Course Concept

Phase 1
- Problem statement
- Sketch by hand
- CAD
- CAE
- Rapid Prototyping / Validation
  - Manufacturing / Test

Phase 2
- Design Optimization
- Optimum solution
- Rapid Prototyping / Validation
  - Manufacturing / Test

Massachusetts Institute of Technology
Course Flow Diagram

Learning/Review
- Design Intro
- CAD/CAM/CAE Intro
- FEM/Solid Mechanics Overview
- Manufacturing Training
- Structural Test “Training”
- Design Optimization

Problem statement
- Hand sketching
- CAD design
- FEM analysis
- Produce Part 1
- Test
- Optimization
- Produce Part 2
- Test
- Final Review

Deliverables
- Design Sketch v1
- Drawing v1
- Analysis output v1
- Part v1
- Experiment data v1
- Design/Analysis output v2
- Part v2
- Experiment data v2

Today
Wednesday
What Is Design Optimization?

Selecting the “best” design within the available means

1. What is our criterion for “best” design?  
   Objective function

2. What are the available means?  
   Constraints  
   (design requirements)

3. How do we describe different designs?  
   Design Variables
Minimize $f(x)$
Subject to $g(x) \leq 0$
$h(x) = 0$
## Constraints

### - Design requirements

#### 2. Requirements

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Constrain</th>
<th>Optimize</th>
<th>Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance ($\delta_1$, $\delta_2$, $f_1$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Displacement $\delta_1 \leq 0.078 \text{ mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Displacement $\delta_2 \leq 0.012 \text{ mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>First natural frequency $f_1 \geq 195 \text{ Hz}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass ($m$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m \leq 0.27 \text{ lbs}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface Quality ($Q$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q \geq 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load Case ($F$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_1 = 50 \text{ lbs} / F_2 = 50 \text{ lbs} / F_3 = 100 \text{ lbs}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The part has to conform to the interface requirements and geometrical boundary conditions shown on page 2 of this document. This requirement cannot be waived.

#### 3. Priorities

Low manufacturing cost is the first priority for this product. Next, the customer cares about light-weighting (low mass) and thirdly, structural performance should be as high as possible. These priorities are shown in the Ishii-matrix below:

<table>
<thead>
<tr>
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<th>Constrain</th>
<th>Optimize</th>
<th>Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Objective Function

- A criterion for best design (or goodness of a design)

2. Requirements

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Constraint</th>
<th>Optimize</th>
<th>Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>□</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>Performance</td>
<td>□</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>Mass</td>
<td>□</td>
<td>□</td>
<td>□</td>
</tr>
</tbody>
</table>

Objective function
Design Variables

Parameters that are chosen to describe the design of a system

Design variables are “controlled” by the designers

The position of upper holes along the design freedom line
For computational design optimization,

Objective function and constraints must be expressed as a function of design variables (or design vector X)

Objective function: $f(x)$

Constraints: $g(x), h(x)$

Cost = $f(\text{design})$

Displacement = $f(\text{design})$

Natural frequency = $f(\text{design})$

Mass = $f(\text{design})$

What is “f” for each case?
Minimize $f(x)$

Subject to $g(x) \leq 0$

$h(x) = 0$

$f(x)$ : Objective function to be minimized
$g(x)$ : Inequality constraints
$h(x)$ : Equality constraints
$x$ : Design variables
Optimization Procedure

Minimize \( f(x) \)
Subject to \( g(x) \leq 0 \)
\( h(x) = 0 \)

Start

Determine an initial design \( (x_0) \)

Evaluate \( f(x), g(x), h(x) \)

Does your design meet a termination criterion?

N

Y

Converge?

Improve Design

Computer Simulation

Change \( x \)

END
Structural Optimization

Selecting the best “structural” design

- Size Optimization
- Shape Optimization
- Topology Optimization
minimize \( f(x) \)
subject to \( g(x) \leq 0 \)
\( h(x) = 0 \)

1. To make the structure strong
   e.g. Minimize displacement at the tip

2. Total mass \( \leq M_C \)

\( \text{Min. } f(x) \)
\( g(x) \leq 0 \)
Size Optimization

minimize \( f(x) \)
subject to \( g(x) \leq 0 \)
\( h(x) = 0 \)

Design variables \((x)\)
\( x : \) thickness of each beam

Number of design variables \((ndv)\)
\( ndv = 5 \)

\( f(x) : \) compliance
\( g(x) : \) mass
Size Optimization

- Shape
- Topology
- Optimize cross sections
Shape Optimization

Design variables (x)

- \( f(x) \): compliance
- \( g(x) \): mass
- \( h(x) = 0 \)

Number of design variables (ndv)

\( ndv = 8 \)

\[ \text{minimize } f(x) \]
\[ \text{subject to } g(x) \leq 0 \]
\[ h(x) = 0 \]

B-spline

Hermite, Bezier, B-spline, NURBS, etc.

\( x \): control points of the B-spline
(position of each control point)
Shape Optimization

Fillet problem

Hook problem

Arm problem
Multiobjective & Multidisciplinary Shape Optimization

Objective function
1. Drag coefficient, 2. Amplitude of backscattered wave

Analysis
1. Computational Fluid Dynamics Analysis
2. Computational Electromagnetic Wave Field Analysis

Obtain Pareto Front

Topology Optimization

Design variables (x)

- \( x \) : density of each cell

Number of design variables (ndv)

- \( ndv = 27 \)

minimize \( f(x) \)
subject to \( g(x) \leq 0 \)
\( h(x) = 0 \)

\( f(x) \) : compliance
\( g(x) \) : mass
Topology Optimization

Short Cantilever problem

Design domain

$E = 100 \text{Gpa}$
$P = 1$

$P$

Initial

Optimized

OPTISHAPE-Topology
Topology Optimization
Topography Optimization

Bridge problem

Minimize \( \int_{\Gamma} F^{i} z^{i} d\Gamma \),

Subject to \( \int_{\Omega} \rho(x) d\Omega \leq M_{o} \),

\( 0 \leq \rho(x) \leq 1 \)

Mass constraints: 35%

Obj = 4.16\times10^{5}

Obj = 3.29\times10^{5}

Obj = 2.88\times10^{5}

Obj = 2.73\times10^{5}
Topology Optimization

DongJak Bridge in Seoul, Korea
What determines the type of structural optimization?

Type of the design variable

*(How to describe the design?)*
Optimum Solution
– Graphical Representation

$f(x)$: displacement
$x$: design variable

Optimum solution ($x^*$)
Optimization Methods

Gradient-based methods

Heuristic methods
Gradient-based Methods

You do not know this function before optimization

No active constraints

Optimum solution \((x^*)\)

(Termination criterion: Gradient=0)
### Gradient-based Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steepest Descent</td>
<td>UNCONSTRAINED</td>
</tr>
<tr>
<td>Conjugate Gradient</td>
<td></td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td></td>
</tr>
<tr>
<td>Newton</td>
<td></td>
</tr>
<tr>
<td>Simplex – linear</td>
<td>CONSTRAINED</td>
</tr>
<tr>
<td>SLP – linear</td>
<td></td>
</tr>
<tr>
<td>SQP – nonlinear, expensive, common in engineering applications</td>
<td></td>
</tr>
<tr>
<td>Exterior Penalty – nonlinear, discontinuous design spaces</td>
<td></td>
</tr>
<tr>
<td>Interior Penalty – nonlinear</td>
<td></td>
</tr>
<tr>
<td>Generalized Reduced Gradient – nonlinear</td>
<td></td>
</tr>
<tr>
<td>Method of Feasible Directions – nonlinear</td>
<td></td>
</tr>
<tr>
<td>Mixed Integer Programming</td>
<td></td>
</tr>
</tbody>
</table>
Global optimum vs. Local optimum

Termination criterion: Gradient=0

No active constraints
Heuristic Methods

- A **Heuristic** is simply a **rule of thumb** that hopefully will find a good answer.

- **Why** use a Heuristic?
  - Heuristics are typically used to solve **complex optimization problems** that are difficult to solve to optimality.
  
  Heuristics are **good at dealing with local optima** without getting stuck in them while searching for the global optimum.

Genetic Algorithm

Principle by Charles Darwin - Natural Selection

Design 10010011110 (chromosome) → decoding → encoding

chromosome

population individuals gene

Generation n selection crossover insertion mutation genetic operators Generation n+1

Radius R=2.57 [m]
Heuristic Methods

- Heuristics Often Incorporate Randomization

- **3 Most Common Heuristic Techniques**
  - Genetic Algorithms
  - Simulated Annealing
  - Tabu Search
Optimization Software

- iSIGHT
- DOT
- Matlab (fmincon)
Topology Optimization Software

- ANSYS
  - Static Topology Optimization
  - Dynamic Topology Optimization
  - Electromagnetic Topology Optimization

Subproblem Approximation Method

First Order Method

Design domain
Topology Optimization Software

- MSC. Visual Nastran FEA

Elements of lowest stress are removed gradually.

Optimization results

Optimization results illustration
Multidisciplinary Design Optimization
Goal: Find a “balanced” system design, where the flexible structure, the optics and the control systems work together to achieve a desired pointing performance, given various constraints.
Boeing Blended Wing Body Concept

Goal: Find a design for a family of blended wing aircraft that will combine aerodynamics, structures, propulsion and controls such that a competitive system emerges - as measured by a set of operator metrics.
Goal: High end vehicle shape optimization while improving car safety for fixed performance level and given geometric constraints

Multidisciplinary Design Optimization

**Aircraft:**
- Aerodynamics
- Propulsion
- Structures
- Controls
- Avionics/Software
- Manufacturing
- others

**Spacecraft:**
- Astrodynamics
- Thermodynamics
- Communications
- Payload & Sensor
- Structures
- Optics
- Guidance & Control

**Automobiles:**
- Engines
- Body/chassis
- Aerodynamics
- Electronics
- Hydraulics
- Industrial design
- others
Multidisciplinary Design Optimization

Design Vector

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{bmatrix}
\]

Simulation Model

Input

Discipline A → Discipline B → Discipline C

Output

Objective Vector

\[
\begin{bmatrix}
    J_1 \\
    J_2 \\
    \vdots \\
    J_z
\end{bmatrix}
\]

Coupling

Optimization Algorithms

Multiobjective Optimization

Numerical Techniques (direct and penalty methods)

Heuristic Techniques (SA, GA)

Approximation Methods

Sensitivity Analysis

Isoperformance

Output Evaluation

Tradespace Exploration (DOE)
Do you want to learn more about MDO?

Take this course!

16.888/ESD.77

Multidisciplinary System Design Optimization (MSDO)

Prof. Olivier de Weck
Prof. Karen Willcox
Do you want to learn more about GA?

Take part in this GA game experiment!

Learn about Genetic Algorithms and earn money during IAP...

Take part in a pedagogical experiment (requiring only 2 hours of your time) where you will learn about Genetic Algorithms and earn $20 for your participation!

When: Thursday, January 29th
1:00 p.m. - 3:00 p.m. (Group 1)
3:00 p.m. - 5:00 p.m. (Group 2)

Where: 33-419

Who may participate: Any undergraduate engineering student at MIT

Please contact Jackie Diley at 324-0092 if you are interested.

Spaces are filling up fast, so please contact us soon!

Refreshments will be served!
Baseline Design

Performance

Natural frequency analysis

Design requirements
Baseline Design

Performance and cost

\[ \delta_1 = 0.070 \text{ mm} \]
\[ \delta_2 = 0.011 \text{ mm} \]
\[ f = 245 \text{ Hz} \]
\[ m = 0.224 \text{ lbs} \]
\[ C = 5.16 \text{ $} \]
Baseline Design

245 Hz

f1 = 245 Hz
f2 = 490 Hz
f3 = 1656 Hz

421 Hz

f1 = 0
f2 = 0
f3 = 0
f4 = 0
f5 = 0
f6 = 0
f7 = 421 Hz
f8 = 1284 Hz
f9 = 1310 Hz
## Design Requirement for Each Team

<table>
<thead>
<tr>
<th>#</th>
<th>Product name</th>
<th>mass (m)</th>
<th>Cost (c)</th>
<th>Disp (δ1)</th>
<th>Disp (δ2)</th>
<th>Nat Freq (f)</th>
<th>Quality</th>
<th>F1 (lbs)</th>
<th>F2 (lbs)</th>
<th>F3 (lbs)</th>
<th>Const</th>
<th>Optim</th>
<th>Acc</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Baseline</td>
<td>0.224 lbs</td>
<td>5.16 $</td>
<td>0.070 mm</td>
<td>0.011 mm</td>
<td>245 Hz</td>
<td>3</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>c</td>
<td>m</td>
<td>δ1, δ2, f</td>
</tr>
<tr>
<td>1</td>
<td>Family economy</td>
<td>20%</td>
<td>-30%</td>
<td>10%</td>
<td>10%</td>
<td>-20%</td>
<td>2</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>c</td>
<td>m</td>
<td>δ1, δ2, f</td>
</tr>
<tr>
<td>2</td>
<td>Family deluxe</td>
<td>10%</td>
<td>-10%</td>
<td>-10%</td>
<td>-10%</td>
<td>10%</td>
<td>4</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>m</td>
<td>c</td>
<td>δ1, δ2, f</td>
</tr>
<tr>
<td>3</td>
<td>Cross over</td>
<td>20%</td>
<td>0%</td>
<td>-15%</td>
<td>-15%</td>
<td>20%</td>
<td>4</td>
<td>50</td>
<td>75</td>
<td>75</td>
<td>m</td>
<td>c</td>
<td>δ1, δ2, f</td>
</tr>
<tr>
<td>4</td>
<td>City bike</td>
<td>-20%</td>
<td>-20%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>3</td>
<td>50</td>
<td>75</td>
<td>75</td>
<td>c</td>
<td>m</td>
<td>δ1, δ2, f</td>
</tr>
<tr>
<td>5</td>
<td>Racing</td>
<td>-30%</td>
<td>50%</td>
<td>0%</td>
<td>0%</td>
<td>20%</td>
<td>5</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>m</td>
<td>δ1, δ2, f</td>
<td>c</td>
</tr>
<tr>
<td>6</td>
<td>Mountain</td>
<td>30%</td>
<td>30%</td>
<td>-20%</td>
<td>-20%</td>
<td>30%</td>
<td>4</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>δ1, δ2, f</td>
<td>m</td>
<td>c</td>
</tr>
<tr>
<td>7</td>
<td>BMX</td>
<td>0%</td>
<td>65%</td>
<td>-15%</td>
<td>-15%</td>
<td>40%</td>
<td>4</td>
<td>75</td>
<td>100</td>
<td>75</td>
<td>δ1, δ2, f</td>
<td>m</td>
<td>c</td>
</tr>
<tr>
<td>8</td>
<td>Acrobatic</td>
<td>-30%</td>
<td>100%</td>
<td>-10%</td>
<td>-10%</td>
<td>50%</td>
<td>5</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>δ1, δ2, f</td>
<td>m</td>
<td>c</td>
</tr>
<tr>
<td>9</td>
<td>Motor bike</td>
<td>50%</td>
<td>10%</td>
<td>-20%</td>
<td>-20%</td>
<td>0%</td>
<td>3</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>δ1, δ2, f</td>
<td>c</td>
<td>m</td>
</tr>
</tbody>
</table>
Design Optimization

Topology optimization

Design domain

Shape optimization
Design Freedom

Volume is the same.

1 bar

$\delta = 2.50 \, mm$

2 bars

$\delta = 0.80 \, mm$

17 bars

$\delta = 0.63 \, mm$
Design Freedom

1 bar
\[ \delta = 2.50 \text{ mm} \]

2 bars
\[ \delta = 0.80 \text{ mm} \]

17 bars
\[ \delta = 0.63 \text{ mm} \]

More design freedom (Better performance)

More complex (More difficult to optimize)
Cost versus Performance

Displacement [mm]

Cost [$]

0 1 2 3 4 5 6 7 8 9
0 0.5 1 1.5 2 2.5 3

1 bar
2 bars
17 bars
Plan for the rest of the course

Class Survey
Jan 24 (Saturday) 7 am – Jan 26 (Monday) 11am

Company tour
Jan 26 (Monday) : 1 pm – 4 pm

Guest Lecture (Prof. Wilson, Bicycle Science)
Jan 28 (Wednesday) : 2 pm – 3:30 pm

Manufacturing Bicycle Frames (Version 2)
Jan 28 (Wednesday) : 9 am – 4:30 pm
Jan 29 (Thursday) : 9 am – 12 pm

Testing
Jan 29 (Thursday) : 10 am – 2 pm

GA Games
Jan 29 (Thursday) : 1 pm – 5 pm

Guest Lecture, Student Presentation (5~10 min/team)
Jan 30 (Friday) : 1 pm – 4 pm
P. Y. Papalambros, Principles of optimal design, Cambridge University Press, 2000


Web-based topology optimization program

Developed and maintained by Dmitri Tcherniak, Ole Sigmund, Thomas A. Poulsen and Thomas Buhl.

Features:

1. 2-D
2. Rectangular design domain
3. 1000 design variables (1000 square elements)
4. Objective function: compliance ($F \times \delta$)
5. Constraint: volume
Web-based topology optimization program

Objective function
- Compliance ($F \times \delta$)

Constraint
- Volume

Design variables
- Density of each design cell
No numerical results are obtained.

Optimum layout is obtained.
Web-based topology optimization program

Absolute magnitude of load does not affect optimum solution
Web-based topology optimization program

http://www.topopt.dtu.dk