be attained by constructing a device that replicates the functional aspects of the reorientation abstraction (3.2), outfitting it with a controller and sensors, and running it through a series of reorientation trials.

The device will replicate the abstracted cat of 3.1 in the following ways: It will be composed of two independent torso-elements joined by a servo, which will provide the primary torque along the twisting axis. Each torso-element will have a variable moment of inertia, to be altered by the extension and retraction of servo-mounted, weighted control legs. To free the device of timing constraints imposed by a finite-height freefall, it will be set up in a suspended orientation, with the twisting axis oriented vertically instead of horizontally. Detailed device descriptions follow in section 4.2.

The project will be completed in three primary phases. The design phase, including development of the device itself, the test setup, instrumentation plans and test matrix, has been completed.

The implementation phase will include construction of the device and test stand, acquisition of controller and data-collection hardware, software code development and full integration of all these sub-systems into the final test setup. This phase will be accomplished during Independent Activities Period and the Spring 2002 semester.

Data collection and analysis will occur upon completion of the implementation phase. Device speed of response and accuracy of deflection will be measured for a variety of net deflection angles and moment of inertia ratios. Analysis of continuous Θ data and the aforementioned results will form the basis for conclusions about the behavior of this reorientation scheme.

4.1 Test Setup

A schematic diagram of the test setup is provided below in Figure 3. The constructed device will be suspended from a test stand, and wired to interface with both a Handyboard microcomputer and a regular desktop running data collection software.

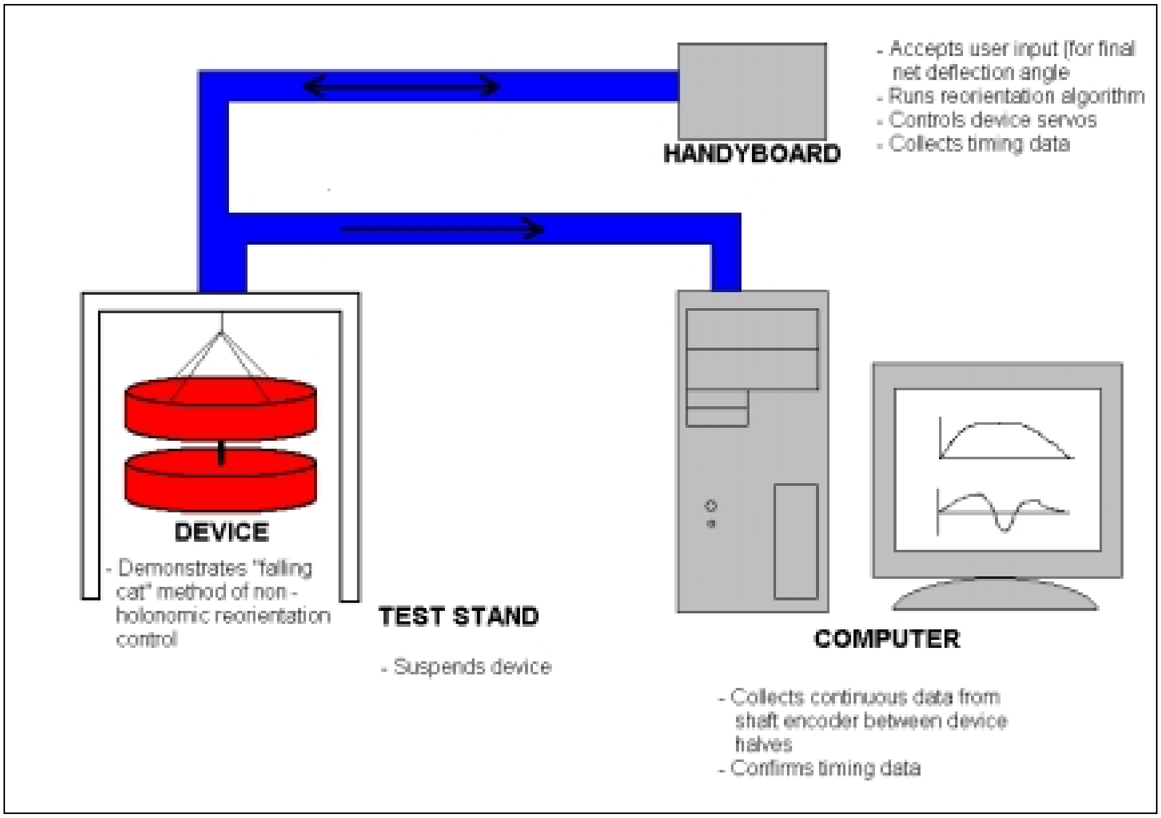


Figure 3 System Overview Schematic

The Handyboard will function as the system controller. The reorientation algorithm will be written in Interactive C and loaded into the Handyboard memory. It will then accept user input for the final net deflection angle and moment of inertia variability, and control the device servos appropriately. The Handyboard is being utilized as a controller for two main reasons. Firstly, it has built-in output ports that can interface directly with servos. In addition, future developments in this area of research may call for the reorienting device to function autonomously, which would require moving the controller onboard. In anticipation of such developments, it was decided to write the code on a platform that is mobile and independent of a full desktop computer.

A desktop computer running continuous data collection software will be wired to a shaft encoder installed on the primary servo of the device. This will allow for continuous collection of **Θ** data.

4.2 Device Design

A rendering of the device to be constructed can be found below in Figures 4 and 5. The first shows an overall view of the complete device, while the second displays only one torso element. The diameter of each torso element will be 30 centimeters. In Figure 4 the moment of inertia control legs for the top torso element are red, while those for the bottom torso element are blue. In both figures, servos are black while aluminum structural elements are beige. The servos and aluminum struts will be attached with bolts and will be position-adjustable to facilitate balancing the device. All wires will exit the device along the axis of rotation so as not to entangle when the device rotates. Detailed and dimensioned views can be found in the Appendix.

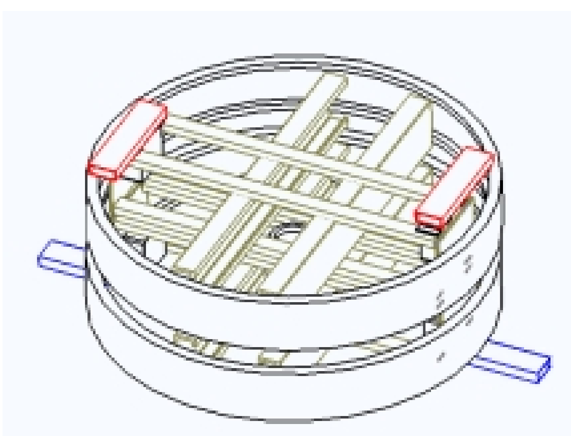


Figure 4 Overall Device Layout

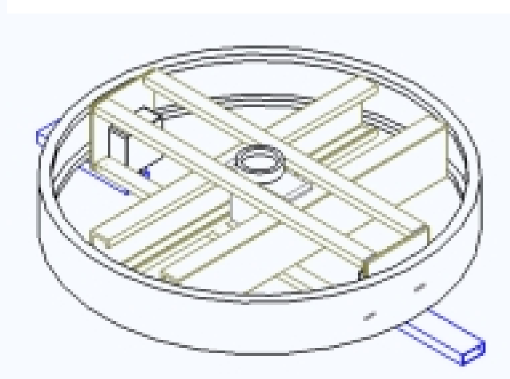


Figure 5 Torso Element Detail

4.3 Test Matrix

Variation between the maximum and minimum values of Moment of Inertia (I_E/I_R = IR)					
		125%	150%	175%	200%
Final Net Deflection (ψ)	45°				
	90°				
	135°				
	180°				
	225°				
	270°				

Measurements for each of the above 24 trials will be comprised of speed of response and deflection accuracy. The final angle between the two torso elements will also be examined, but only as a check since it should always equal zero indicating the device has returned to a relaxed state. The behavior of this angle with respect to time will be recorded in addition to the variation of torso-element moments of inertia as potential tools for further analysis.

4.4 Measurements and Data Acquisition

Final Net Deflection (ψ) – This will be varied in increments of 45 degrees from 45 to 270 degrees. Accuracy will be measured using a shaft encoder. This variable will be held constant, or independent.

Variability of Torso Element Moment of Inertia (IR) – This will range in increments of 25% from 125 to 200 percent. This value is the ratio between the baseline value for the torso element moment of inertia, and the maximum value (when the control-leg is fully extended). This variability will be controlled by control-leg weighting, and angle of extension. Measurements could be made using control-leg angle of extension, but this data eventually reduces to torso-element moment of inertia anyway. This variable will also be held constant, or independent.

System speed of response – This variable will always be dependent and will be measured using the Handyboard built-in timer.

Angle between torso elements (**Θ**) – This is not an actual variable but is a parameter that will be measured as an indicator of proper algorithm function. After each deflection this angle should be zero (indicating both torso-elements point in the same direction). Though the final value for this angle should always be zero, its behavior with respect to time may provide insight during analysis of system performance.

Both the angle of deflection and the moment of inertia variability are at times independent or constant. In one series of measurements the angle deflected will be held constant while the moment of inertia variability is varied. For each variability system speed of response and angular accuracy will be measured. In this case the moment of inertia variability is the independent variable, and measured accuracy and speed of response are the dependent variables. In the second series of measurements the moment of inertia variability will be held constant, and the angle of deflection will be varied (and will thereby be the independent variable).

All instrument calibrations will be performed per manufacturer instructions. Torso element moment of inertia values will be calculated by measuring dynamic response to a known torque.

4.5 Error Analysis

There are two potential types of error in instrumentation, bias and uncertainty. The former is a systematic error and proper calibration should nullify it. Uncertainty, on the other hand may affect the collected data. There is no expected error due to hysterisis or the order of the tests. All important data is static or timing-based and can be determined from the device's final state. The data to be acquired via continuous sampling (the behavior of the angle between torso elements and variation of element moments of inertia) is taken only as a potential tool for further analysis. If the data necessitates it, uncertainty analysis may be done based on measured baseline values.

5 Planning

5.1 Facilities and Resources

Assembly of the complete test setup and testing will occur in laboratory space provided in the MIT Course 16 hanger. All manufactured parts will be machined in the MIT Gelb Laboratory.

5.2 Safety Concerns

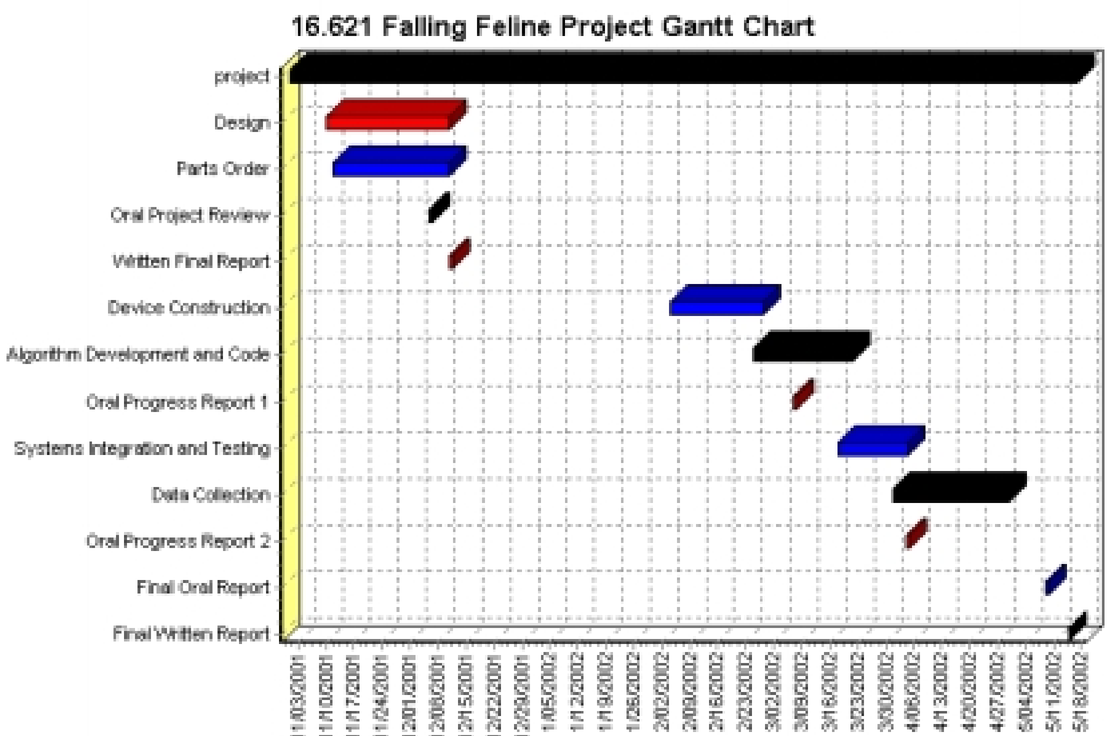
There are no concerns for human safety in the vicinity of the test apparatus because there are no sources of threat or danger. All voltages involved are on the order of magnitude of a battery, and no motions will occur quickly enough to cause injury. The only real pressing safety concern is for the safety of the constructed device itself. It will be suspended for the purposes of data collection, and care will be taken to ensure it remains suspended and does *not* itself ever undergo a freefall.

5.3 Budget

The proposed budget for complete execution of this project is outlined below.

Item	#	Price	Status	
Handyboard Kit	2	$24	Ordered – Douglas Electronics	
Primary Servo HS-805BB	1	$63	Ordered – Tower Hobbies	
Secondary Servo HS300B	4	$20	Ordered – Tower Hobbies	
Digital Compass	1	$63	Ordered – Honeywell Int'l	
Structural Components		N/A	Supplied by MIT	
Test Stand	1	N/A	Supplied by MIT	
Shaft Encoder	1	$39	Ordered – USDigital.com	
Total Cost:		$293		

5.4 Schedule



6 Conclusion

The reorientation mechanism demonstrated by a cat in freefall has great potential as an alternative to current reorientation schemes. Proposed here is a means of learning more about specific relationships involved in non-holonomic moment of inertia reorientation control. All plans are complete and parts are on order. Device construction, test setup and data acquisition are already to be started next semester.

7 Acknowledgements

I would like to acknowledge Hector Ayuso, my partner in this endeavor, and Professor Charles Coleman, our advisor, for all his help and tutelage. Thank you as well to Professors Earll Murman, Kim Blair, Andrea McKenzie and Col. Pete Young for their assistance throughout.

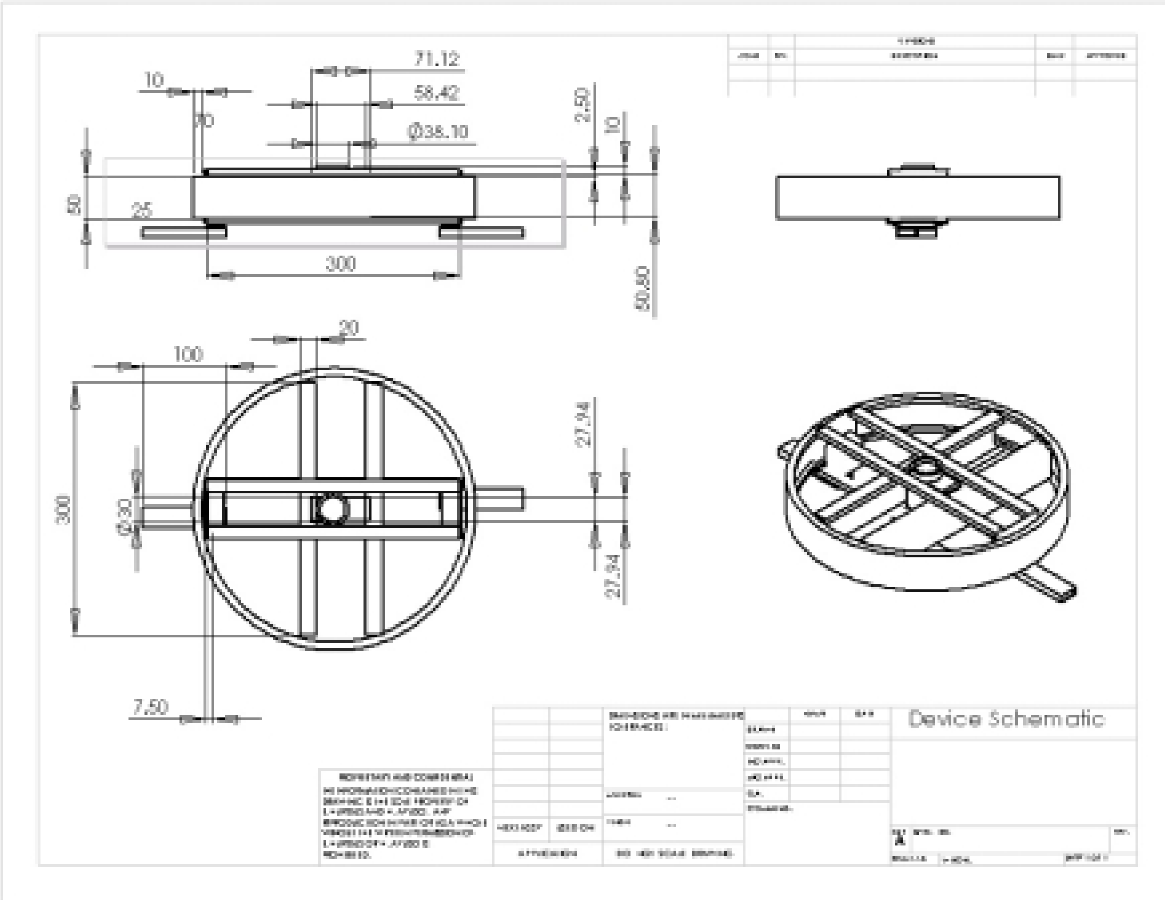
8 References

[1] Desapio, Vincent and Antonio Layan, "A Dynamical Explanation of a Cat's Ability to Re-orient Itself during Freefall," Stanford University, http://www.stanford.edu/~alayon/index.htm, September 24, 2001

[2] Fernandes, Chris and Leonid Gurvitz and Zexiang Li, "Near-Optimal Nonholonomic Motion Planning for a System of Coupled Rigid Bodies." IEEE Transactions of Automatic Control, March, 1994.

[3] Frohlich, Cliff, "The Physics of Somersaulting and Twisting." Scientific American, March, 1980.

9 Appendix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Device Schematic