Block 3:

Elasticity

OVERALL OBJECTIVES FOR BLOCK **M3**

Through participation in the lectures, recitations, and work associated with Block M3, it is intended that you will be able to………

- ….**employ** a continuum version of the constitutive law of elasticity
- ….**explain** the factors that contribute to material properties in terms of behavior that can and cannot be controlled
- ….**discuss** the limits of the model of linear elasticity for materials
- ….**summarize** the key components to solve problems of elasticity

Unit M3.1

The Role of Material Properties

Readings:

A & J 1, 2

16.001/002 -- "Unified Engineering"
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

LEARNING OBJECTIVES FOR UNIT **M3.1**

Through participation in the lectures, recitations, and work associated with Unit M3.1, it is intended that you will be able to………

- ….**explain** what a material property is and represents
- ….**cite** various material properties and types thereof
- ….**indicate** how material properties are used in material selection for structural applications and **employ** such

Thus far, we have learned about “universal” concepts (e.g.)

- Equilibrium
  - of discrete systems
  - of continuum
- Deformation behavior
  - strain-displacement
- Stress, strain and transformation

These concepts are “*universal*” in that they always apply (given the limiting assumptions we placed). But, there is also behavior which depends upon the particular material (think about constitutive behavior). We therefore need to look at materials and their properties.

Let’s first consider….

Classes of Materials

The choice of materials is a critical part of the design process. Development of new materials (can) radically alters the way in which systems (structures, aircraft, etc.) are designed as well as in the resulting performance.

--> Consider:

- Historically:

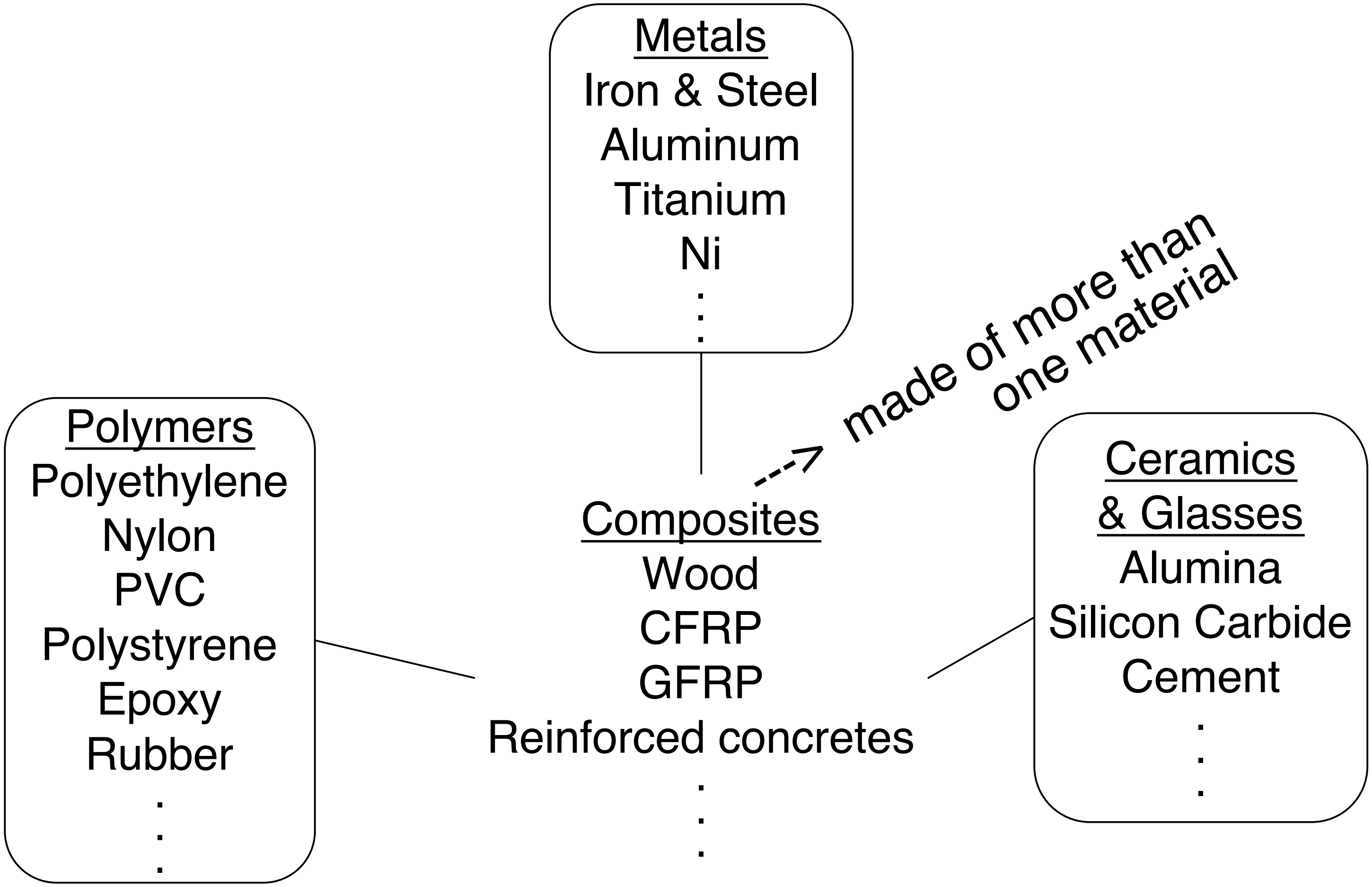
  Stone Age --> Bronze Age --> Iron Age --> ?
- Airframes:

  Wood & fabric --> Aluminum Alloy $\overset{?}{\text{-->}}$ Composites
- Turbine Blades:

  Steel --> Nickel Alloys $\overset{?}{\text{-->}}$ Ceramics
- Buildings/Bridges:

  Rock --> Wood --> Steel $\overset{?}{\text{-->}}$ Composites

--> Materials can generally be categorized into 3 classes:



Metals
Iron & Steel
Aluminum
Titanium
Ni
⋮

Polymers
Polyethylene
Nylon
PVC
Polystyrene
Epoxy
Rubber
⋮

Composites
Wood
CFRP
GFRP
Reinforced concretes
⋮

--> made of more than one material

Ceramics & Glasses
Alumina
Silicon Carbide
Cement
⋮

The boundaries between the different classes may be "blurred", but it is useful to be able to generalize to be able to get a feel (i.e., ballpark estimate) for the behavior of a material of a certain class.

--> Think of examples of characterizing material behavior

We need to look at

Types of Properties

The properties of material can be grouped into several categories as follows:

- Economic:
  - Price
  - Availability
- Bulk Mechanical:
  - Density
  - Modulus
  - Damping
  - Strength (yield, ultimate)
  - Fracture (toughness)
  - Hardness
  - Fatigue/longevity
  - Creep

- Bulk Non-Mechanical:
  - Thermal
  - Optical
  - Magnetic
  - Resistance/electrical
  - Chemical
  - Environmental
- Surface:
  - Oxidation
  - (General) Corrosion
  - Friction
  - Abrasion and wear
- Production:
  - Manufacturability
  - Joining
  - Finishing
- Aesthetic:
  - Appearance
  - Texture
  - Feel

(and there are others)

Note: Properties are not a constant. They change with various parameters (e.g., temperature)

With all these various types of properties, choosing a material could be somewhat confusing, so let's consider:

Use of Material Properties in Design (and Material Selection)

--> First think about the general process of engineering

- Understand science and understand need (what is function)
- Model
- Analyze
- Design
- Get good answers
- Communicate

iterate

Use material properties as "quantifiable parameters"

Now need to apply this to structures and the materials of which they are made

Design of structural element depends on:

- functional requirements
- geometry
- material properties
- processing issues

**** --> Material and structure design are integrally linked <-- ****

- Choice of a different material (can) change the structural configuration

and

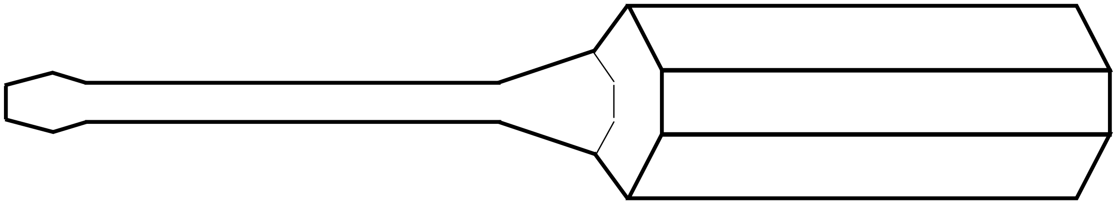
- Choice of a structural configuration (can) change(s) the material choice(s)

--> Materials and structures are selected for:

- primary/driving requirements (what is function?)
- secondary requirements (what is function?)
- manufacturing, processing, assembly considerations
- cost

--> Let’s consider some examples…..

Example 1 - Screwdriver



Consider function of each section and therefore need

Shaft & blade	Handle
High modulus (rubber blade?)	Appearance
High strength (lead blade?)	Ease of fabrication
High toughness (glass blade?)	Price
High hardness	⇒ PMMA (plexiglass)
⇒ High Carbon Steel	

Example 2 - Turbofan blades

Performance demands:

- High modulus
- High strength
- High toughness
- Fatigue resistance
- Surface wear resistance (e.g., water droplets)
- Corrosion resistance
- Low density
- Impact resistance (e.g., birdstrike)

- Generally have seen titanium alloy
- Rolls Royce in 1960s’ tried to use CFRP (graphite epoxy) but failure (and almost bankruptcy) due to birdstrike
- After 3 more decades of development and work to model and understand, there is a turbofan blade that has received FAA certification. (GE)

Example 3 - Turbine blades

Performance demands:

(much like turbofan blades minus impact resistance requirements but with…)

high temperature resistance (T ~ 1050° C)

⇒ oxidation resistance
creep resistance

⇒ use nickel-based alloys
⇒ work toward ceramics

--> But what about COST!

Cost is always the overriding issue. The question is how much one is willing to pay for a certain level of performance.

⇒ **Tradeoffs**

--> Contributors to cost

- Commodity price (raw material price on open market)
- Manufacturing (depends upon form wanted)
- Environmental cost (disposal, recycling, processing, etc.)
- Engineering cost (including R&D…how much to design…costs more if less experience w/ material)

--> Generally work to find a <u>figure of merit</u>:

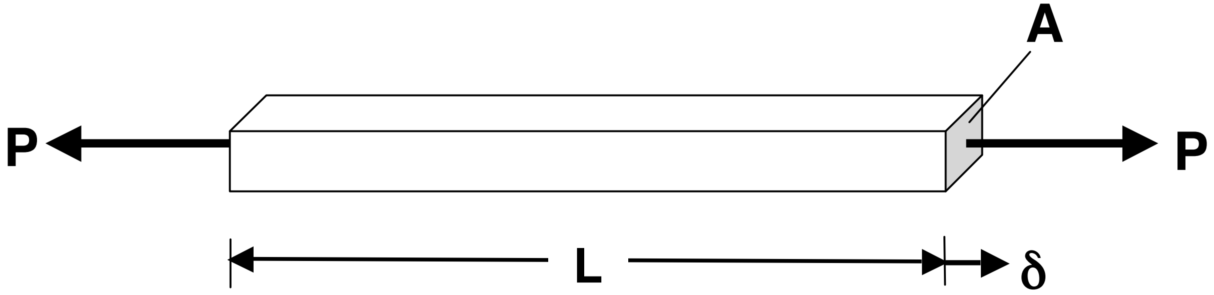
$$\text{Figure of merit: } \left(\begin{array}{c}\text{geometrical}\\ \text{parameters}\end{array}\right)\left(\begin{array}{c}\text{material}\\ \text{parameters}\end{array}\right)\left(\begin{array}{c}\text{constraint}\\ \text{parameters}\end{array}\right)$$

(depends on “objective”)

(may include cost)

depends on model(s) of behavior of system

<u>Example</u>: Carry a fixed load (P) with a fixed deformation (δ) of a given length (L) bar; minimize weight/cost combo



P is fixed
L is fixed
δ Is fixed (constrains ε)

- Stiffness consideration

$$\sigma = \overset{\text{modulus}}{E}\varepsilon$$

with: $\sigma = \dfrac{P}{A}$ and $\varepsilon = \dfrac{\delta}{L}$

$$\Rightarrow \frac{P}{A} = E\frac{\delta}{L}$$

$$\Rightarrow A = \frac{PL}{E\delta}$$

- Figures of merit:

  - weight $= LA\underset{\text{density}}{\rho} = \dfrac{L^2 P\rho}{E\delta} = \underset{\text{constraints}}{\left(\dfrac{P}{\delta}\right)}\underset{\text{material}}{\left(\dfrac{\rho}{E}\right)}\underset{\text{geometrical}}{L^2}$

  - cost $= \underset{\text{cost/lb}}{\$} \cdot \dfrac{L^2 P\rho}{E\delta}$

$\Rightarrow$ Choice depends on “value” of minimizing weight versus cost of material

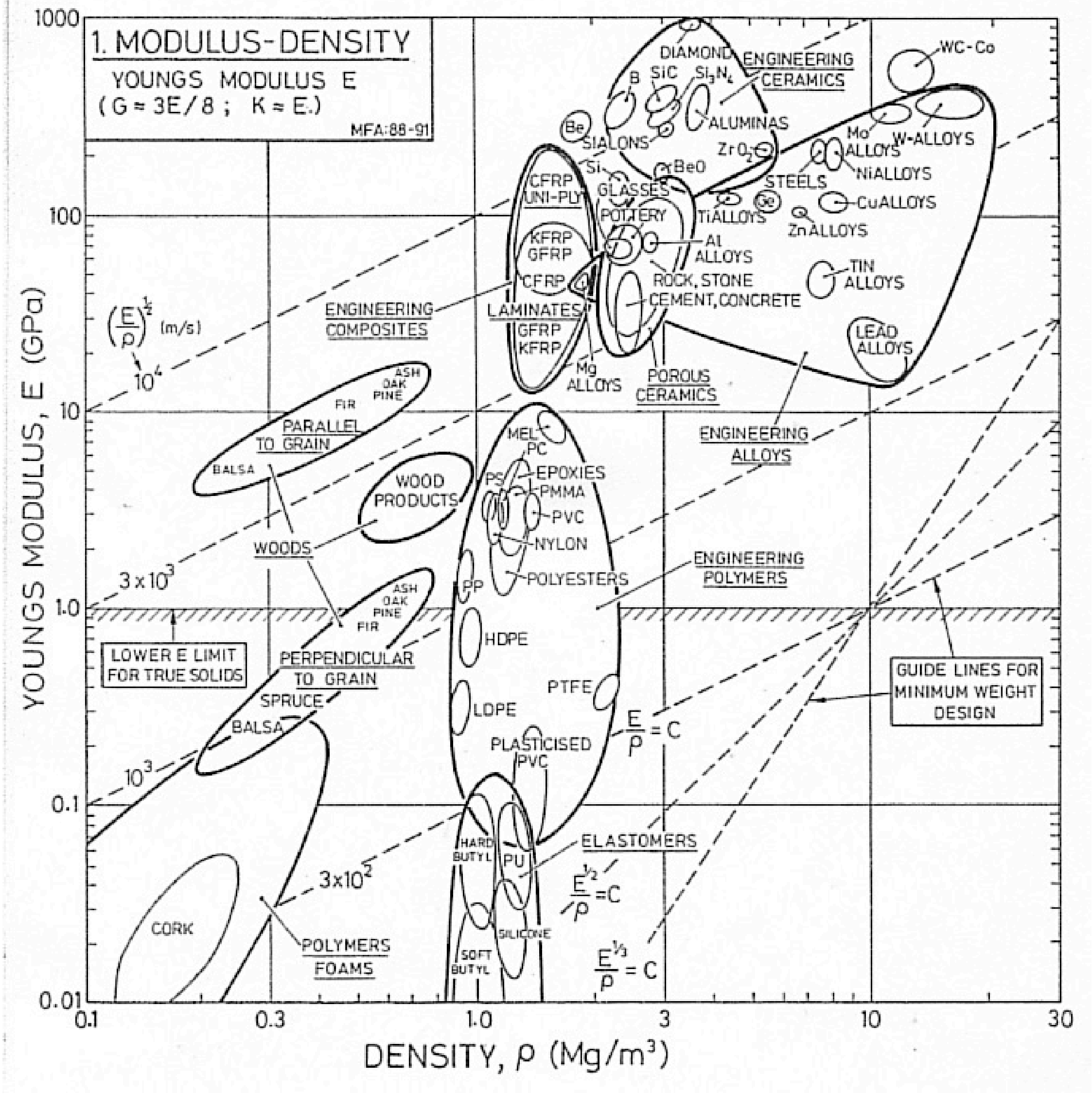
Choosing materials is aided by use of…..

Material Property Charts

- Plot one important material property on one axis versus another important material property on other axis
- Classes of materials often cluster

Reference:
Materials Selection in Mechanical Design,
M.F. Ashby, Pergamon Press,
Oxford, 1992.

Figure M3.1- Example chart: Modular Density



from: *Materials Selection in Mechanical Design,*
M.F. Ashby, Pergamon Press, 1992 (p. 76).

Thus far we've talked about properties without defining them. But just exactly…

What is a "Property?"

First, what is it not…?…

It is not necessarily an inherent "truth" of the material

This makes more sense by thinking of generally what it is…

Often it is

- empirical in nature
- linked implicitly to a model
- limited by assumption

or in general…

Definition: A (mechanical) material property is (often) a quantifiable engineering approximation of the behavior of a material system.

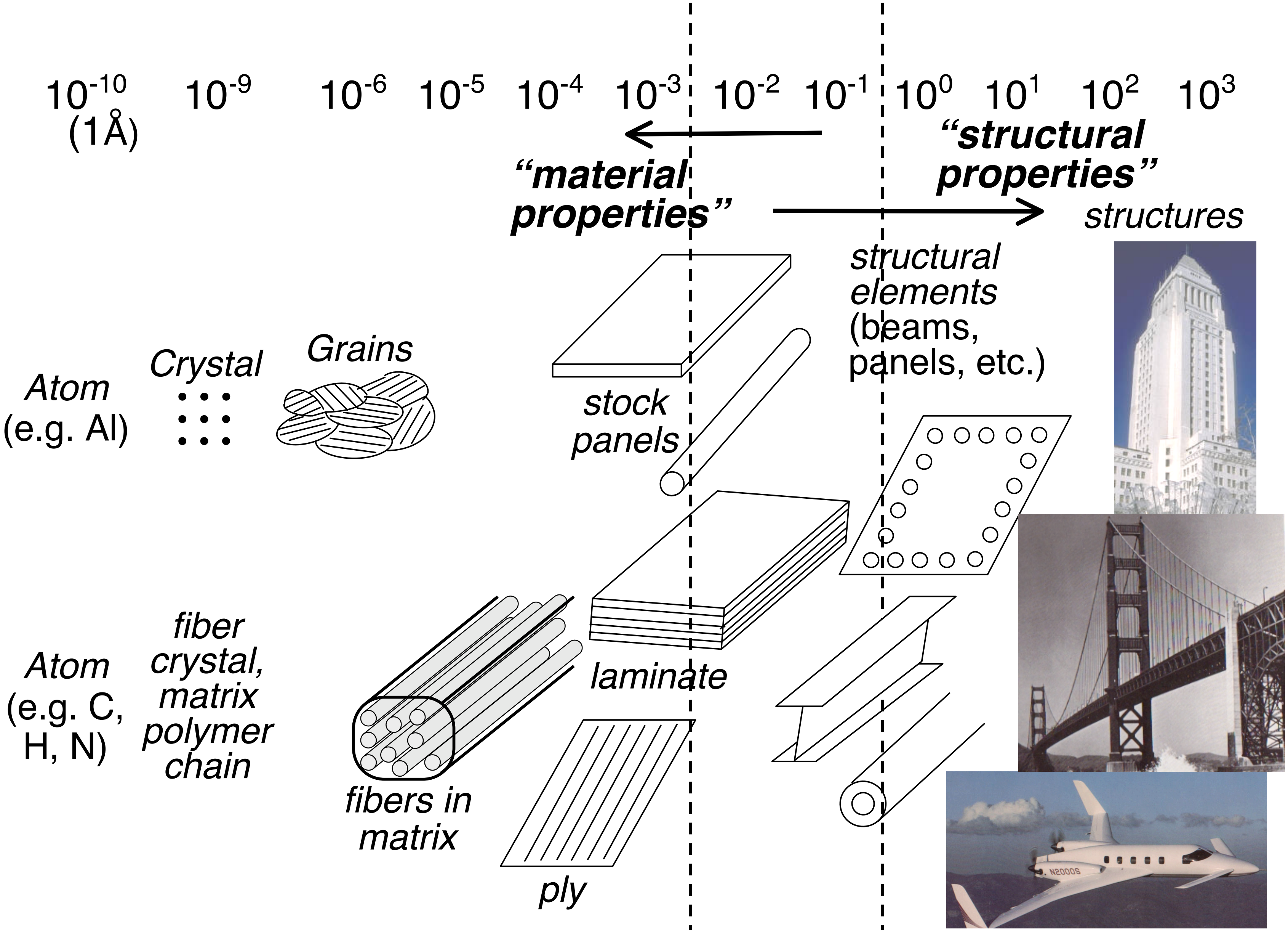
Keys: quantifiable -- need to be able to express in numbers
approximation -- there are limits, assumptions, etc.

--> Recognizing these limits and assumptions is critical in using material properties

It is also important to recognize that "properties" depend upon the length/scale at which we view/characterize a material/structure. We, therefore, need to consider…

The Role of Scale

There are many "(length) scales" in dealing with a material/structure. In fact it is a continuum:



LENGTH [m]

10^{-10} (1Å) 10^{-9} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}

"material properties" ←

→ ***"structural properties"***

Metal

Atom (e.g. Al)

Crystal

Grains

stock panels

structural elements (beams, panels, etc.)

structures

Composite

Atom (e.g. C, H, N)

fiber crystal, matrix polymer chain

fibers in matrix

ply

laminate

Consider the composite material…..

- --> on some level, the composite material is a structure of fiber and matrix
- --> on some level we model it as a homogeneous material (do not recognize fiber and matrix)
- --> In a similar way on some level, a structural truss can be modeled as a material (continuum) by homogenizing (averaging) the behavior to get the stiffness of the truss

This all depends upon the scale at which one views/models the behavior

Consider two important categories:

Microscopic (analysis)

Consider the internal structure (often on a "microscopic" basis)

Macroscopic (analysis)

Characterize behavior on a "macroscopic" scale by averaging the behavior without regard for the internal (often "microscopic") structure

And there are other levels (nano, meso, …)

Note that: scale always plays an important role, but even more so in failure

So we can see that material properties are important. Since we've already looked at stress and strain separately, let's next consider the material properties (property) that relate stress and strain.