16.82 CONTROLS FALL 2003

TWO HALF-HOUR LECTURES

OUTLINE:

REFERENCE (SEE BELOW)

L1

OBJECTIVES OF CONTROL
- METRICS — 1-5, 1-6
- INFORMATION

DOMINANT POLES
- SIGNIFICANCE — 1-13 → 2-4
- ANALYSIS

ROOT LOCUS TECHNIQUES
- BACKGROUND — 2-5 → 2-17
- MODIFICATION / SYNTHESIS

L2

REALIZABLE CONTROLLERS
- IMPLICATIONS FOR ESTIMATION & SENSING

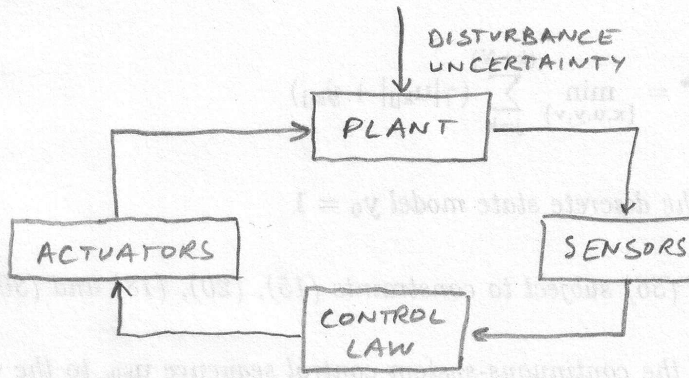
NESTED LOOP APPROACH
- "SEPARATION"

TIME DELAY — 7

SUMMARY

REFERENCE: PROF. HOW, "INTRODUCTION TO CONTROL DESIGN TECHNIQUES" LECTURE NOTES, 1998 (AVAILABLE AT http://www.mit.edu/~16.82)

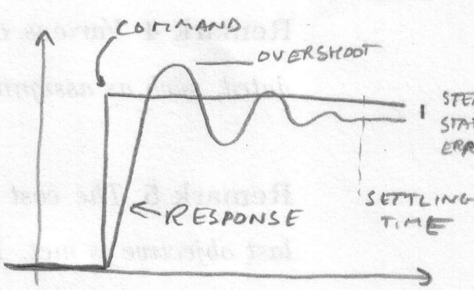
"CLOSED - LOOP SYSTEM"



- WE ARE GIVEN THE PLANT
- WE DESIGN THE "CONTROL SYSTEM"
  (THE SENSORS, CONTROL LAW AND ACTUATORS)
- OUR GOAL IS TO ACHIEVE REQUIREMENTS FOR THE DYNAMICS OF THE CLOSED -LOOP SYSTEM

TYPICAL REQUIREMENTS:

- QUANTITATIVE
  - STEP RESPONSE SETTLING TIME $\leq T_1$
  - OVERSHOOT ON STEP $\leq X\%$
  - STEADY STATE ERROR $\leq Y$
- QUALITATIVE
  - USE MINIMUM FUEL
  - STAY AS "CLOSE" AS POSSIBLE TO REFERENCE



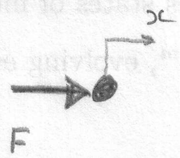
ANALYSIS

EXAMPLE: 1-D POINT MASS

ACTUATOR: FORCE ON THE MASS

SENSOR: POSITION OF THE MASS

(FOR SIMPLICITY, USE MASS = 1)



EQUATION OF MOTION: $\ddot{x} = F$

FOR CONTROL ANALYSIS, LAPLACE TRANSFORM IS MORE USEFUL (BACKGROUND REF: 1-10)

$$X(s) = \frac{1}{ms^2} F(s)$$

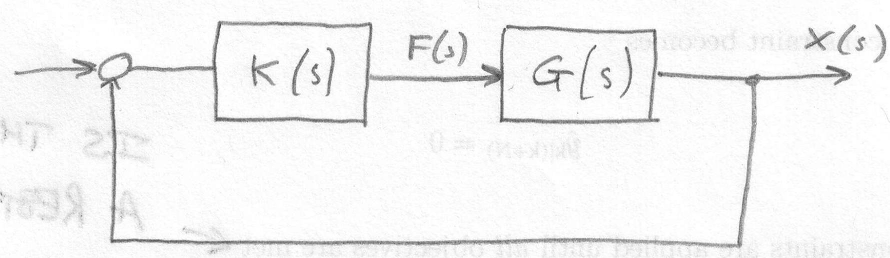
THIS IS OUR PLANT TRANSFER FUNCTION

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2}$$

THIS IS A FIRST PRINCIPLES MODEL. AN ALTERNATIVE IS TO USE SYSTEM ID METHODS. IN PRACTICE, TRY AND COMBINE THE BEST OF BOTH

WE CAN NOW MODEL OUR CLOSED LOOP SYSTEM IN LAPLACE FORM:

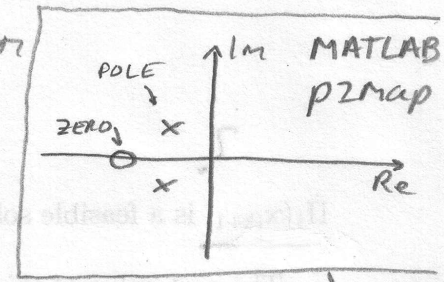
MATLAB "tf"



WHERE $K(s)$ IS OUR CONTROL LAW IN LAPLACE FORM

SIGNIFICANCE OF POLES

WE HAVE THE LAPLACE FORM OF A SYSTEM



$$P(s) = \frac{N(s)}{D(s)}$$

CAN FIND ITS <u>POLES</u> (VALUES OF s GIVING $D(s) = 0$)

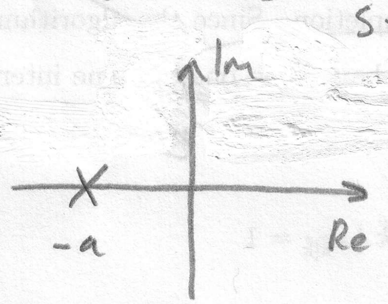
AND ITS <u>ZEROES</u> (VALUES OF s GIVING $N(s) = 0$)

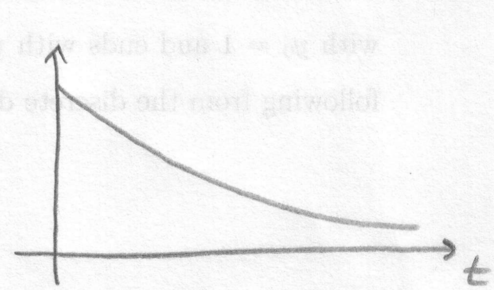
EACH POLE DETERMINES A COMPONENT OF THE SYSTEM RESPONSE. THE ZEROES ARE A FUNCTION OF HOW THEY MIX. WE COULD USE PARTIAL FRACTIONS AND SEPARATE:

$$P(s) = \frac{A}{s+a} + \frac{B}{s+b} + \frac{C}{s+c} + \cdots$$

TWO IMPORTANT FORMS: <u>IMPULSE RESPONSES</u>

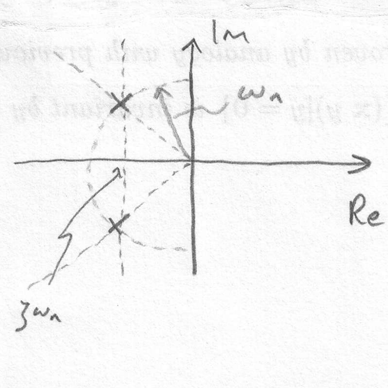
1) POLE ON REAL AXIS: $\frac{1}{s+a} \Rightarrow e^{-at}$ EXPONENTIAL DECAY

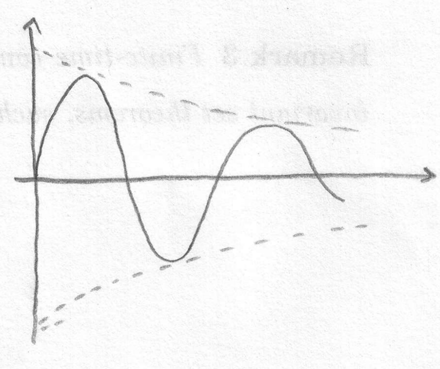




2) COMPLEX CONJUGATE POLE PAIR $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow$ DAMPED SINUSOID $e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t\right)$

$\boxed{\text{MATLAB sgrid}}$





EXAMPLE

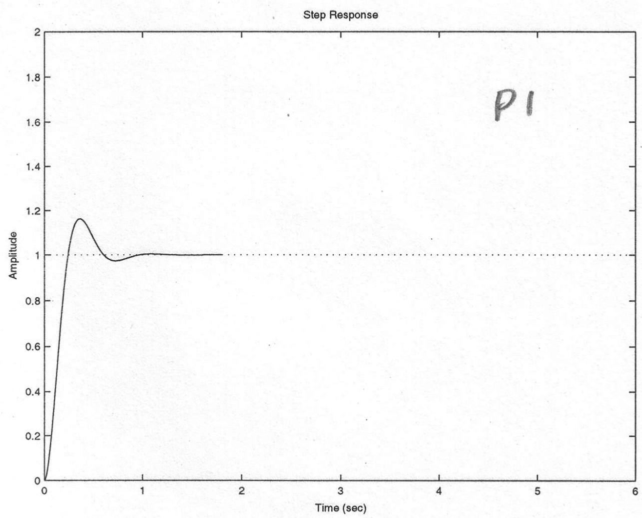
SYSTEM $P(s) = \dfrac{s^2 + 110s + 200}{s^3 + 11s^2 + 110s + 100}$

PARTIAL FRACTION EXPANSION

$$P(s) = \underbrace{\frac{1}{s+1}}_{P1(s)} + \underbrace{\frac{100}{s^2+10s+100}}_{P2(s)}$$

NOW LOOK AT STEP RESPONSES (AT LEFT ←)

CAN <u>SUM</u> THE CONTRIBUTIONS OF THE POLES.



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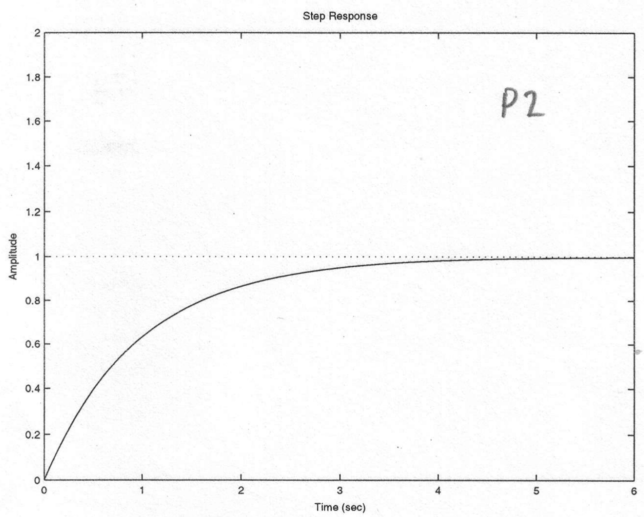

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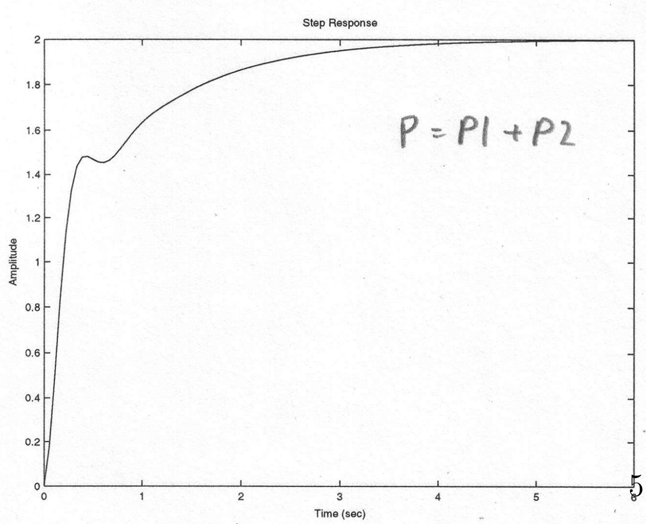


FOR CONJUGATE POLE PAIR, THE METRICS (TIME DOMAIN) CAN BE RELATED TO THE POLE POSITION (LAPLACE DOMAIN) BY COMMON FORMULAE (SEE 1-15)

STEP SETTLING TIME $= \dfrac{4.6}{\xi \omega_n}$

STEP PEAK OVERSHOOT $= e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$

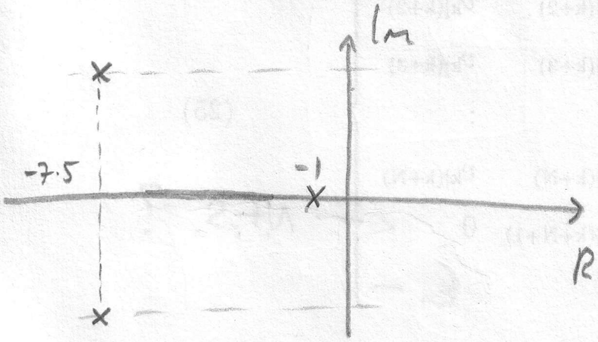
DOMINANT POLES

$\boxed{\text{A SYSTEM'S RESPONSE IS DOMINATED BY THE POLE OR POLE PAIR CLOSEST TO THE IMAGINARY AXIS *}}$

* CAUTION: WATCH OUT FOR NEARBY ZEROES. (SEE 2-1)

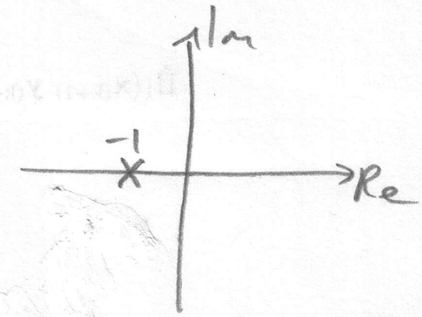
SO

$$\frac{1}{(s+1)(s^2+15s+100)}$$



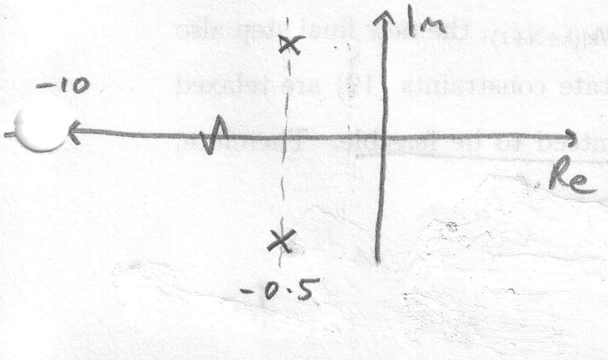
BEHAVES VERY MUCH LIKE

$$\frac{1}{s+1}$$



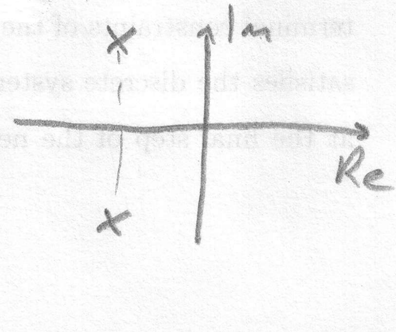
AND

$$\frac{1}{(s+10)(s^2+s+1)}$$

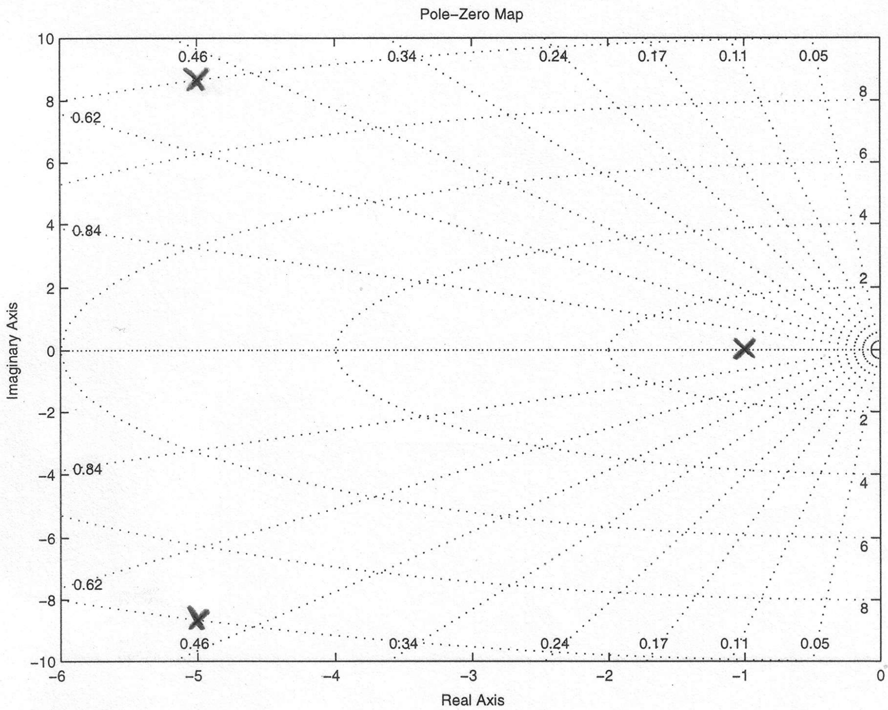


$\sim$

$$\frac{1}{s^2+s+1}$$



EXAMPLE

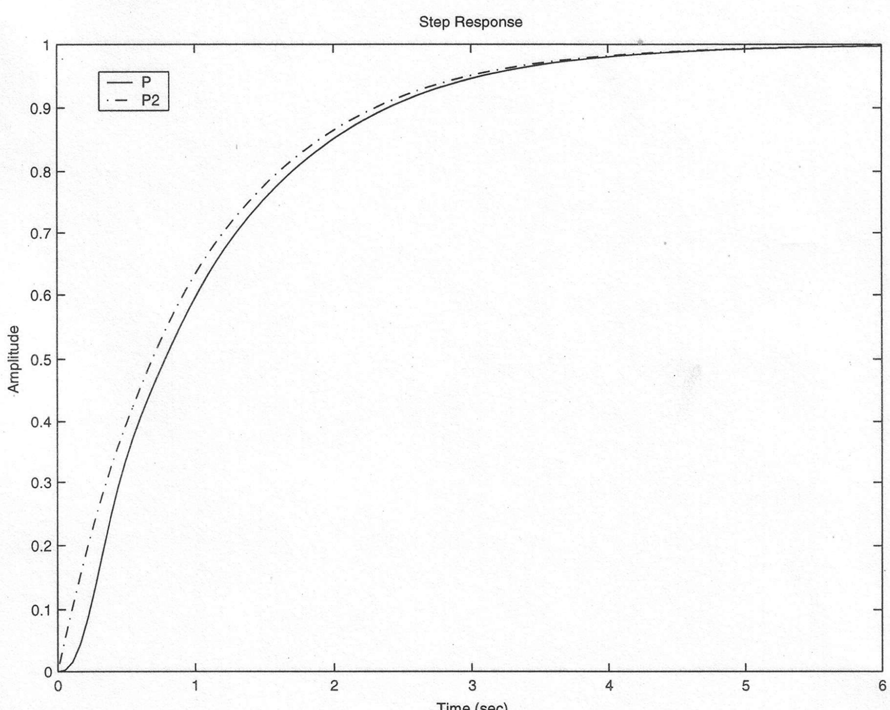


$$P = \frac{100}{(s+1)(s^2+10s+100)}$$

$$P1 = \frac{100}{(s^2+10s+100)}$$

$$P2 = \frac{1}{(s+1)}$$

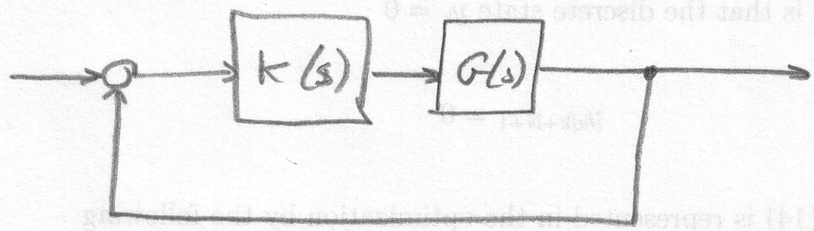
$$P = P1 \times P2$$



STEP RESPONSES OF P AND P2 VERY SIMILAR

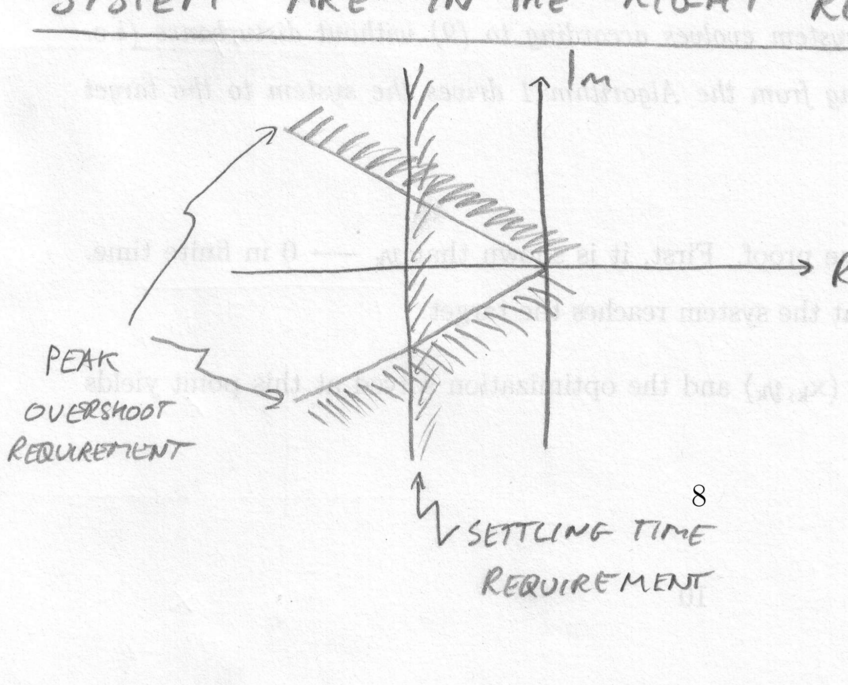
$\Rightarrow$ TO ANALYZE P, JUST CONSIDER DOMINANT POLE P2

BACK TO OUR CONTROL PROBLEM



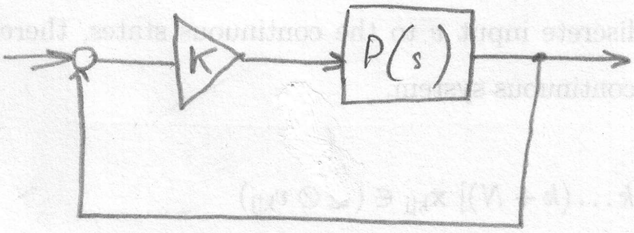
1. A SYSTEM'S RESPONSE IS DOMINATED BY THE POLE(S) CLOSEST TO IMAGINARY AXIS
2. THE POLES OF A SYSTEM CAN BE RELATED TO ITS TIME-DOMAIN RESPONSE (SETTLING TIME etc.)

SO, WE CAN RELATE OUR CONTROL REQUIREMENTS TO RESTRICTIONS ON POLE LOCATIONS. THE CONTROL PROBLEM IS NOW TO CHOOSE K(s) SUCH THAT THE DOMINANT POLES OF THE CLOSED LOOP SYSTEM ARE IN THE RIGHT REGIONS.



WHERE ARE THE CLOSED LOOP POLES ?

THE ROOT-LOCUS DIAGRAM TELLS US WHERE THE CLOSED-LOOP POLES ARE AS A FUNCTION OF FEEDBACK GAIN



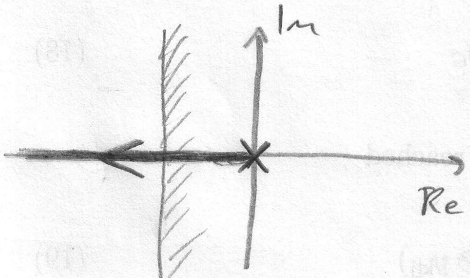
P(s) INCLUDES K(s) AND G(s) (CONTROL & PLANT)

(BACKGROUND: SEE 2-5 ONWARDS)

START WITH ROOT LOCUS OF PLANT ALONE, G(s)

EXAMPLE 1 SINGLE INTEGRATOR $G(s) = \frac{1}{s}$

MATLAB rlocus



APPLY SUFFICIENT GAIN TO MOVE POLE TO THE LEFT OF THE REQUIREMENT

SIMPLE SCALAR GAIN DOES THE TRICK :

"PROPORTIONAL FEEDBACK"

MATLAB

```
P = tf(1, [1 0])
rlocus(P)
rlocfind(P)
```

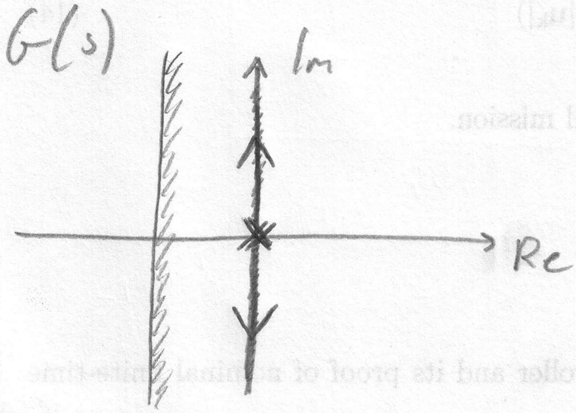
[click to left of req.]

[displays a gain: use at least this much]

THAT WAS EASY! THE LOCATION WAS ALREADY ON THE ROOT LOCUS. NOT ALWAYS SO...

EXAMPLE 2 DOUBLE INTEGRATOR $G(s) = \frac{1}{s^2}$

$G(s)$



PROBLEM! NO GAIN VALUE WILL MOVE POLES WHERE WE WANT THEM.

ANSWER: ADD DYNAMIC ELEMENTS (POLES OR ZEROES) TO CONTROLLER $K(s)$

TYPICAL STRATEGY FOR ADDING DAMPING: PD CONTROL

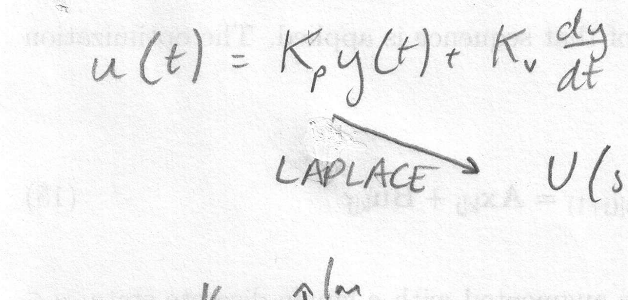
PD = PROPORTIONAL PLUS DERIVATIVE

$\boxed{\text{BACKGROUND: 3-4 ON}}$

$$u(t) = K_p y(t) + K_v \frac{dy}{dt}$$

$$\xrightarrow{\text{LAPLACE}} U(s) = K_p Y(s) + K_v s Y(s)$$

$$= K_v \underbrace{\left(s + \frac{K_p}{K_v}\right)}_{\text{ADDING A ZERO}} Y(s)$$



$\boxed{\text{NEXT LECTURE: IMPLICATIONS OF ADDING A ZERO}}$

SUMMARY

OUTLINED A PROCEDURE FOR CONTROLLER SYNTHESIS:

1) MODEL THE PLANT
2) DRAW ITS ROOT LOCUS
3) MARK ON THE REQUIREMENTS FOR DOMINANT POLES
4) CAN A PROPORTIONAL GAIN DO THE JOB?

- YES ⇒ GREAT, FIND IT & USE IT
- NO ⇒ NEED TO ADD POLES/ZEROES
  TRY STANDARD FORMS (PD, PI, PID)

REMARKS

- THIS IS A METHOD FOR SYNTHESIS. THERE ARE OTHERS (LOOP-SHAPING WITH BODE PLOTS, STATE SPACE POLE PLACEMENT. SEE REF)
- APPROACH BASED ON ASSUMPTIONS
  - A1) NEGLECT ALL BUT DOMINANT POLES
  - A2) BELIEVE THE MODEL

ALWAYS CHECK ASSUMPTIONS e.g. SIMULATE, TEST