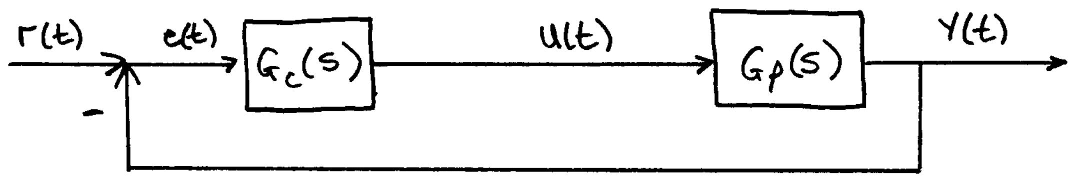
16.82 - Fall 2003

Digital Implementation of Controllers

DIGITAL CONTROL - PART 1

- OUR CONTROL PICTURE SO FAR:

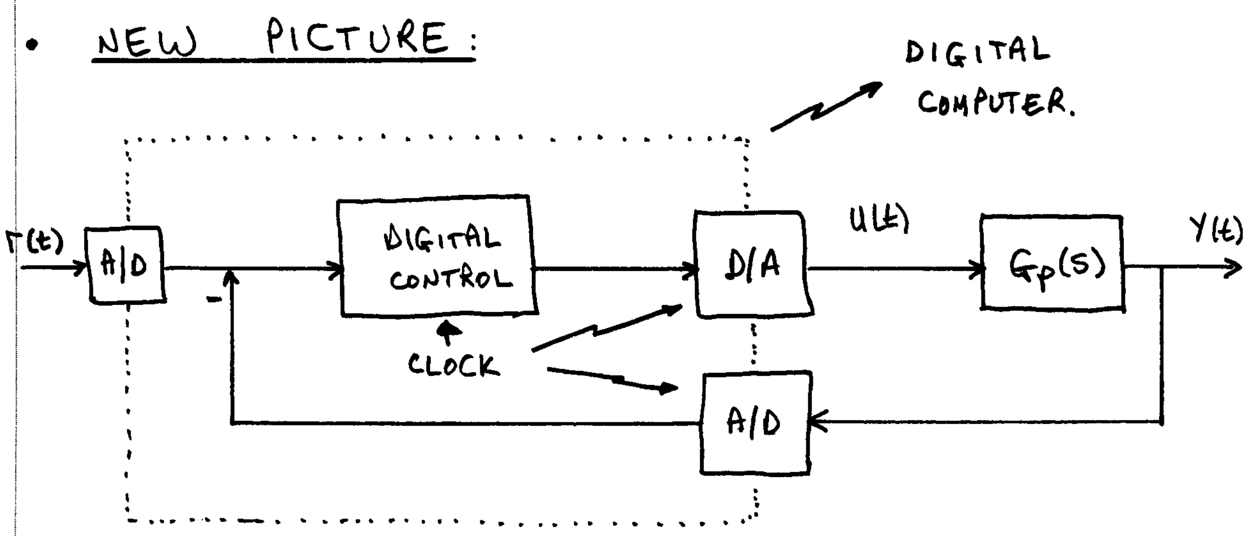


- YOU CAN IMPLEMENT THIS CONTROLLER USING ANALOG CIRCUITS - EXAMPLES IN BOOK

  ⇒ EXCELLENT CHOICE FOR VERY HIGHBANDWITH CONTROLLERS.

- MANY ADVANTAGES IF WE IMPLEMENT $G_c(s)$ USING A DIGITAL COMPUTER.
  - INCREASED FLEXIBILITY (EASIER TO MODIFY)
  - CAN EASILY INCLUDE LOGIC
  - EASIER TO HANDLE NONLINEARITIES.

- NEW PICTURE:



DIGITAL CONTROL MECHANICS

- DIGITAL / DISCRETE CONTROL RUNS ON A CLOCK

  ⇒ ONLY USES THE SIGNALS AT DISCRETE INSTANTS IN TIME.

  - CONTINUOUS $r(t)$ SAMPLED AT FIXED PERIODS IN TIME $\quad r(KT_s)$

    T_s - SAMPLING PERIOD (FIXED)

    K - INTEGER.

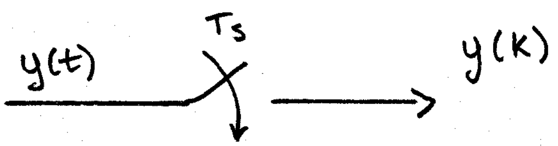
- MUST ALSO GET INFORMATION INTO AND OUT OF COMPUTER → A/D, D/A

- A/D - 2 STEPS

  i) CONVERT PHYSICAL SIGNAL (VOLTAGE) TO A BINARY NUMBER

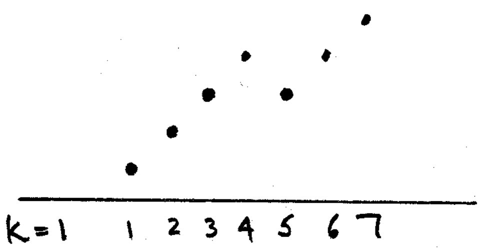
     - AN APPROXIMATION SINCE WE TYPICALLY ONLY HAVE 12-16 BITS TO COVER ±10V RANGE.

  ii) SAMPLE CONTINUOUS SIGNAL $y(t)$ EVERY T_s SECONDS $\quad y(t) \rightarrow y(K)$



  - SAMPLER CLEARLY IGNORES MUCH OF CTS SIGNAL $y(t)$.

- **D/A – 2 STEPS AS WELL**
  1. BINARY TO ANALOG
  2. DISCRETE (ONLY AT KT_s) TO CONTINUOUS.



$u(K) \rightarrow \bullet$

$u(t)$?

EASIEST WAY IS TO JUST HOLD $u(K)$ FOR PERIOD T_s

- CALLED A <u>ZERO-ORDER HOLD</u> (ZOH) SINCE FUNCTION HELD WITH A ZEROTH ORDER POLYNOMIAL.

- WE MUST DETERMINE WHAT IMPACT THIS "SAMPLE AND HOLD" WILL HAVE ON THE LOOP TRANSFER FUNCTION

- SET $r(t) = 0$, CONTROL LAW $= 1$ $[u(K) = y(K)]$

  A/D $\rightarrow$ SAMPLE

  D/A $\rightarrow$ HOLD (ZOH)

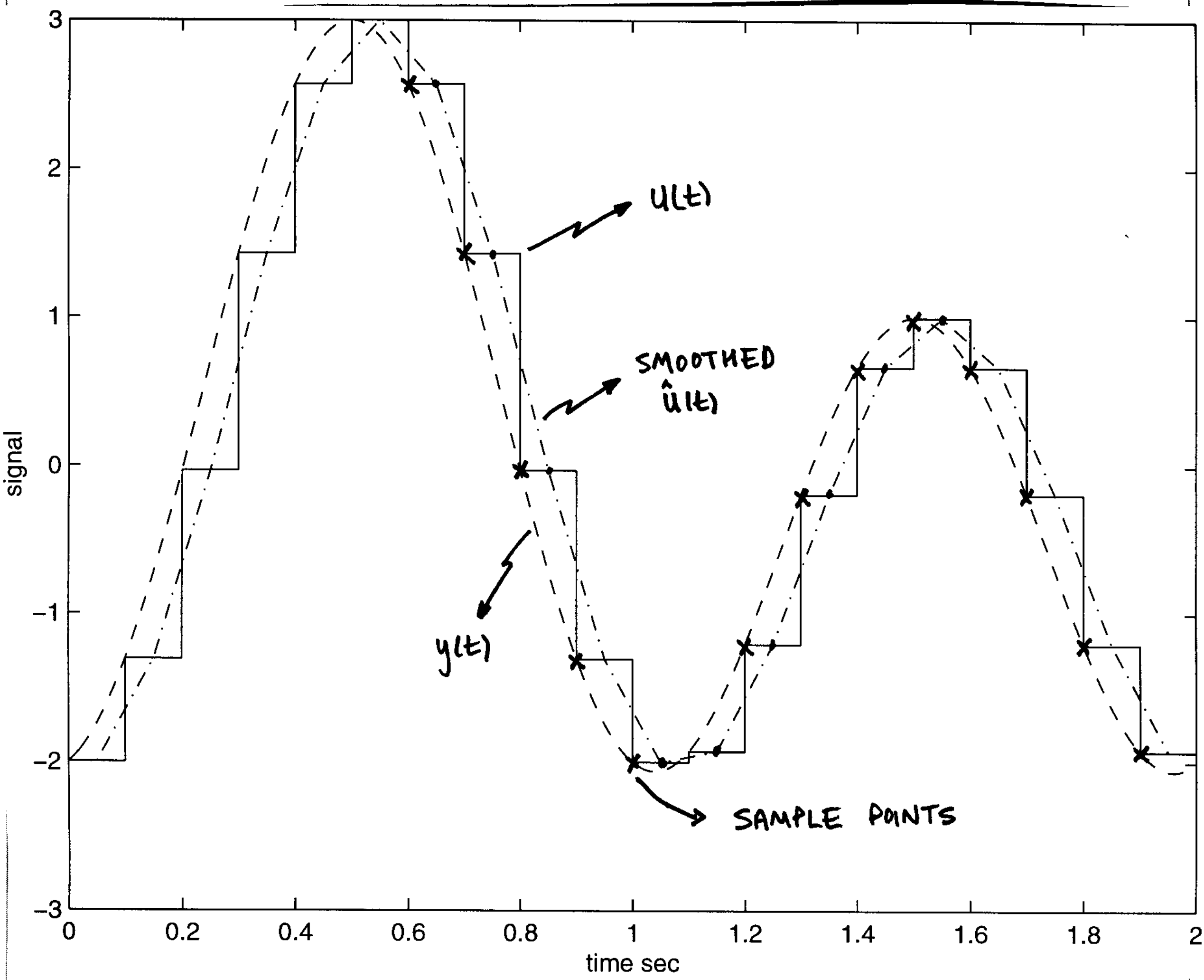
$y(t) \longrightarrow$ A/D $\longrightarrow$ CONTROL $\longrightarrow$ D/A $\longrightarrow u(t)$

$y(t) \longrightarrow \overset{T_s}{\diagup} \xrightarrow{u(K) = y(K)}$ ZOH $\longrightarrow u(t)$

- **SAMPLE AND HOLD**

$$y(t) \longrightarrow \diagup \longrightarrow \text{ZOH} \longrightarrow u(t)$$

- MOST BASIC PIECE OF DIGITAL CONTROL ANALYSIS
- CAN INVESTIGATE TRANSFER FUNCTION $\frac{U(s)}{Y(s)}$ ANALYTICALLY.
- ALSO INSIGHTFUL TO LOOK AT THE CHANGE TO BASIC SIGNALS.

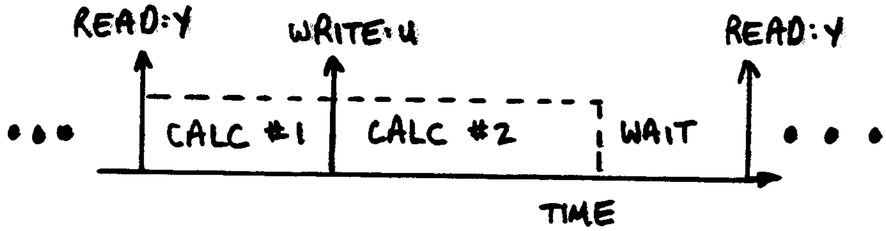


- u(t) HAS STANDARD BOX CAR SHAPE
- SO "SMOOTHED" u(t) BY CONNECTING MID-POINTS $\rightarrow \hat{u}(t)$

$\Rightarrow$ SAMPLE + HELD y(t) LOOKS LIKE y(t), BUT DELAY OBVIOUS.

- EFFECTIVE DELAY OF SAMPLE AND HOLD $\sim \frac{T_s}{2}$ ON AVERAGE.
  - $\Rightarrow$ BIG PROBLEM IF T_s LARGE.
  - $\rightarrow$ SO WHY NOT MAKE T_s SMALL?

- SAMPLING RATE T_s IS HOW LONG WE HAVE TO COMPUTE THE CONTROL COMMAND GIVEN THE MEASUREMENTS WE MAKE



- USUALLY "WAIT" PERIOD IS SHORT, BUT LENGTH OF CALC #1, CALC #2, A/D, D/A OPERATIONS DEPEND ON COMPLEXITY OF ALGORITHM AND QUALITY OF COMPUTER EQUIPMENT.

$\uparrow$ QUALITY $\Rightarrow$ $\uparrow$ COST !!

- WE WILL TYPICALLY ASSUME THAT $\omega_s \simeq$ 10-20 ω_{BW}

SAMPLING FREQUENCY.

CONTROL LAW

- BASIC COMPENSATOR $G_c = K_c\left(\frac{s+z}{s+p}\right) = \frac{u(s)}{e(s)}$

- EQUIVALENT TO DIFFERENTIAL EQUATION

$$\dot{u} + pu = K_c(\dot{e} + ze)$$

- NOT USEFUL FOR DIGITAL IMPLEMENTATION.

$\Rightarrow$ MUST APPROXIMATE DERIVATIVES WITH:

$$\dot{u}\Big|_{t=KT_s} \cong \frac{1}{T_s}\left[u((K+1)T_s) - u(KT_s)\right] = \frac{u_{K+1} - u_K}{T_s}$$

- FORWARD APPROXIMATION, OTHERS EXIST.

- THEN $\dot{u} + pu = K_c(\dot{e} + ze)$ BECOMES:

$$\frac{1}{T_s}(u_{K+1} - u_K) + p\,u_K = K_c\left(\frac{1}{T_s}(e_{K+1} - e_K) + ze_K\right)$$

OR

$$u_{K+1} = (1 - pT_s)u_K + K_c e_{K+1} - K_c(1 - zT_s)e_K$$

- RECURSIVE SO GOOD TO DO ON A COMPUTER

- CALLED A <u>DIFFERENCE</u> <u>EQUATION</u>

COMPUTER CODE

1) GIVEN PREVIOUS INFORMATION u_K, e_K

   AND NEW INFORMATION $(y_{K+1}, r_{K+1}) \rightarrow e_{K+1}$

2) USE DIFFERENCE EQUATION TO FIND u_{K+1}

$$\text{LET} \quad u_{OLD} = (1 - pT_s)\,u_K - K_c(1 - zT_s)\,e_K$$

$$\Rightarrow \quad u_{K+1} = K_c\, e_{K+1} + u_{OLD}$$

- DEFINE $\gamma_1 = 1 - pT_s$

  $\gamma_2 = -(1 - zT_s)\,K_c$

- CODE

  INITIALIZE
  START LOOP
  - SAMPLE A/D'S $\{\text{READ } y_{K+1}, r_{K+1}\}$
  - COMPUTE $e_{K+1} = r_{K+1} - y_{K+1}$
  - UPDATE $u_{K+1} = K_c\, e_{K+1} + u_{OLD}$
  - OUTPUT TO D/A'S $\{\text{WRITE } u_{K+1}\}$
  - UPDATE $u_{OLD} = \gamma_1 u_{K+1} + \gamma_2 e_{K+1}$
  - WAIT $\{K \rightarrow K+1\}$

  END LOOP

$\Rightarrow$ OUTPUT CONTROL AS SOON AFTER READ AS POSSIBLE. COULD WRITE u_{K+1} AT END OF THE WAIT SO THE DELAY IS LONGER, BUT FIXED.

SUMMARY

1) USING DIGITAL COMPUTER INTRODUCES SOME EXTRA DELAY
   - SAMPLE + HOLD $\sim T_s/2$ DELAY ← VERY IMP.
   - HOLDING $u(k)$ TO END OF LOOP $\sim T_s$ DELAY

$\Rightarrow$ DELAY $\sim \frac{T_s}{2} \leftrightarrow \frac{3T_s}{2}$

2) WITH $\omega_s \doteq 20\,\omega_{BW}$, DELAY EFFECTS ARE SMALL AND CTS/DISCRETE CONTROLLERS ARE VERY SIMILAR.

3) C2DM.M OFFERS SIMPLE WAY TO DISCRETIZE $G_c(s)$.

EMULATION

1. FIND DESIRED SYSTEM CHARACTERISTICS
   ω_c, PM, ζ, $\zeta\omega_n$, ω_{BW}, ... $\Rightarrow$ PICK $\omega_s = \frac{2\pi}{T_s}$
2. ADD THE $\frac{T_s}{2}$ DELAY TO YOUR SYSTEM $G_p(s)$ TO ACCOUNT FOR ZOH. $\Rightarrow \hat{G}_p(s) = e^{-sT_s/2}G_p(s)$
   $\Rightarrow$ USUALLY MEANS ADDING TO PM $\left(\frac{\omega_c T_s}{2}\cdot\frac{180^\circ}{\pi}\right) \Rightarrow \frac{\omega_c}{\omega_s}\cdot 180^\circ$
3. DESIGN $G_c(s)$ TO MEET SPECS WITH $\hat{G}_p(s)$
4. CONVERT $G_c(s)$ TO DIFFERENCE EQUATION
5. WRITE CODE.