16.810 (16.682)

Engineering Design and Rapid Prototyping

Design Optimization

- Structural Design Optimization

Instructor(s)

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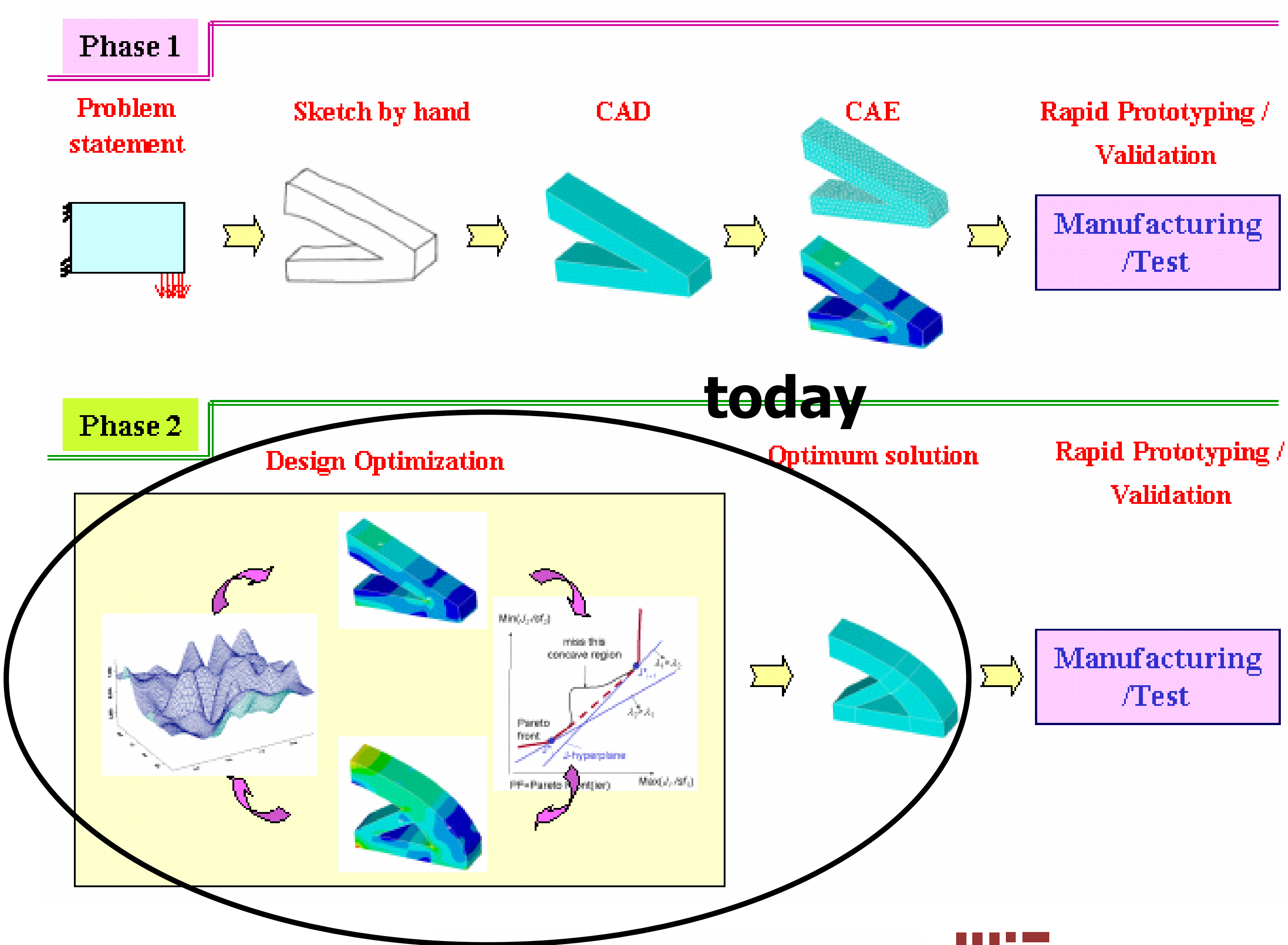
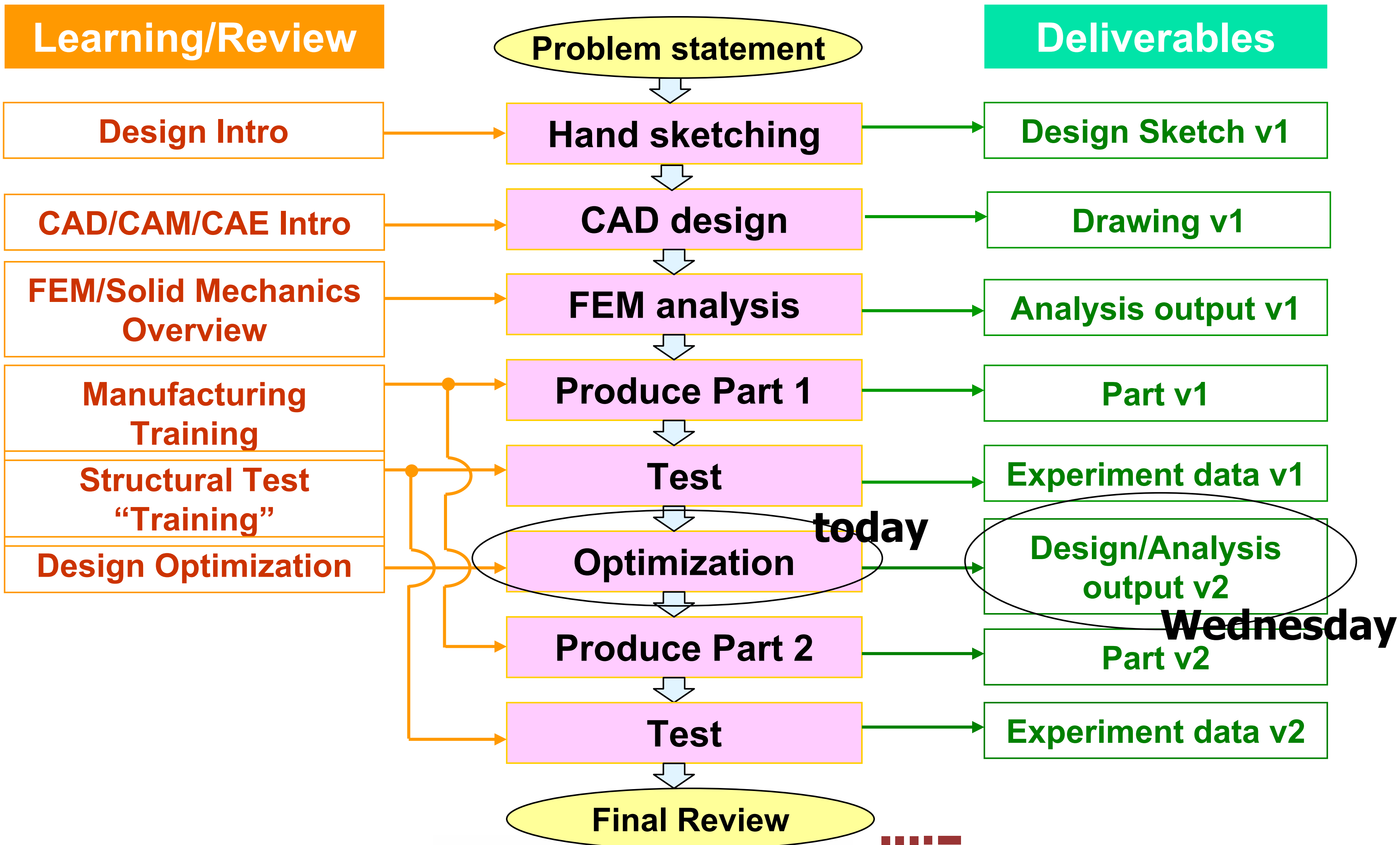
Course Concept



Course Flow Diagram



What Is Design Optimization?

Selecting the "best" design within the available means

1. What is our criterion for "best" design? — **Objective function**
2. What are the available means? — **Constraints (design requirements)**
3. How do we describe different designs? — **Design Variables**

Optimization Statement

$$\begin{aligned} \text{Minimize} \quad & f(\mathbf{x}) \\ \text{Subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$

Constraints

- Design requirements

2. Requirements

Manufacturing Cost (C): $C \leq 3.6\$$ /part

Performance (δ_1, δ_2, f_1): Displacement $\delta_1 \leq 0.078$ mm
Displacement $\delta_2 \leq 0.012$ mm
First natural frequency $f_1 \geq 195$ Hz

Mass (m): $m \leq 0.27$ lbs

Surface Quality (Q): $Q \geq 2$

Load Case (F): F1 = 50 lbs / F2 = 50 lbs / F3 = 100 lbs

The part has to conform to the interface requirements and geometrical boundary conditions shown on page 2 of this document. This requirement cannot be waived.

3. Priorities

Low manufacturing cost is the first priority for this product. Next, the customer cares about light-weighting (low mass) and thirdly, structural performance should be as high as possible. These priorities are shown in the Ishii-matrix below:

Attribute	**Constrain**	**Optimize**	**Accept**
Cost	■		
Performance			■
Mass		■	

Inequality constraints

Equality constraints

Objective Function

- A criterion for best design (or goodness of a design)

2. Requirements

Manufacturing Cost (C): $C \leq 3.6\$$ /part

Performance (δ_1, δ_2, f_1): Displacement $\delta_1 \leq 0.078$ mm
Displacement $\delta_2 \leq 0.012$ mm
First natural frequency $f_1 \geq 195$ Hz

Mass (m): $m \leq 0.27$ lbs

Objective function

Surface Quality (Q): $Q \geq 2$

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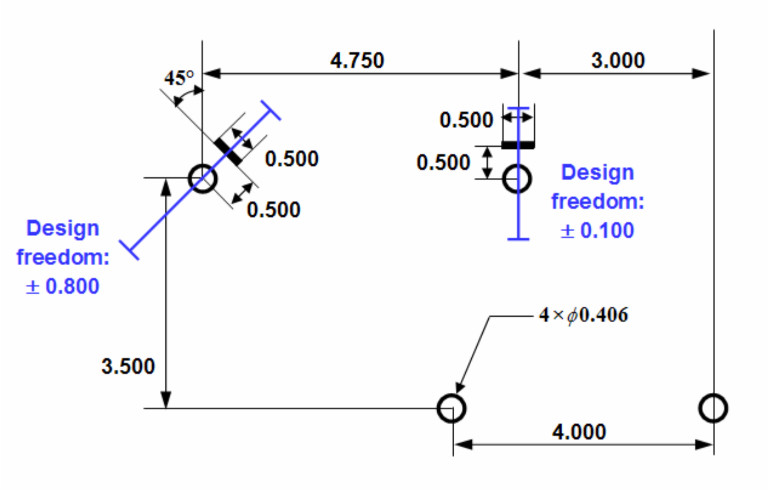
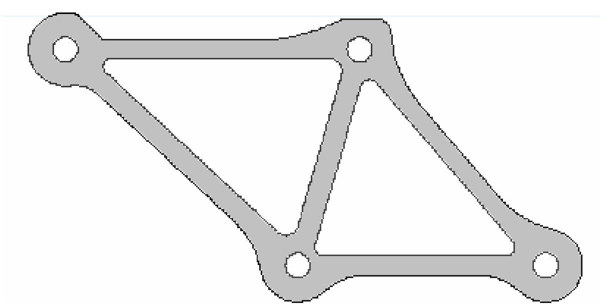
Attribute	**Constrain**	**Optimize**	**Accept**
Cost	■		
Performance			■
Mass		■	

Design Variables

Parameters that are chosen to describe the design of a system

⇨ Design variables are "controlled" by the designers





The position of upper holes along the design freedom line

Design Variables

For computational design optimization,

⇨ **Objective function and constraints must be expressed as a function of design variables (or design vector X)**

Objective function: $f(\mathbf{x})$ Constraints: $g(\mathbf{x}), h(\mathbf{x})$

Cost = f(design)

Displacement = f(design)

Natural frequency = f(design)

Mass = f(design)

What is "f" for each case?

Optimization Statement

$$\text{Minimize} \quad f(\mathbf{x})$$

$$\textit{Subject to} \quad g(\mathbf{x}) \le 0$$

$$h(\mathbf{x}) = 0$$

$f(\mathbf{x})$: **Objective function to be minimized**

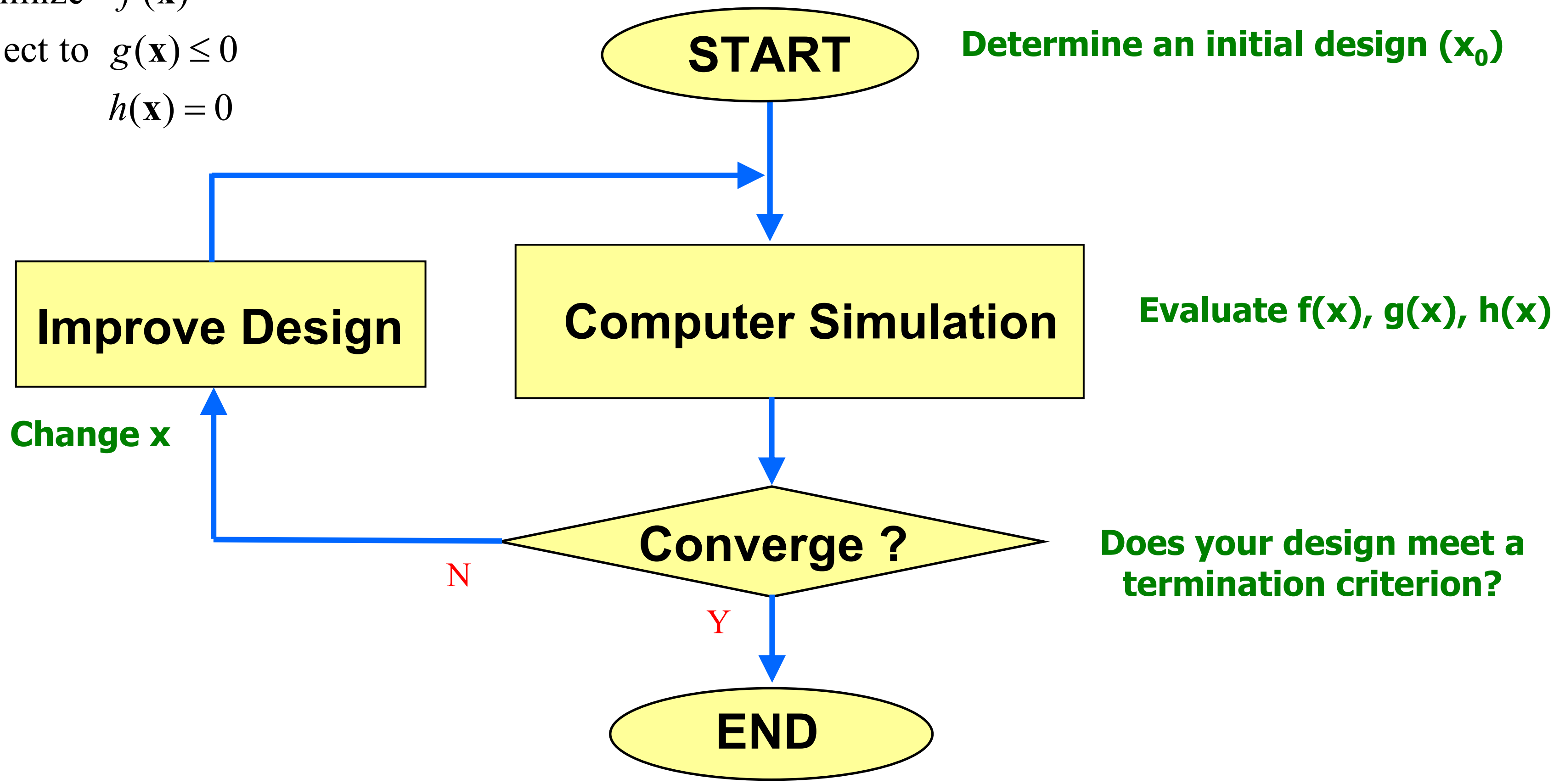
$g(\mathbf{x})$: **Inequality constraints**

$h(\mathbf{x})$: **Equality constraints**

$\mathbf{x}$: **Design variables**

Optimization Procedure

$$\text{Minimize} \quad f(\mathbf{x})$$
$$\text{Subject to} \quad g(\mathbf{x}) \leq 0$$
$$h(\mathbf{x}) = 0$$



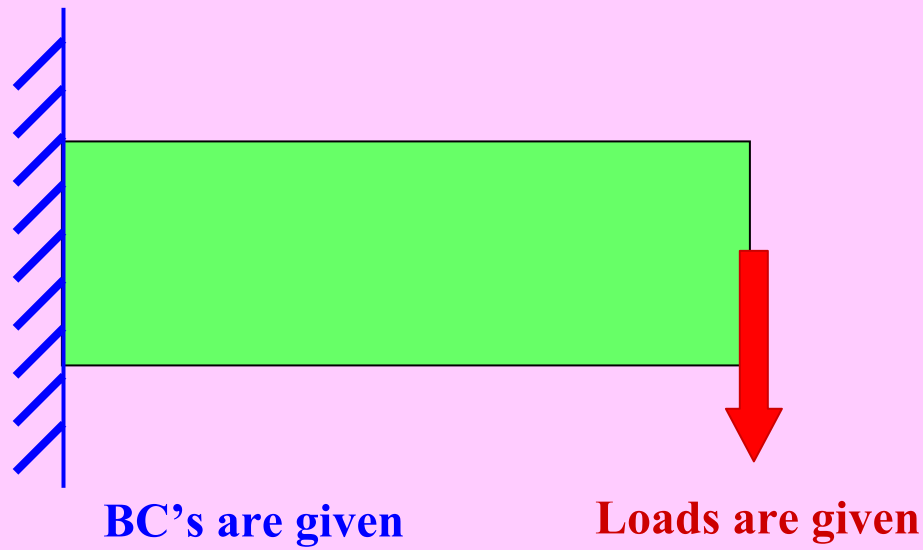
Structural Optimization

Selecting the best "structural" design

- Size Optimization
- Shape Optimization
- Topology Optimization

Structural Optimization

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$

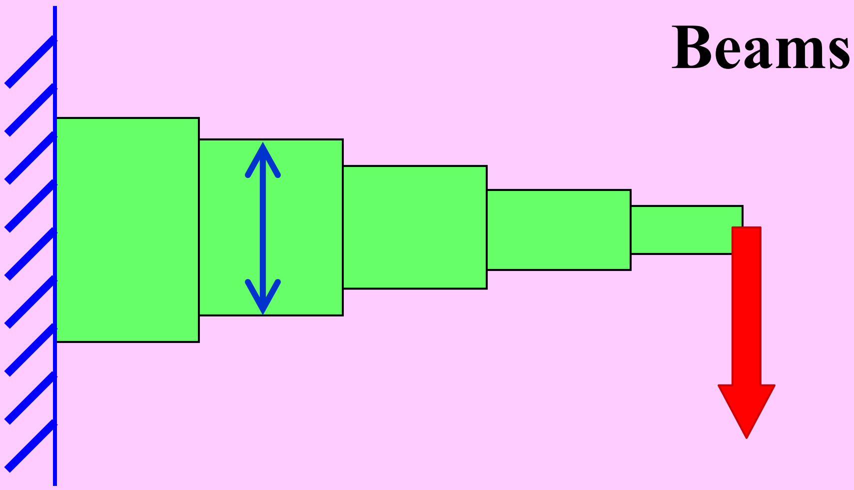


1. To make the structure strong
   e.g. Minimize displacement at the tip → *Min.* $f(x)$

2. Total mass $\le M_C$ → $g(x) \le 0$

Size Optimization

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leq 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : thickness of each beam

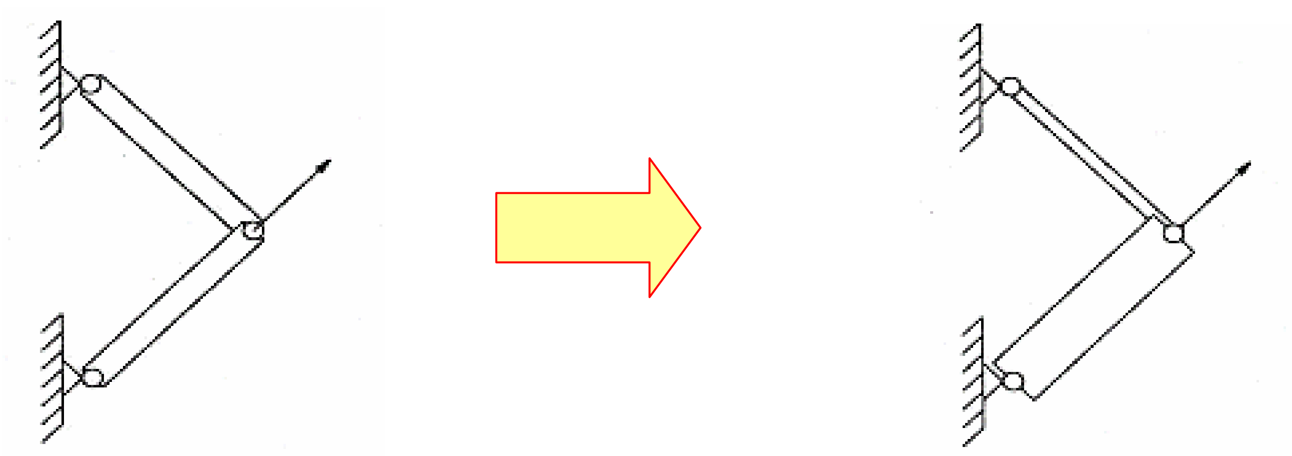
***f*(x) : compliance**

***g*(x) : mass**

Number of design variables (ndv)

ndv = 5

Size Optimization

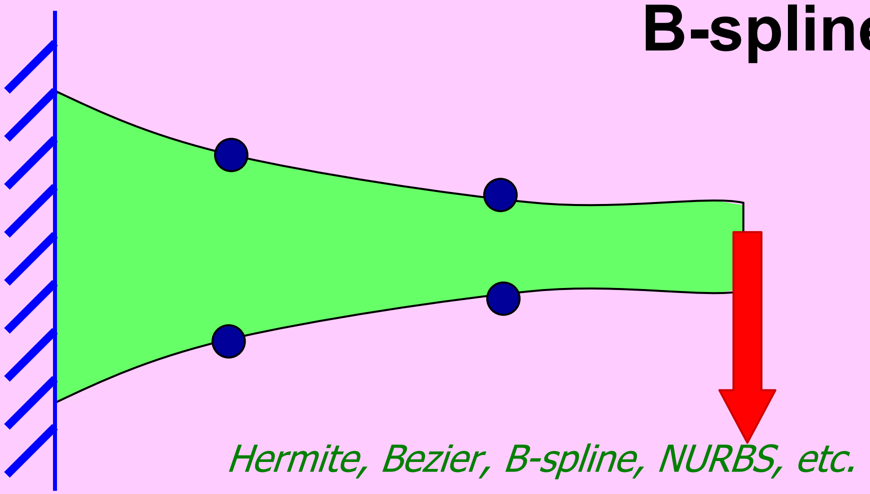


- Shape
  Topology } are given

- Optimize cross sections

Shape Optimization

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leq 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : control points of the B-spline

(position of each control point)

f(x) : compliance

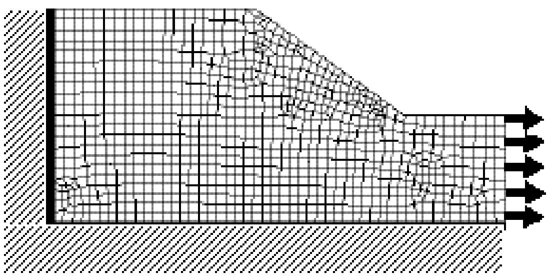
g(x) : mass

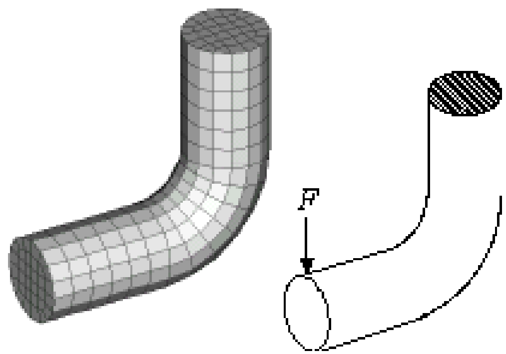
Number of design variables (ndv)

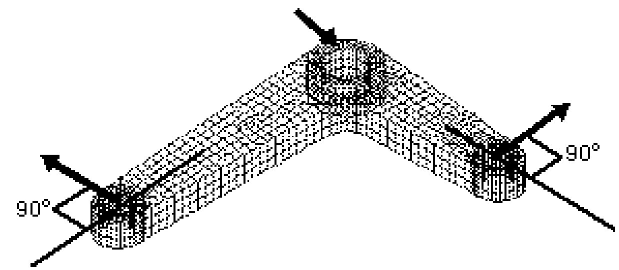
ndv = 8

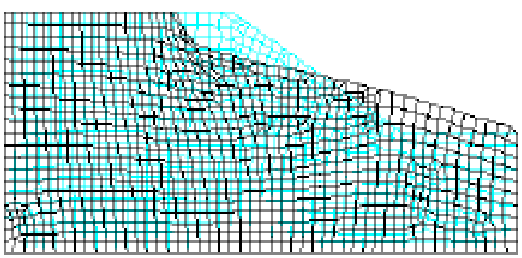
Shape Optimization

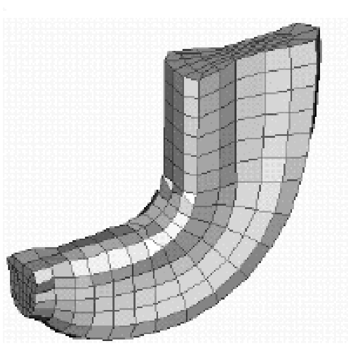
Fillet problem | **Hook problem** | **Arm problem**

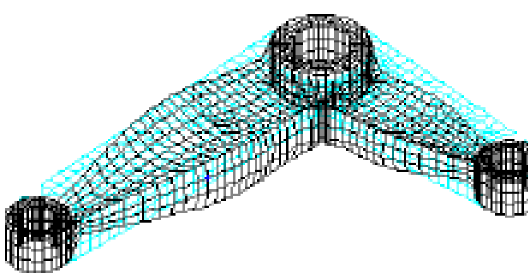












Shape Optimization

Multiobjective & Multidisciplinary Shape Optimization

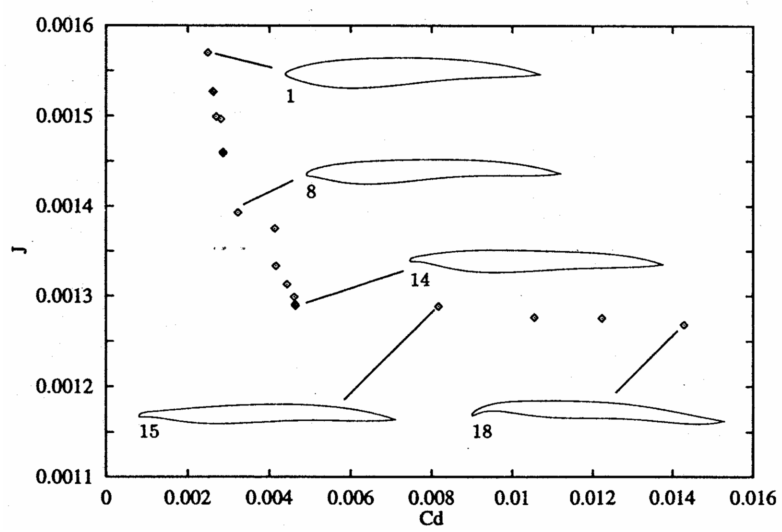
Objective function

1. Drag coefficient,   2. Amplitude of backscattered wave

Analysis

1. Computational Fluid Dynamics Analysis
2. Computational Electromagnetic Wave Field Analysis

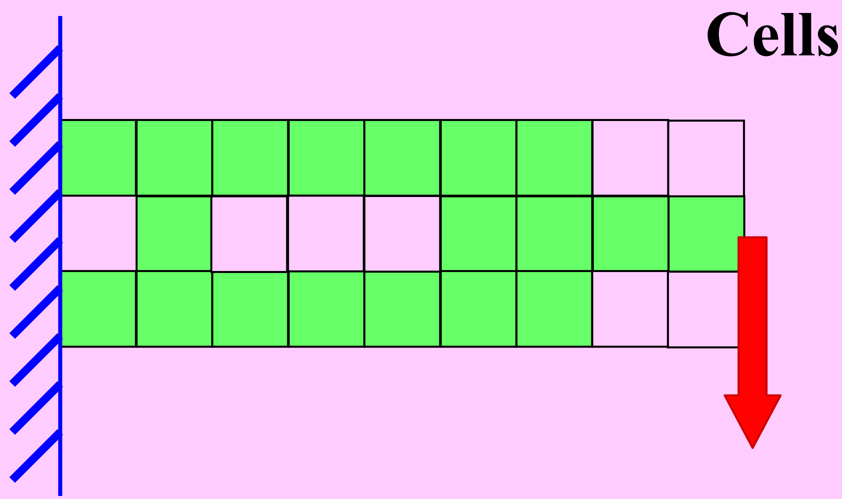
Obtain Pareto Front



Raino A.E. Makinen et al., "Multidisciplinary shape optimization in aerodynamics and electromagnetics using genetic algorithms," International Journal for Numerical Methods in Fluids, Vol. 30, pp. 149-159, 1999

Topology Optimization

$$\begin{aligned} &\text{minimize} && f(\mathbf{x}) \\ &\text{subject to} && g(\mathbf{x}) \le 0 \\ & && h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : density of each cell

Number of design variables (ndv)

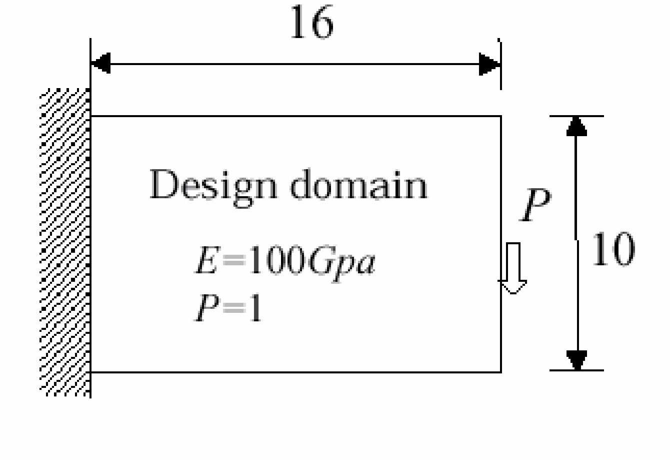
ndv = 27

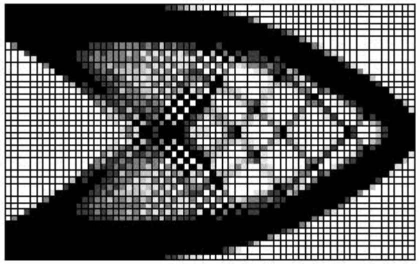
***f*(x) : compliance**

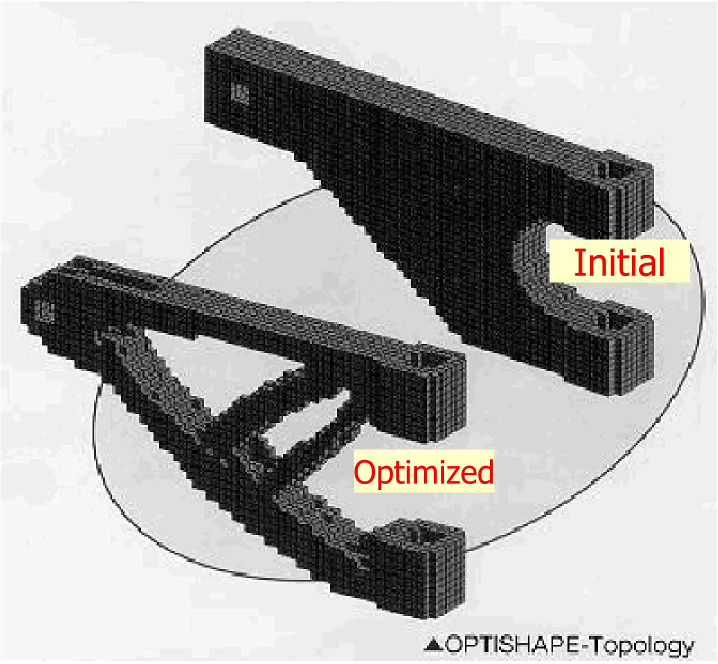
***g*(x) : mass**

Topology Optimization

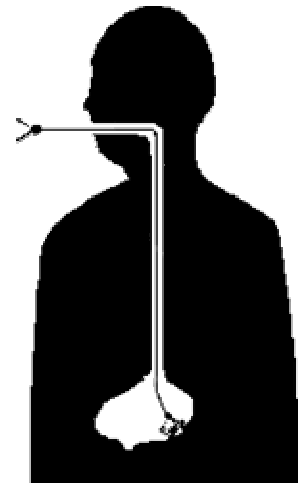
Short Cantilever problem

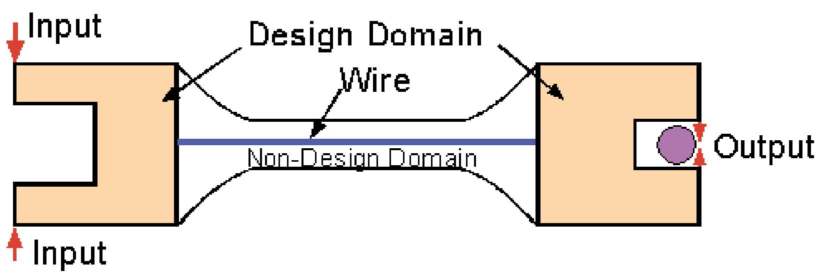


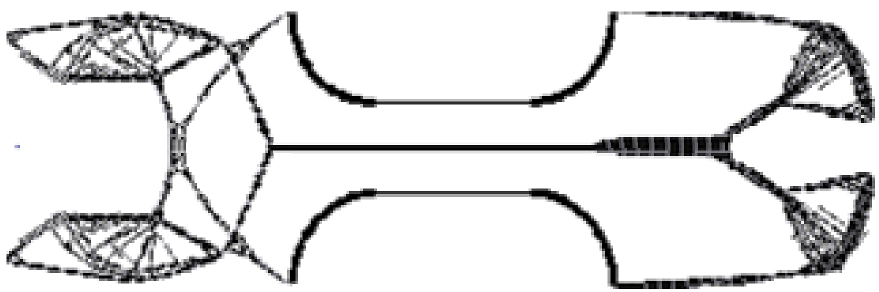




Topology Optimization

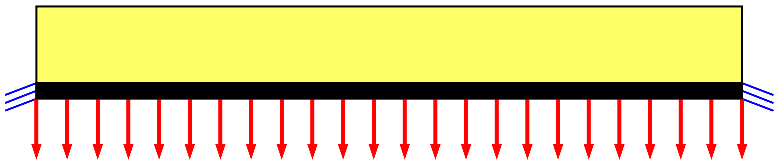






Topology Optimization

Bridge problem



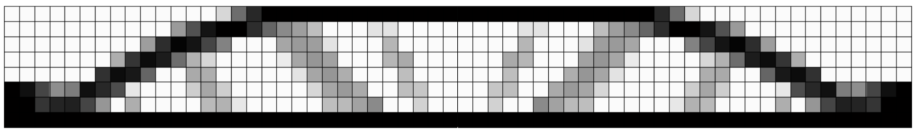
Distributed loading

$$\text{Minimize} \quad \int_{\Gamma} F^i z^i d\Gamma,$$

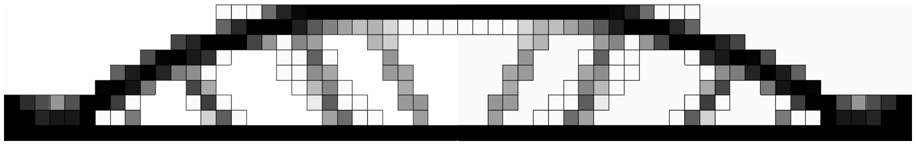
$$\text{Subject to} \quad \int_{\Omega} \rho(x) d\Omega \leq M_o,$$

$$0 \leq \rho(x) \leq 1$$

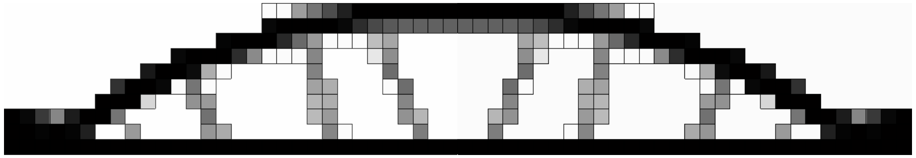
Mass constraints: 35%



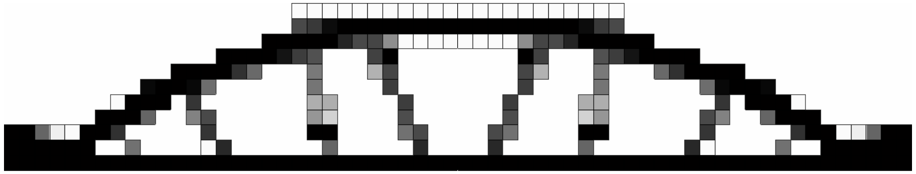
Obj = 4.16×10^5



Obj = 3.29×10^5



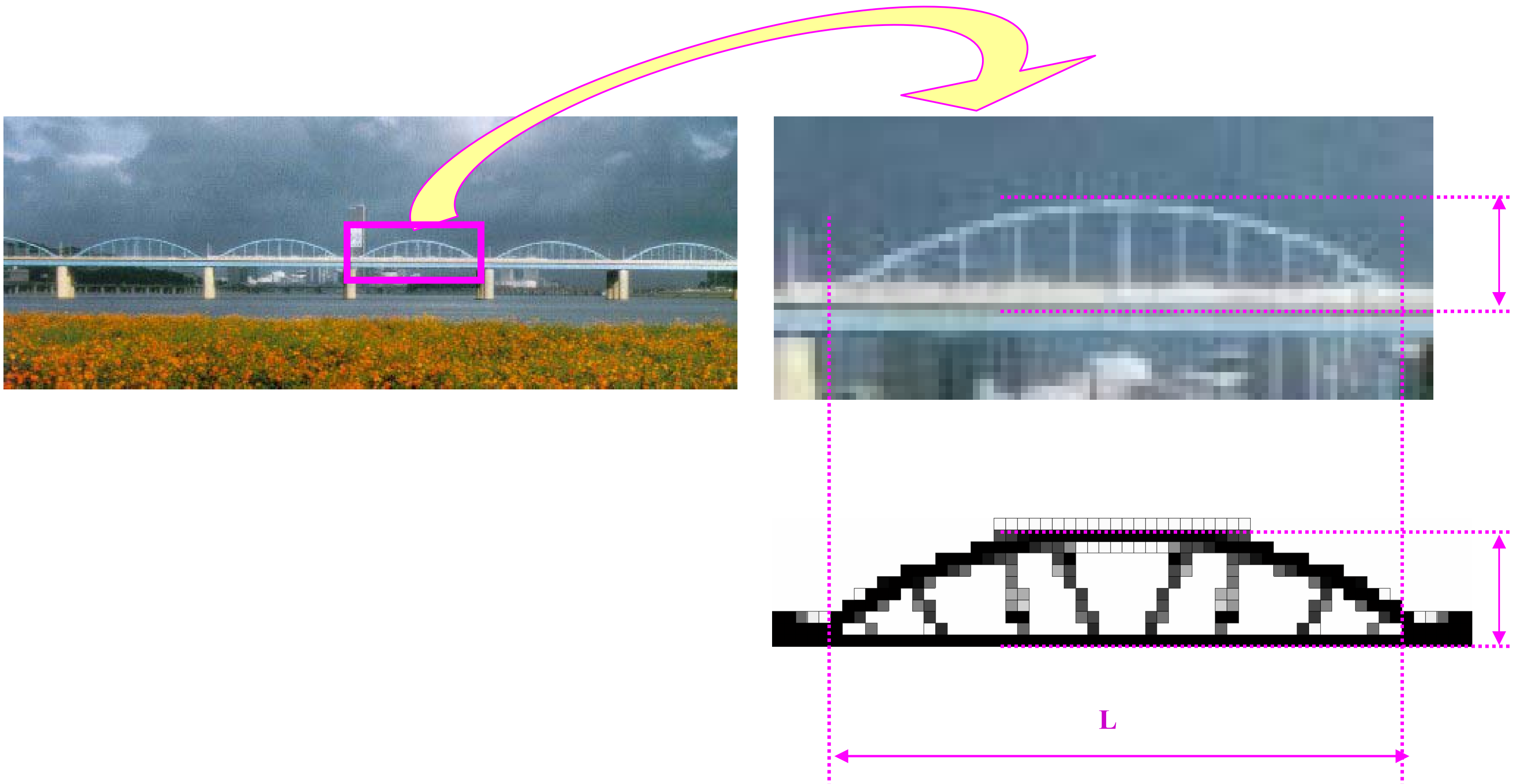
Obj = 2.88×10^5



Obj = 2.73×10^5

Topology Optimization

DongJak Bridge in Seoul, Korea



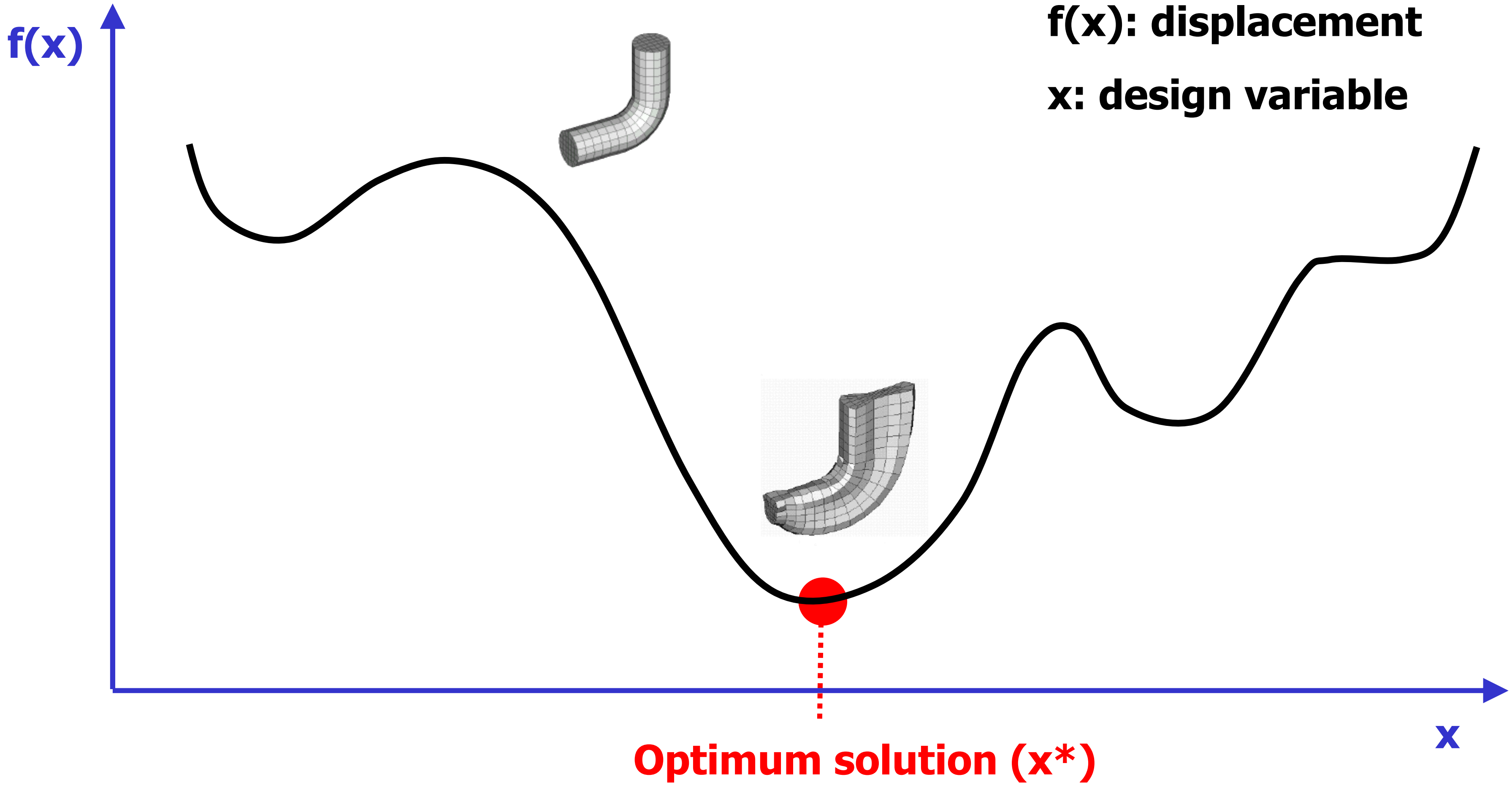
Structural Optimization

What determines the type of structural optimization?

Type of the design variable

(How to describe the design?)

Optimum Solution
– Graphical Representation

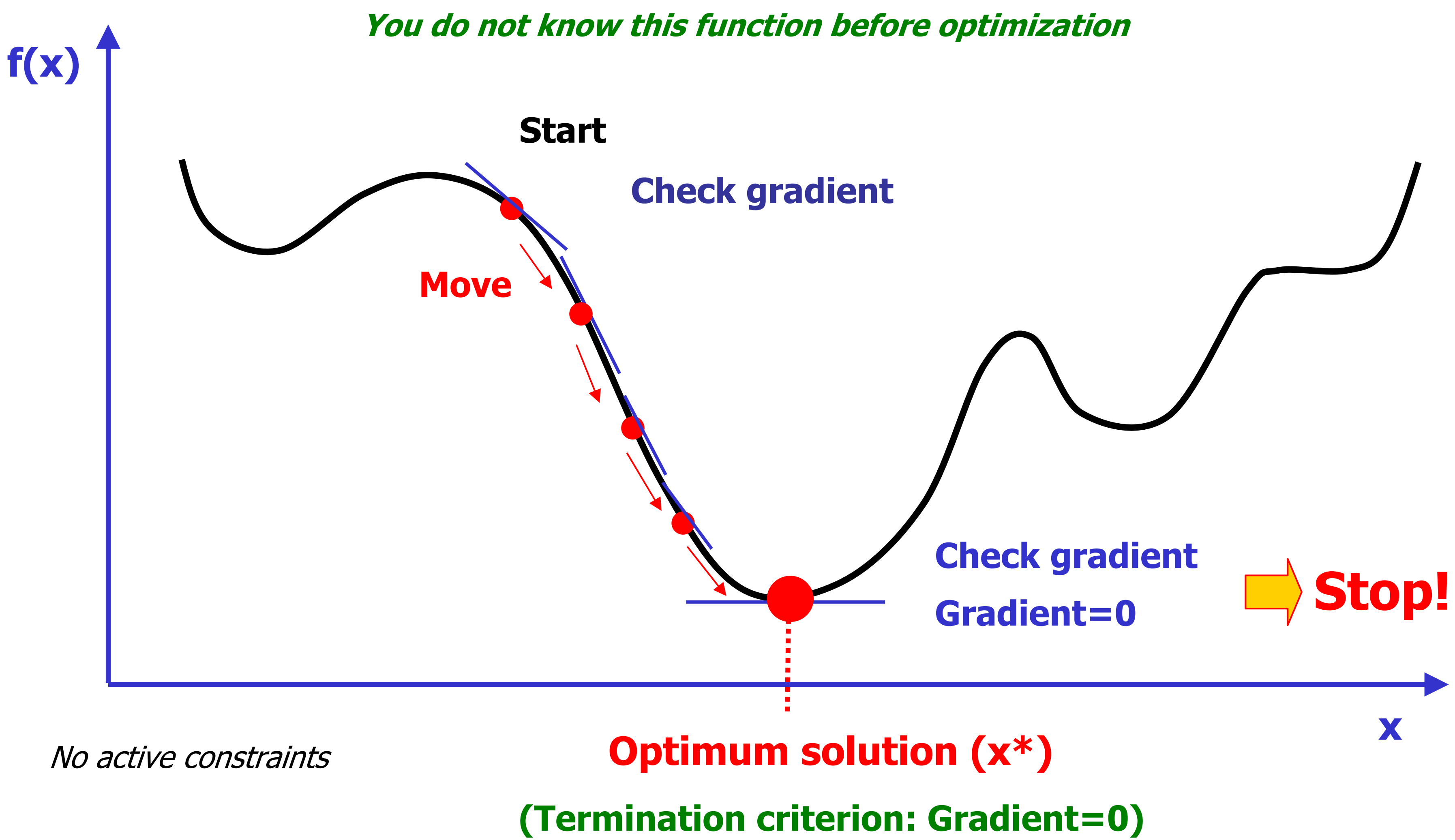


Optimization Methods

Gradient-based methods

Heuristic methods

Gradient-based Methods



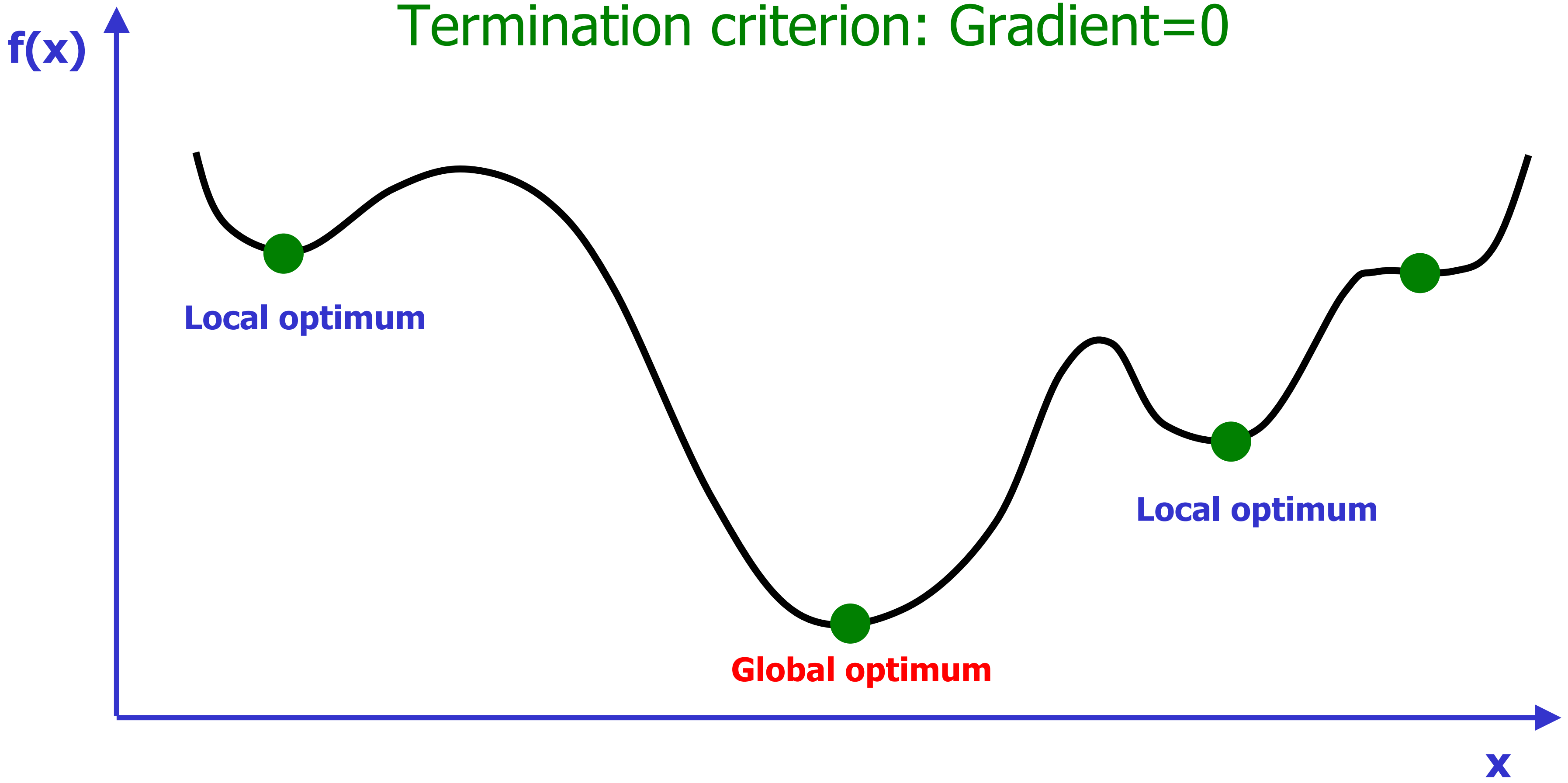
No active constraints

Gradient-based Methods

Steepest Descent Conjugate Gradient Quasi-Newton Newton	**UNCONSTRAINED**
Simplex – linear SLP – linear SQP – nonlinear, expensive, common in engineering applications Exterior Penalty – nonlinear, discontinuous design spaces Interior Penalty – nonlinear Generalized Reduced Gradient – nonlinear Method of Feasible Directions – nonlinear Mixed Integer Programming	**CONSTRAINED**

Global optimum vs. Local optimum

Termination criterion: Gradient=0



No active constraints

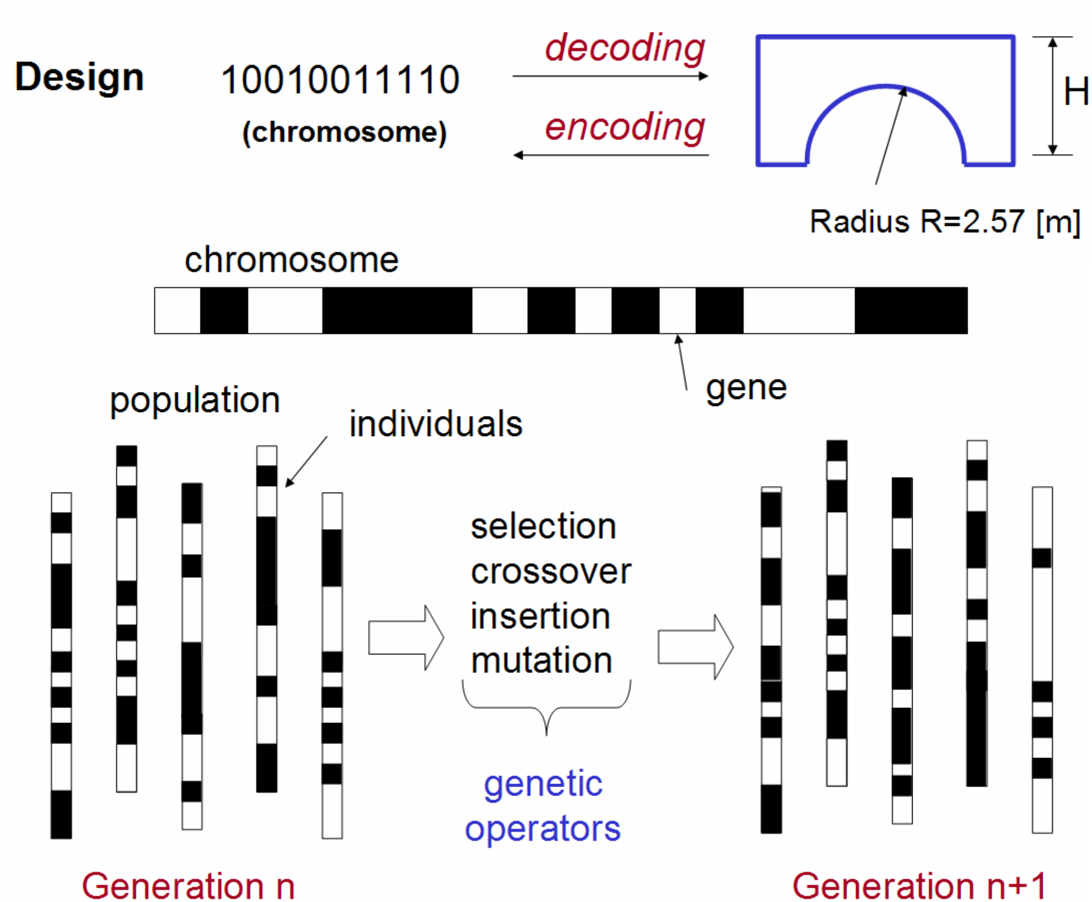
Heuristic Methods

- A **Heuristic** is simply a **rule of thumb** that hopefully will find a good answer.
- **Why** use a Heuristic?
  - Heuristics are typically used to solve **complex optimization problems** that are difficult to solve to optimality.
- Heuristics are **good at dealing with local optima** without getting stuck in them while searching for the global optimum.

Schulz, A.S., "Metaheuristics," 15.057 Systems Optimization Course Notes, MIT, 1999.

Genetic Algorithm

Principle by Charles Darwin - Natural Selection



Heuristic Methods

- Heuristics Often Incorporate Randomization

- **3 Most Common Heuristic Techniques**
  - Genetic Algorithms
  - Simulated Annealing
  - Tabu Search

Optimization Software

- **iSIGHT**
- **DOT**
- **Matlab (fmincon)**

Topology Optimization Software

- **ANSYS**

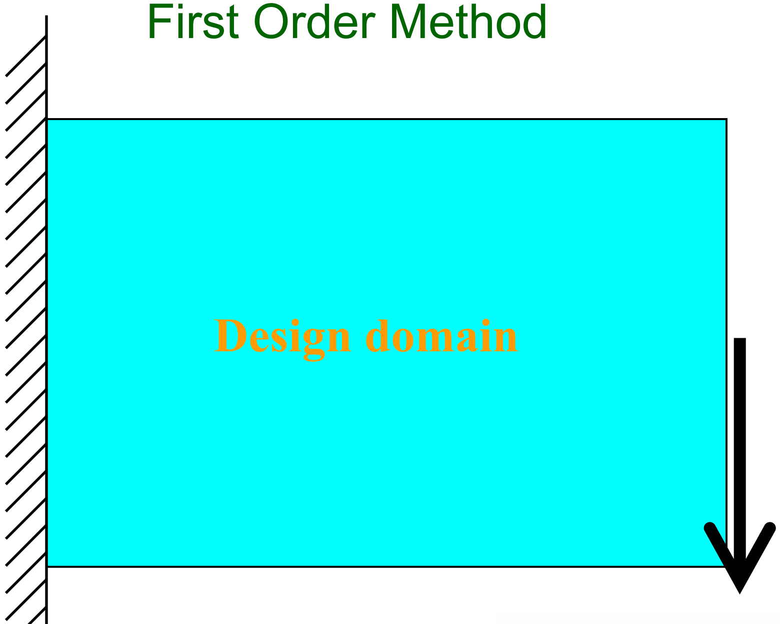
  Static Topology Optimization

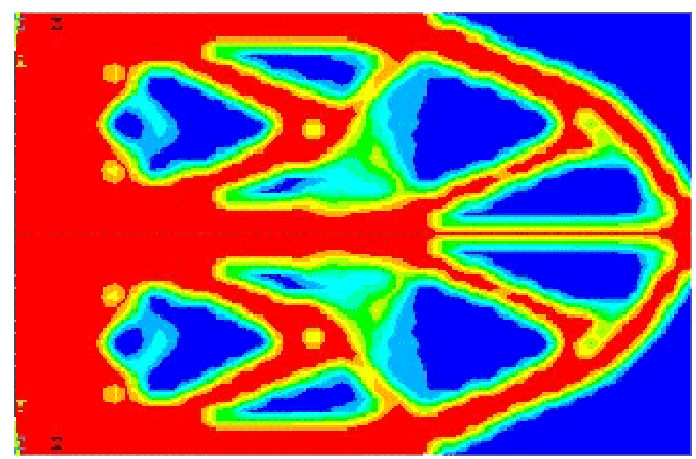
  Dynamic Topology Optimization

  Electromagnetic Topology Optimization

  Subproblem Approximation Method

  First Order Method



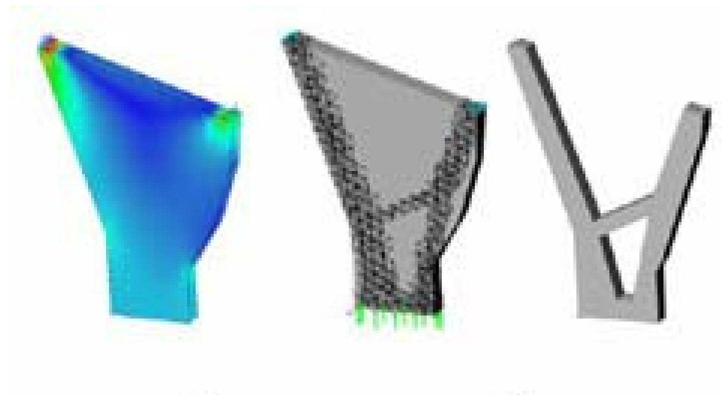


16.810 Topology Optimization Software

- **MSC. Visual Nastran FEA**

Elements of lowest stress are removed gradually.

Optimization results



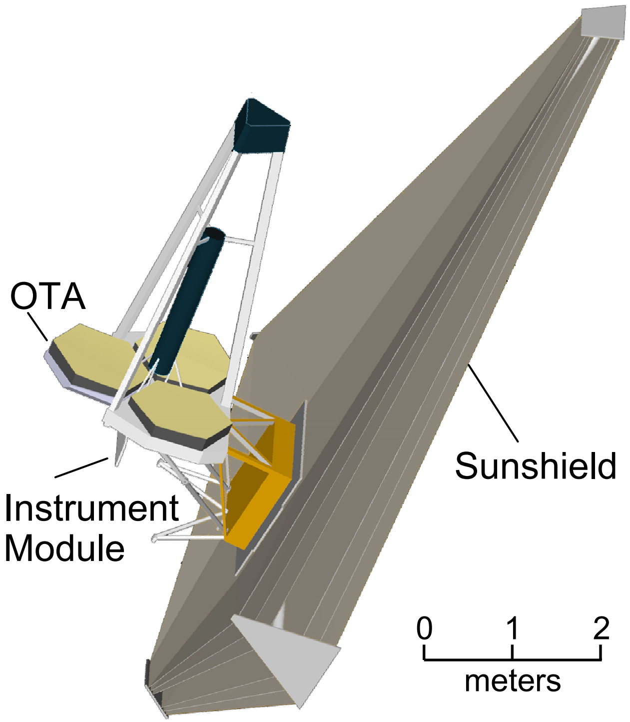
Optimization results illustration

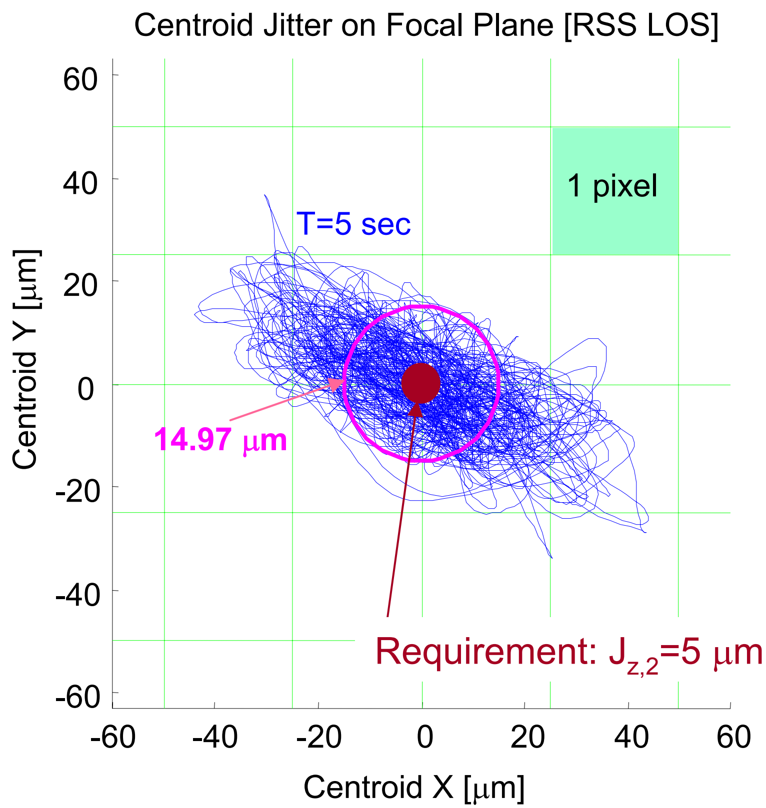
MDO

Multidisciplinary Design Optimization

16.810 Multidisciplinary Design Optimization

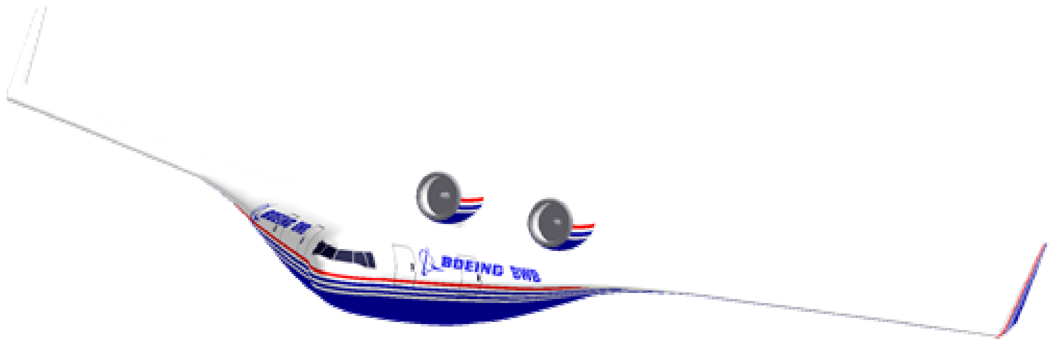
NASA Nexus Spacecraft Concept

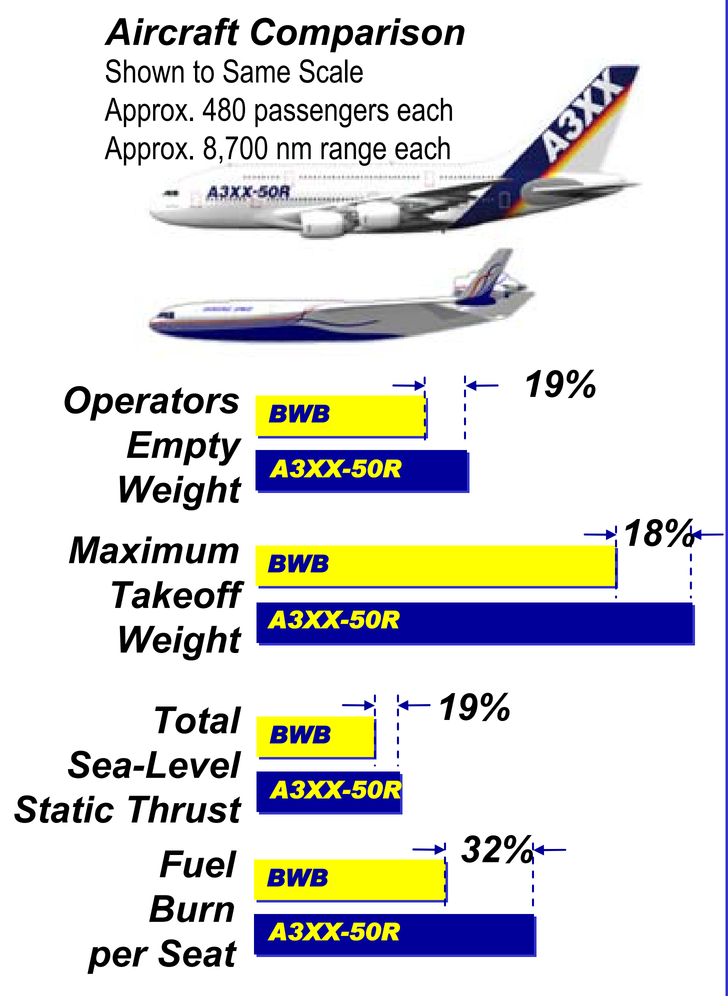




<u>Goal:</u> Find a "balanced" system design, where the flexible structure, the optics and the control systems work together to achieve a desired pointing performance, given various constraints

Boeing Blended Wing Body Concept

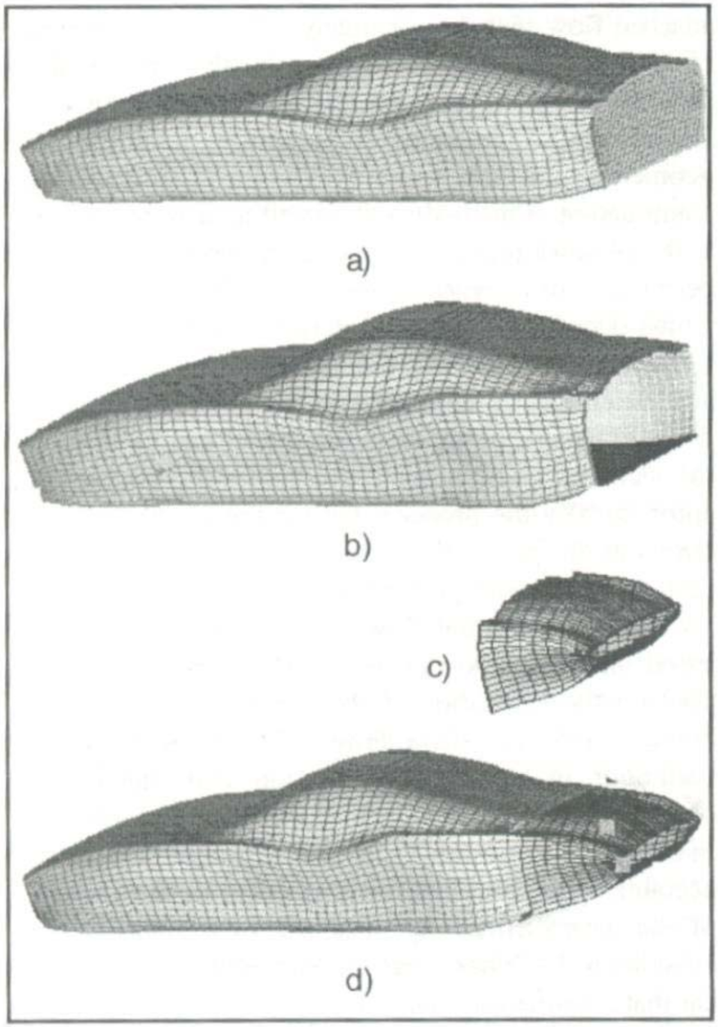




<u>Goal</u>: Find a design for a family of blended wing aircraft that will combine aerodynamics, structures, propulsion and controls such that a competitive system emerges - as measured by a set of operator metrics.

Multidisciplinary Design Optimization

Ferrari 360 Spider



<u>Goal:</u> High end vehicle shape optimization while improving car safety for fixed performance level and given geometric constraints

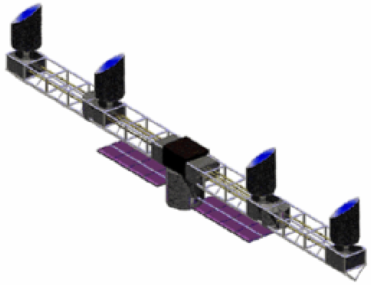
Reference: G. Lombardi, A. Vicere, H. Paap, G. Manacorda, "Optimized Aerodynamic Design for High Performance Cars", AIAA-98-4789, MAO Conference, St. Louis, 1998

16.810 Multidisciplinary Design Optimization

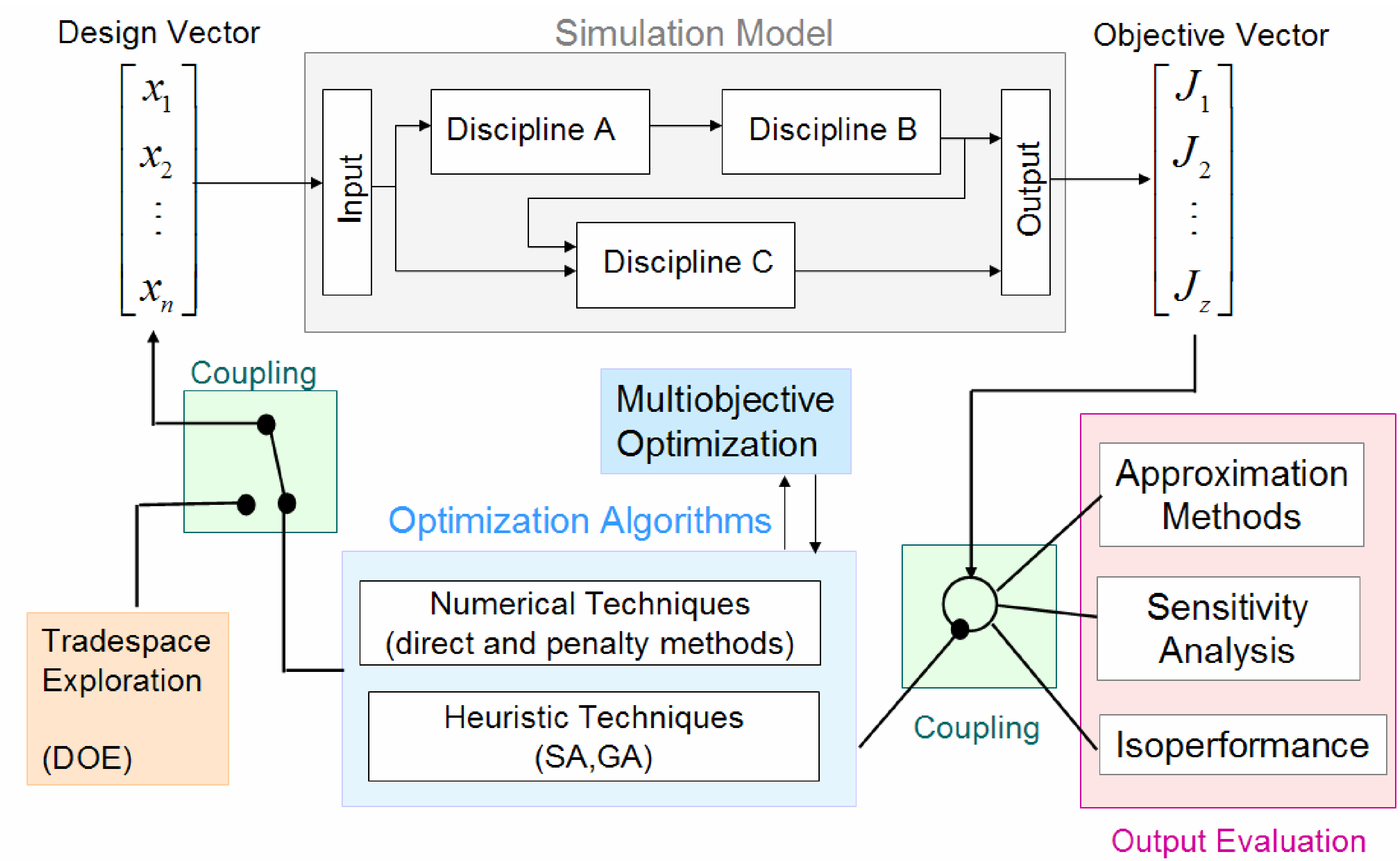
<u>Aircraft:</u>
Aerodynamics
Propulsion
Structures
Controls
Avionics/Software
Manufacturing
others

<u>Spacecraft:</u>
Astrodynamics
Thermodynamics
Communications
Payload & Sensor
Structures
Optics
Guidance & Control

<u>Automobiles:</u>
Engines
Body/chassis
Aerodynamics
Electronics
Hydraulics
Industrial design
others



16.810 Multidisciplinary Design Optimization



Do you want to learn more about MDO?

Take this course!

16.888/ESD.77

Multidisciplinary System Design Optimization (MSDO)

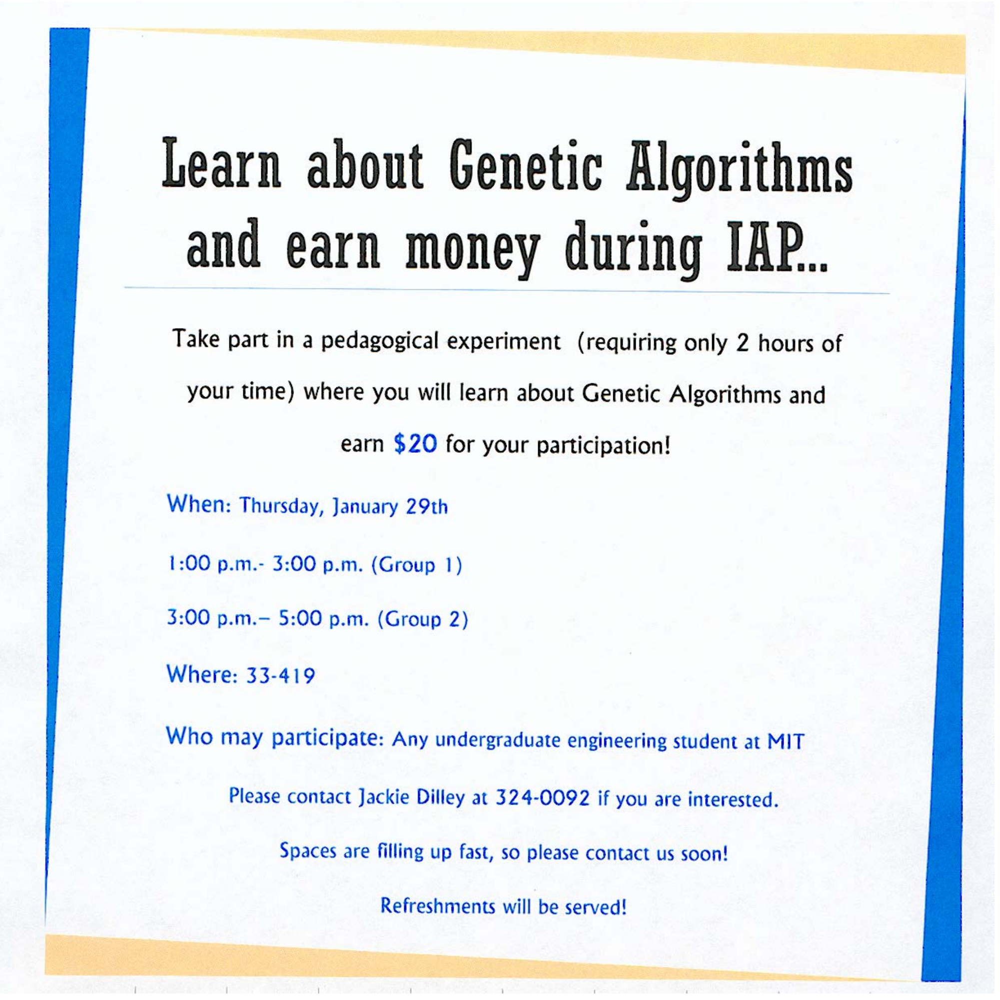
Prof. Olivier de Weck

Prof. Karen Willcox

Genetic Algorithm

Do you want to learn more about GA?

Take part in this GA game experiment!



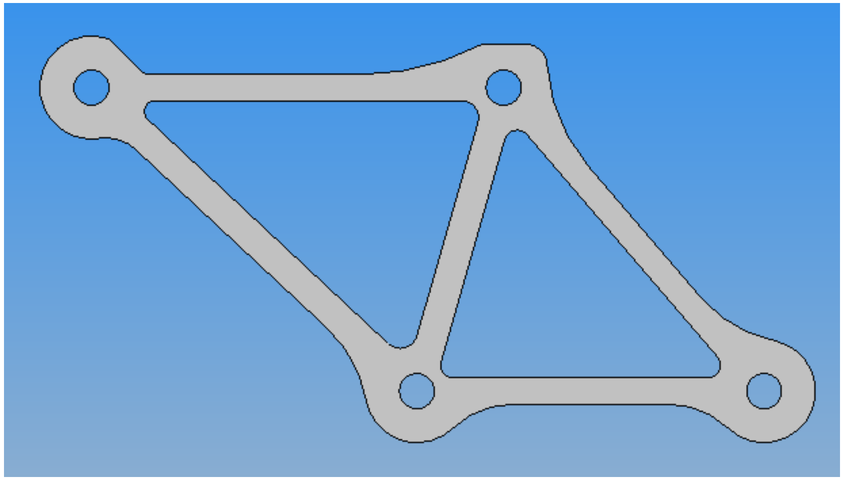
Baseline Design

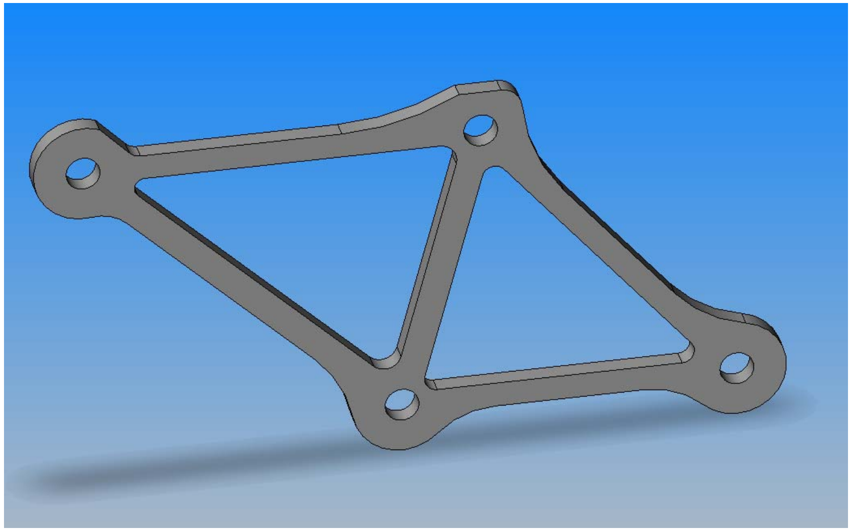
Performance

Natural frequency analysis

Design requirements

Baseline Design





Performance and cost

$$\delta_1 = 0.070\ mm$$

$$\delta_2 = 0.011\ mm$$

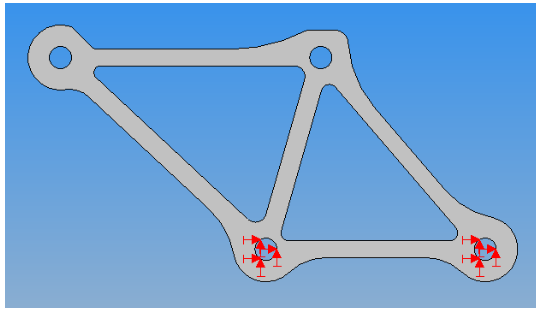
$$f = 245\ Hz$$

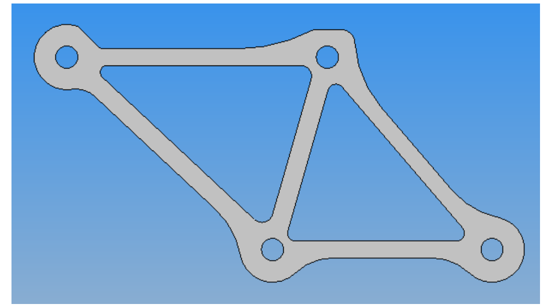
$$m = 0.224\ lbs$$

$$C = 5.16\ \$$$

Baseline Design

245 Hz



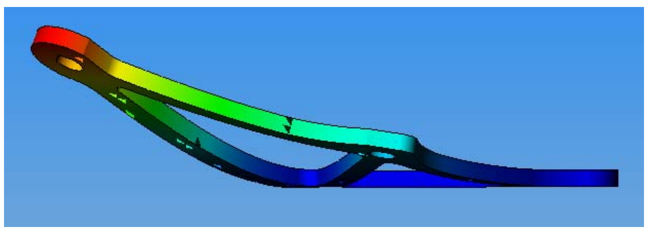


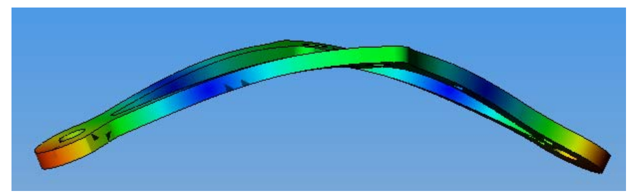
421 Hz

f1=245 Hz

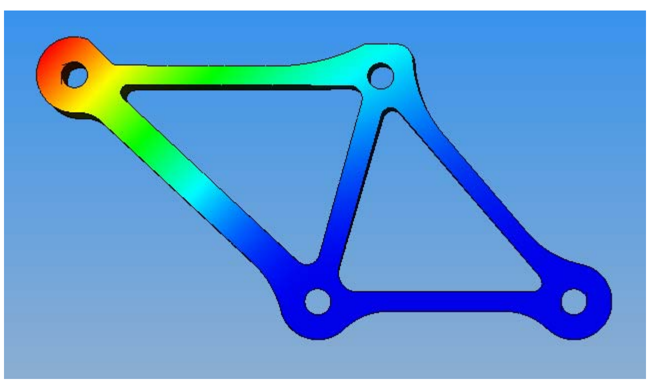
f2=490 Hz

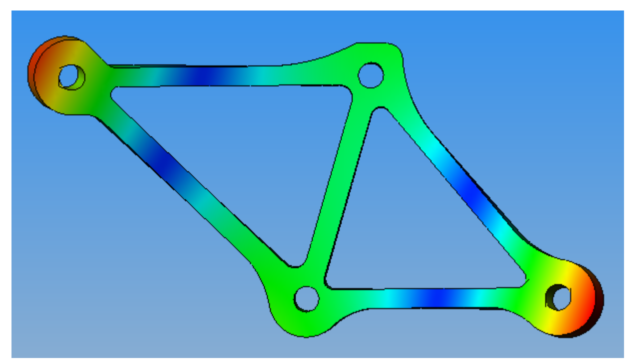
f3=1656 Hz





f1=0
f2=0
f3=0
f4=0
f5=0
f6=0
f7=421 Hz
f8=1284 Hz
f9=1310 Hz



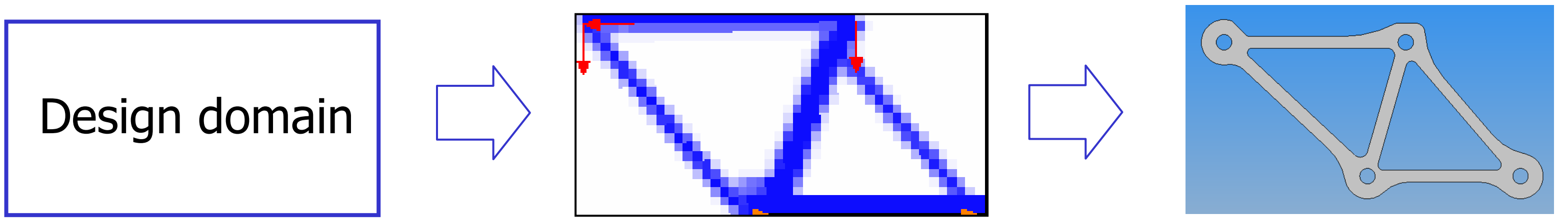


Design Requirement for Each Team

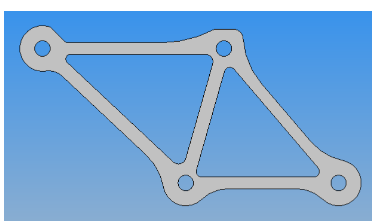
#	Product name	mass (m)	Cost (c)	Disp (δ1)	Disp (δ2)	Nat Freq (f)	Qual ity	F1 (lbs)	F2 (lbs)	F3 (lbs)	Const	Optim	Acc
0	Base line	0.224 lbs	5.16 $	0.070 mm	0.011 mm	245 Hz	3	50	50	100	c	m	δ1, δ2,f
1	Family economy	20%	-30%	10%	10%	-20%	2	50	50	100	c	m	δ1, δ2,f
2	Family deluxe	10%	-10%	-10%	-10%	10%	4	50	50	100	m	c	δ1, δ2,f
3	Cross over	20%	0%	-15%	-15%	20%	4	50	75	75	m	c	δ1, δ2,f
4	City bike	-20%	-20%	0%	0%	0%	3	50	75	75	c	m	δ1, δ2,f
5	Racing	-30%	50%	0%	0%	20%	5	100	100	50	m	δ1, δ2, f	c
6	Mountain	30%	30%	-20%	-20%	30%	4	50	100	50	δ1, δ2,f	m	c
7	BMX	0%	65%	-15%	-15%	40%	4	75	100	75	δ1, δ2,f	m	c
8	Acrobatic	-30%	100%	-10%	-10%	50%	5	100	100	100	δ1, δ2,f	m	c
9	Motor bike	50%	10%	-20%	-20%	0%	3	50	75	100	δ1, δ2,f	c	m

Design Optimization

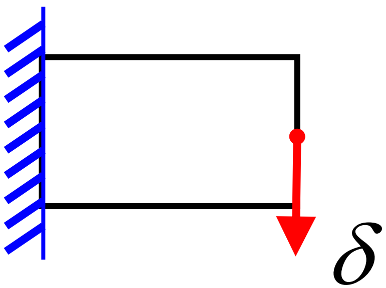
Topology optimization



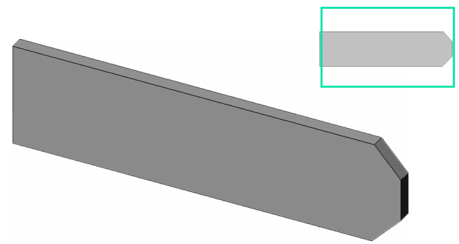
Shape optimization



Design Freedom

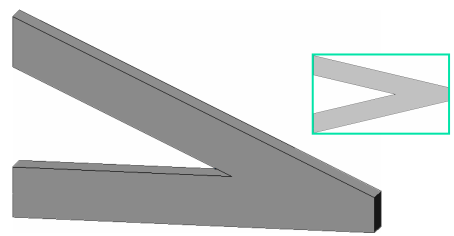


1 bar



$\delta = 2.50\ mm$

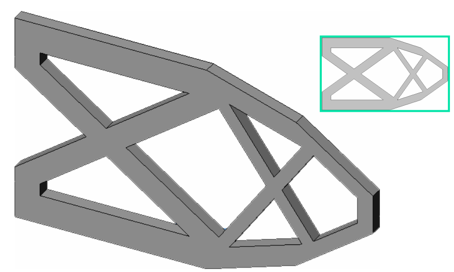
2 bars



$\delta = 0.80\ mm$

Volume is the same.

17 bars



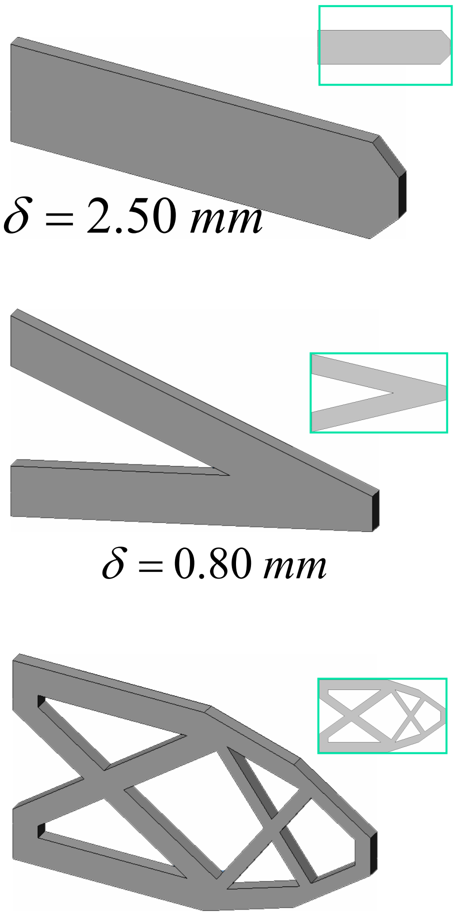
$\delta = 0.63\ mm$

Design Freedom

1 bar

2 bars

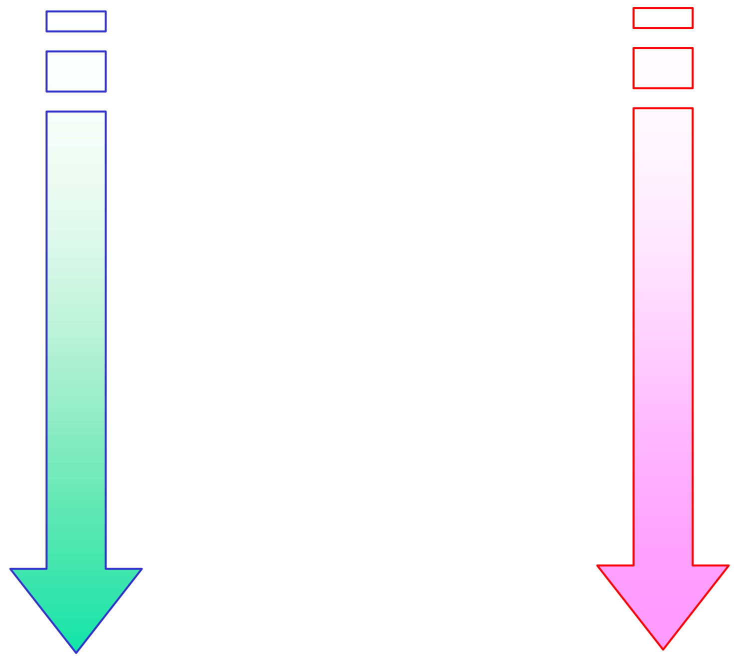
17 bars



$\delta = 2.50\ mm$

$\delta = 0.80\ mm$

$\delta = 0.63\ mm$



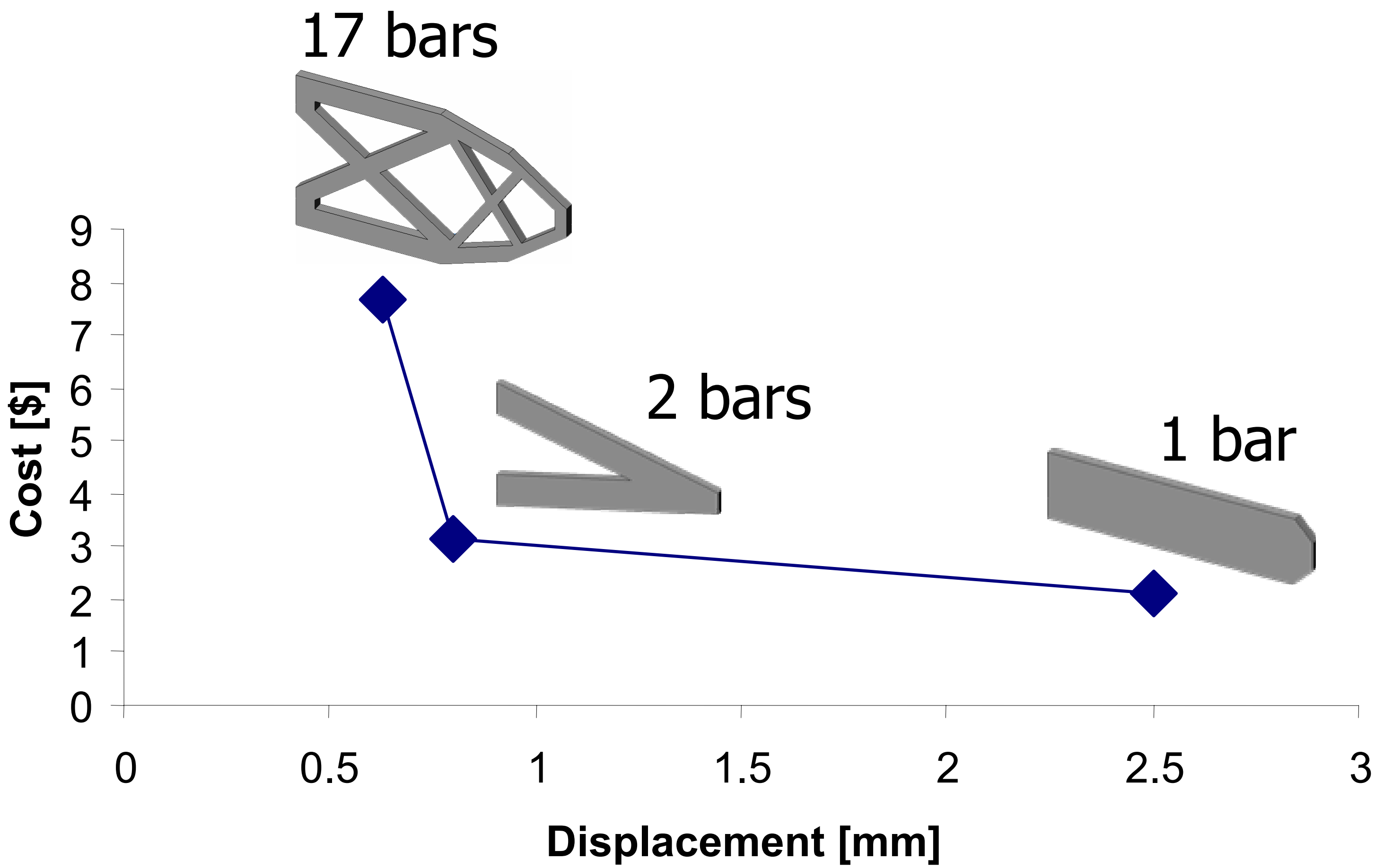
More design freedom

(Better performance)

More complex

(More difficult to optimize)

16.810 Cost versus Performance



Plan for the rest of the course

Class Survey

Jan 24 (Saturday) 7 am – Jan 26 (Monday) 11am

Company tour

Jan 26 (Monday) : 1 pm – 4 pm

Guest Lecture (Prof. Wilson, Bicycle Science)

Jan 28 (Wednesday) : 2 pm – 3:30 pm

Manufacturing Bicycle Frames (Version 2)

Jan 28 (Wednesday) : 9 am – 4:30 pm

Jan 29 (Thursday) : 9 am – 12 pm

Testing

Jan 29 (Thursday) : 10 am – 2 pm

GA Games

Jan 29 (Thursday) : 1 pm – 5 pm

Guest Lecture, Student Presentation (5~10 min/team)

Jan 30 (Friday) : 1 pm – 4 pm

References

P. Y. Papalambros, Principles of optimal design, Cambridge University Press, 2000

O. de Weck and K. Willcox, Multidisciplinary System Design Optimization, MIT lecture note, 2003

M. O. Bendsoe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," comp. Meth. Appl. Mech. Engng, Vol. 71, pp. 197-224, 1988

Raino A.E. Makinen et al., "Multidisciplinary shape optimization in aerodynamics and electromagnetics using genetic algorithms," International Journal for Numerical Methods in Fluids, Vol. 30, pp. 149-159, 1999

Il Yong Kim and Byung Man Kwak, "Design space optimization using a numerical design continuation method," International Journal for Numerical Methods in Engineering, Vol. 53, Issue 8, pp. 1979-2002, March 20, 2002.

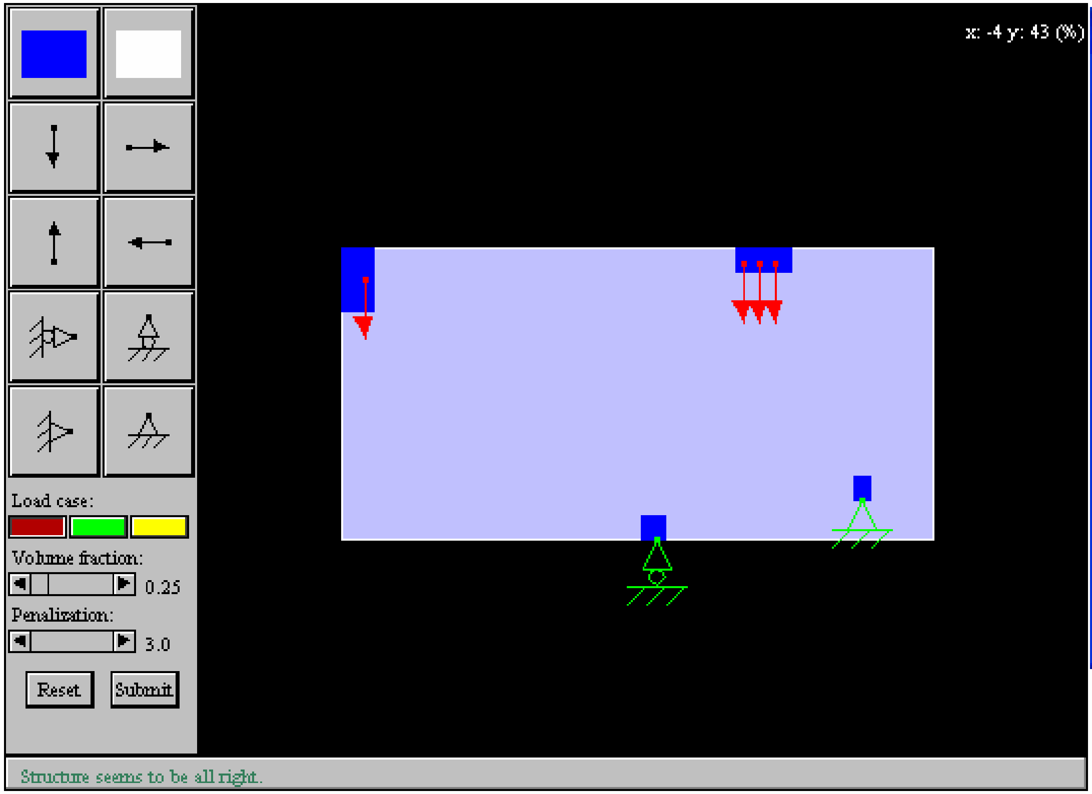
Web-based topology optimization program

Developed and maintained by Dmitri Tcherniak, Ole Sigmund, Thomas A. Poulsen and Thomas Buhl.

Features:

1. 2-D
2. Rectangular design domain
3. 1000 design variables (1000 square elements)
4. Objective function: compliance ($F \times \delta$)
5. Constraint: volume

Web-based topology optimization program



Objective function

-Compliance ($F \times \delta$)

Constraint

-Volume

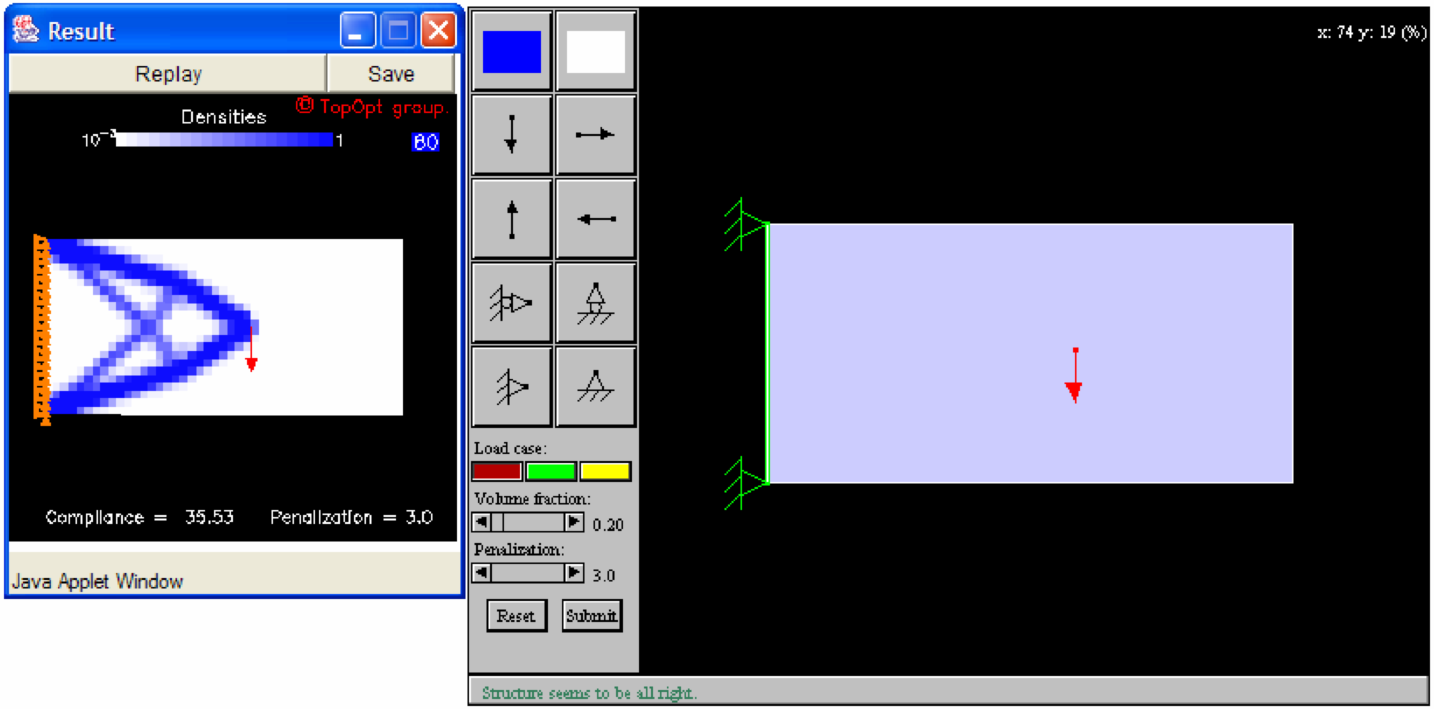
Design variables

- Density of each design cell

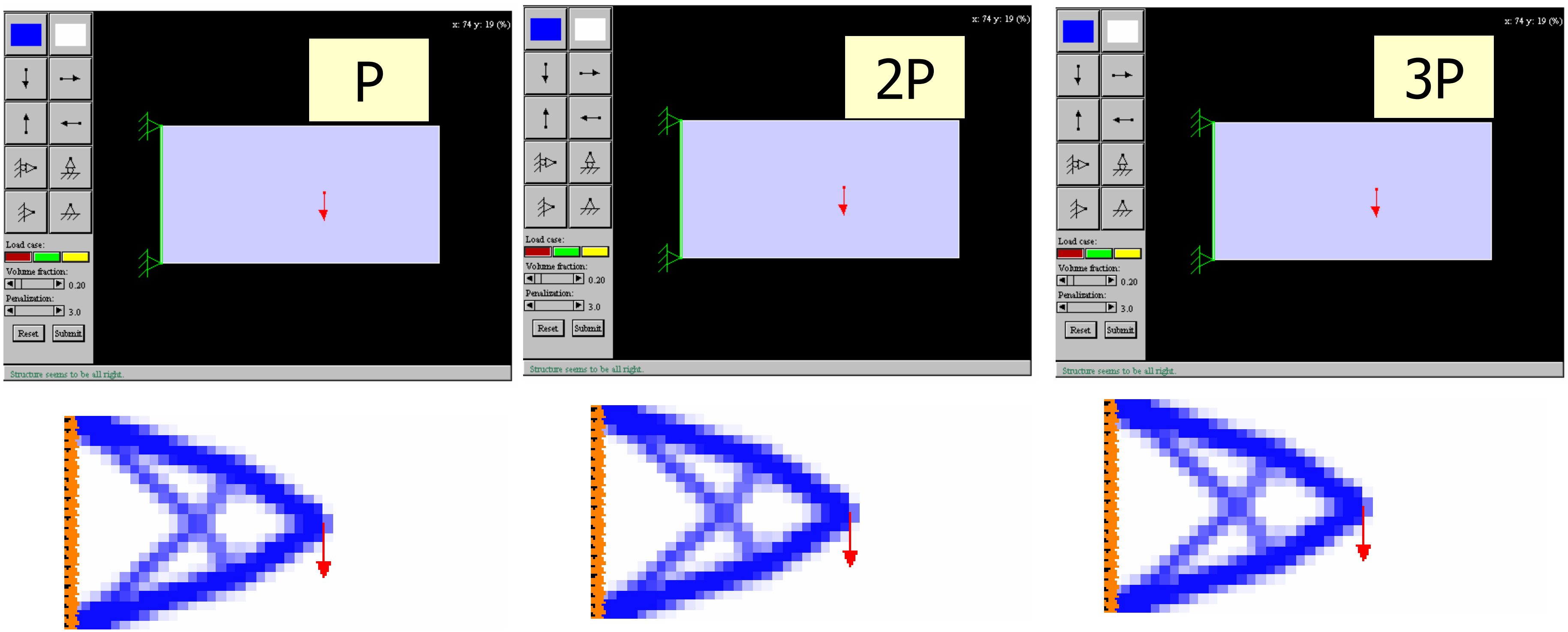
Web-based topology optimization program

No numerical results are obtained.

Optimum layout is obtained.



Web-based topology optimization program



Absolute magnitude of load does not affect optimum solution

Web-based topology optimization program

http://www.topopt.dtu.dk