Unit M3.4
Stress-Strain Behavior Revisited

Readings:

16.001/002 -- "Unified Engineering"
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

LEARNING OBJECTIVES FOR UNIT **M3.4**

Through participation in the lectures, recitations, and work associated with Unit M3.4, it is intended that you will be able to………

- ….**explain** three basic phenomena associated with stress-strain behavior (basic response, classes of behavior, nonlinear response) in terms of atomic factors
- ….**describe** the role of scale in elastic response and the concept of “effective moduli”

Given our qualitative understanding of the structure of materials, we can now go back and give an…..

Explanation of Phenomena and Behavior

We'll consider three basic phenomena: basic deformation response, classes of stress-strain behavior, and nonlinear stress-strain response

--> Basic Deformation Response

Consider pulling (or pushing) on a material

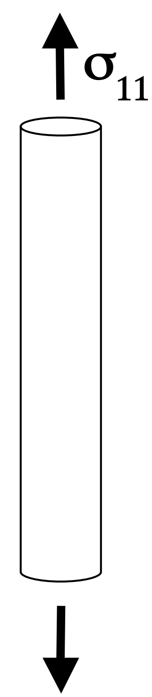
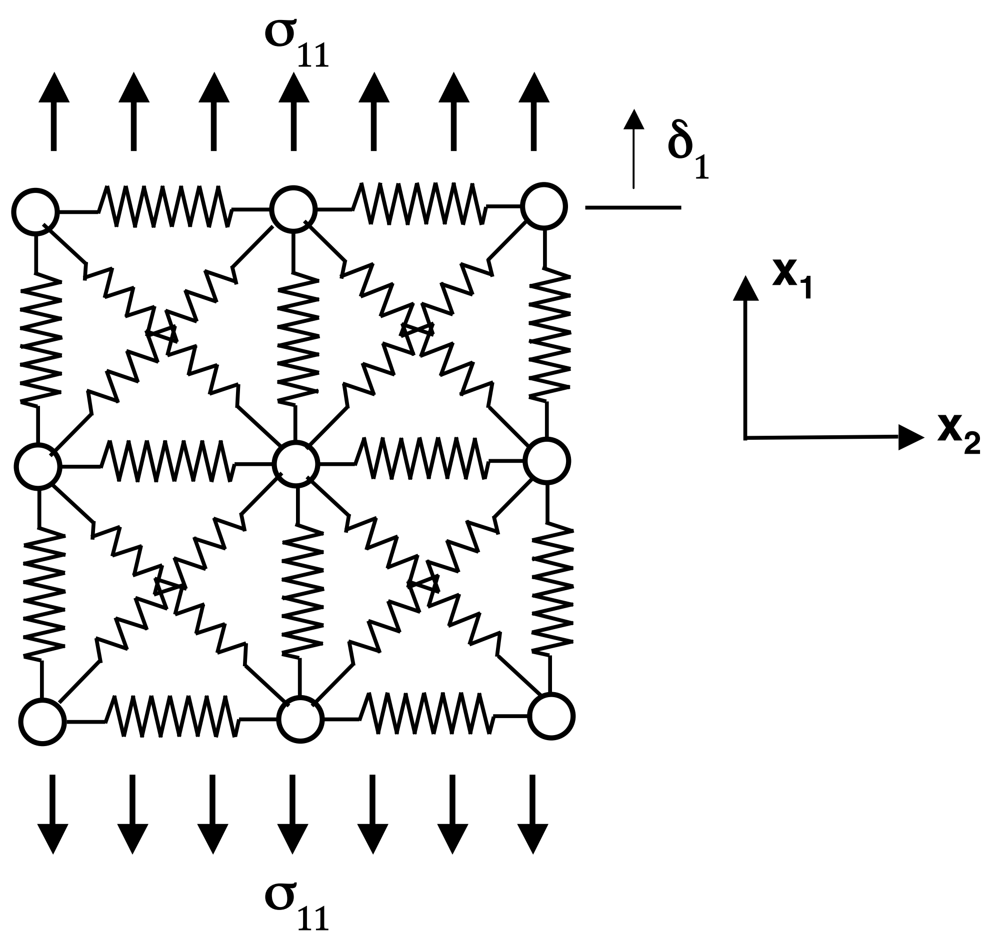


Figure M3.4-1 **Basic atomic arrangement with bonds:**

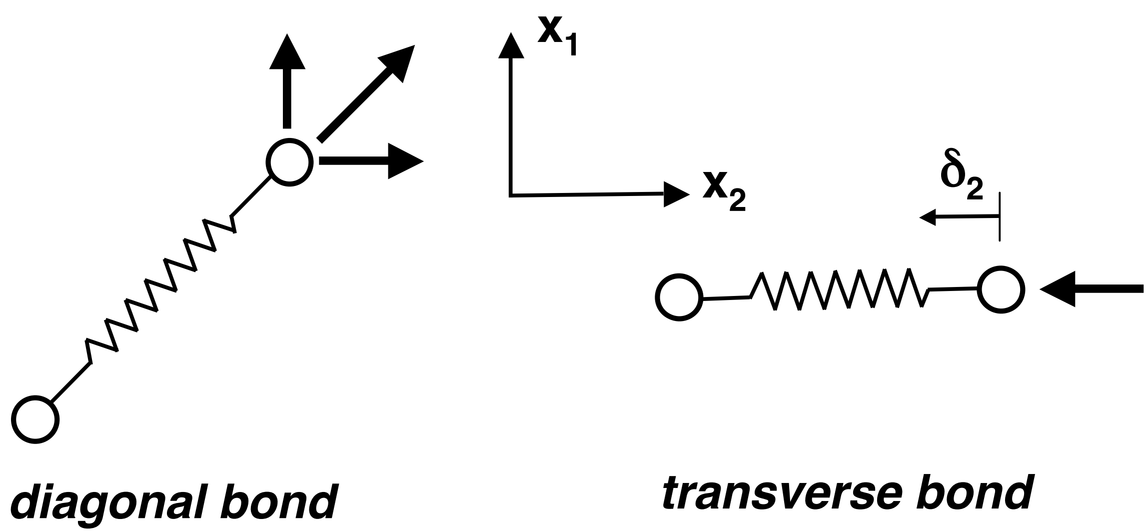


The first response is that elongation occurs in the x_1 - direction so that the force (stress) applied is balanced by the "spring loads" in the bonds

--> This gives E (modulus)

The “diagonal” bonds would lengthen causing a force there. This cannot totally happen since there is a component of this in the x_2 - direction

Figure M3.4-2 **Forces and resulting deformation in various bonds**



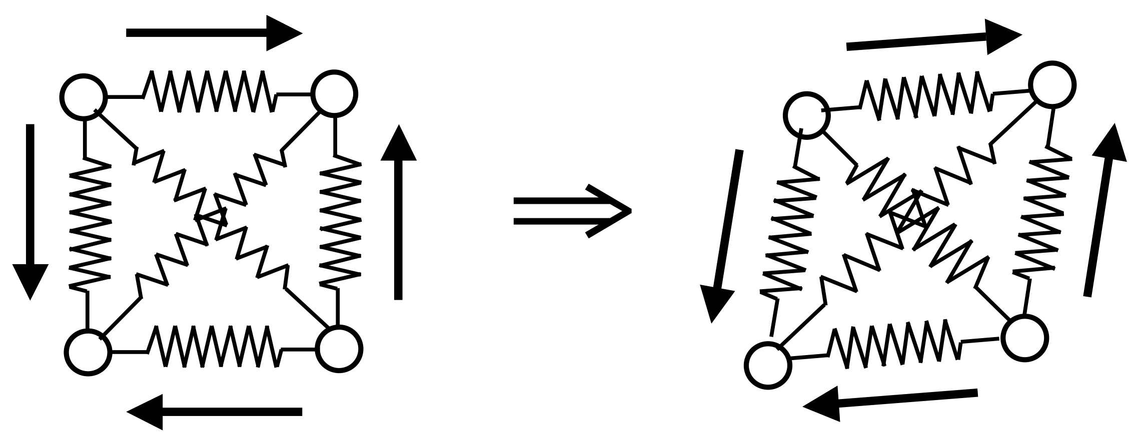
Thus, the material contracts in x_2 until the (tensile) forces in the diagonal bonds are countered by the compressive forces in the bonds in the x_2 - direction.

--> This gives Poisson’s ratio (ν)

Note: The same effect occurs for a compressive load/stress, all the signs just change.

- For shear loading a balance of forces again occurs resulting in shear deformation:

Figure M3.4-3 **Model of shearing at atomic level**

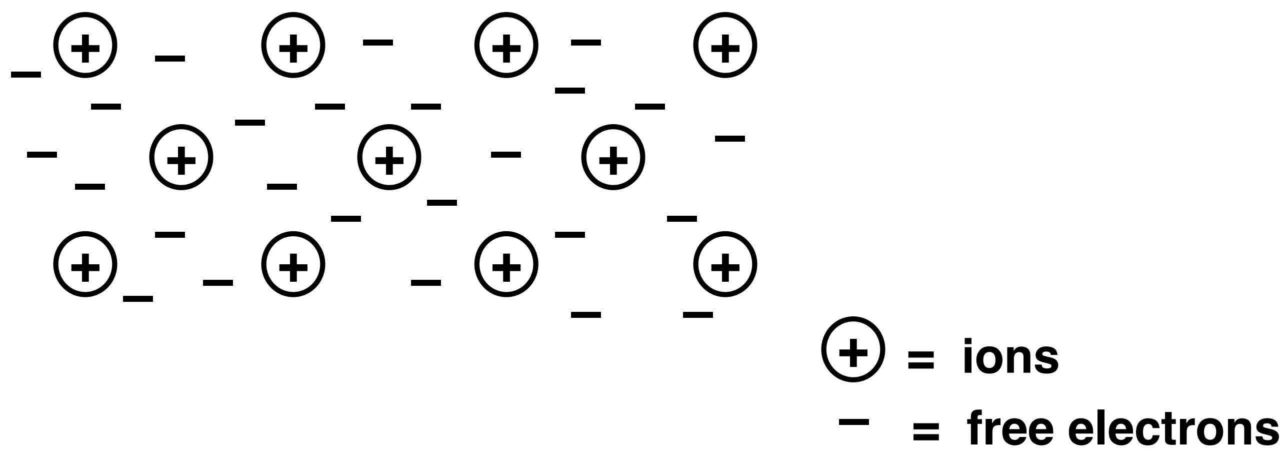


--> Classes of stress-strain behavior

The macroscopic response depends on the arrangement of the microstructure

Isotropic

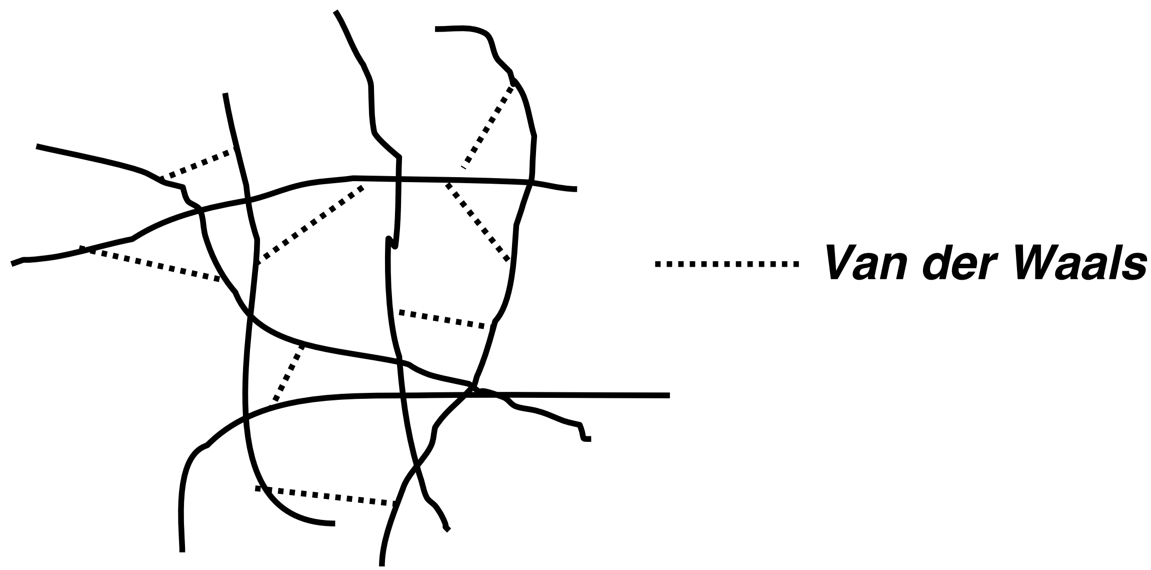
- Metallic Bonds



On scale above atomic, same structure in all directions.

$\Rightarrow$ response will be the same no matter how one pulls/pushes

- Polymer chains



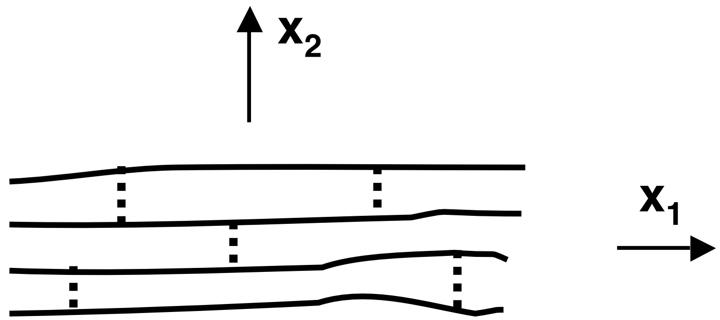
There is a clear directionality to the polymer chains, but at a scale above this, the general distribution of the chains and secondary bonds gives no preferred response.

Note: Role of Scale (we'll come back to this)

Orthotropic

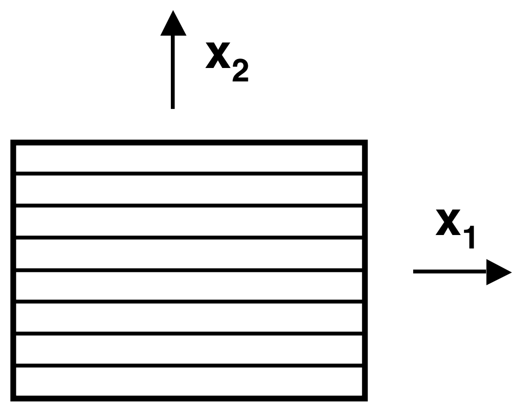
Now there are preferred directions

- covalent bonds
- oriented polymers



Primary bonds at work in x_1; secondary bonds at work in x_2

- composites with unidirectional fibers



Fibers control in x_1; matrix more important in x_2

<u>Note</u> (again): Role of Scale

--> Nonlinear stress-strain behavior

There are two contributors to nonlinear stress-strain behavior (just as there are two contributors in general at the microstructural level)

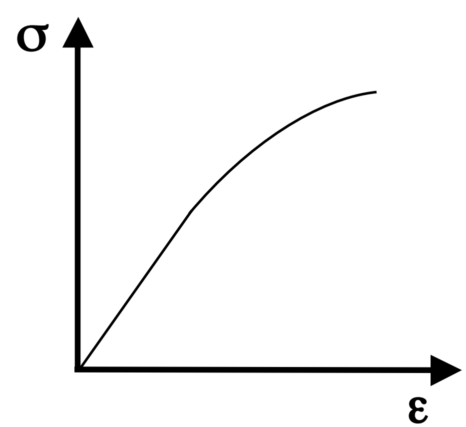
1. Basic Bond

We saw that the basic bond stiffness is:

$$S = \frac{dF}{dr} = \frac{d^2U}{dr^2}$$

where U is the bond energy.

There is a region around the equilibrium distance, r_0, where S is linear; outside this region it is nonlinear (generally softening behavior)

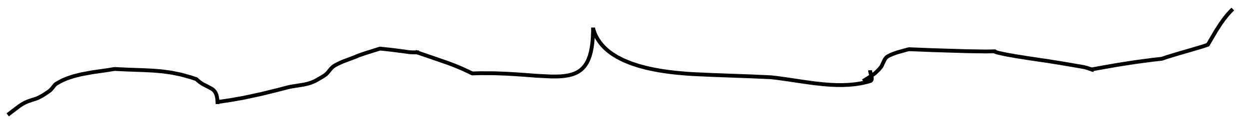


(Note: There are some other mechanisms at work which we shall look at in the second term)

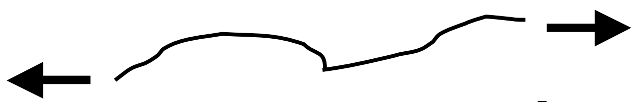
2. Arrangement of atoms and bonds

For example, carbon fibers show a stiffening in stress-strain. Why?

Basic carbon fiber is made up of carbon chains which are twisted:

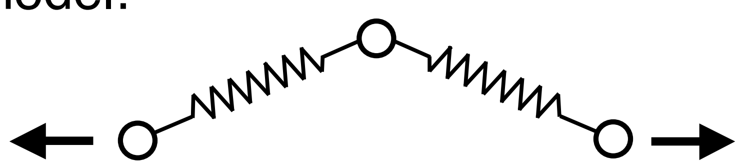


As load is applied, these straighten out

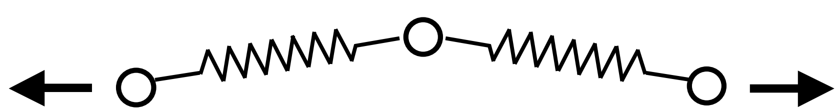


to

Think of easy model:

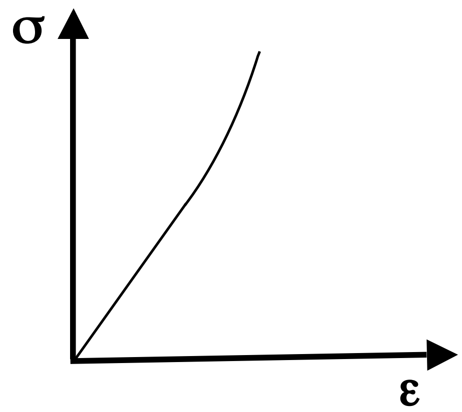


to



Stiffening of structure as bonds (springs) align with load

gives



Let's now revisit….

The Role of Scale ("Effective Moduli")

We have clearly seen how the behavior we characterize will depend on the scale (length) at which we observe/characterize.

Examples

- attraction and repulsion at electron/proton level versus atom level
- atomic bonding to material response

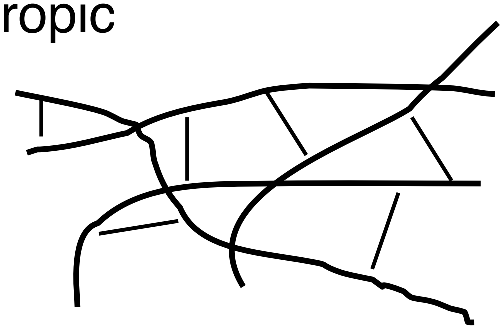
- crystals and grains and polymer chains can be orthotropic (have directionality) but they can be distributed in an overall material such that homogeneous (averaged) response on macroscopic level is isotropic

--> best example (with several scales) is a fibrous polymer composites:

- polymer chains are directional

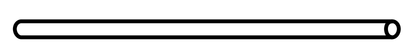
10^{-9} m

- distribution of polymer chains with secondary links makes polymer (matrix) isotropic



10^{-9} to 10^{-7} m

- fiber is directional



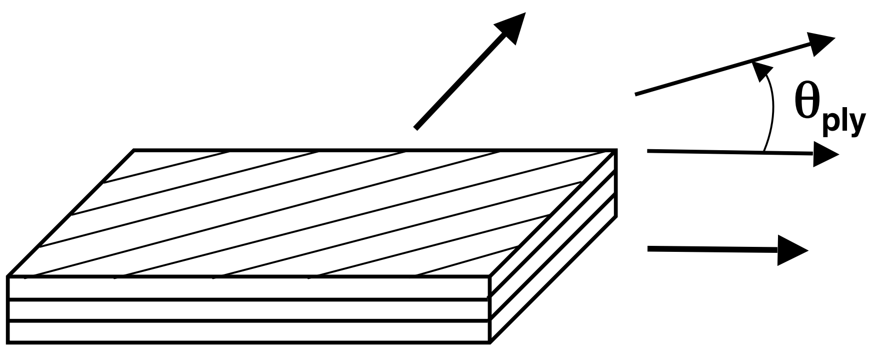
10^{-7} m

- aligned fiber composite is directional, use Rule of Mixtures to get orthotropic elastic constant of ply

10^{-7} to 10^{-5} m

E_L, E_T, ν_{LT}, etc.

- add up (laminated) plies in many directions and get laminate properties



10^{-5} to 10^{-3} m

How? (*as approximation*)

1. Transform all elastic properties to common axis system

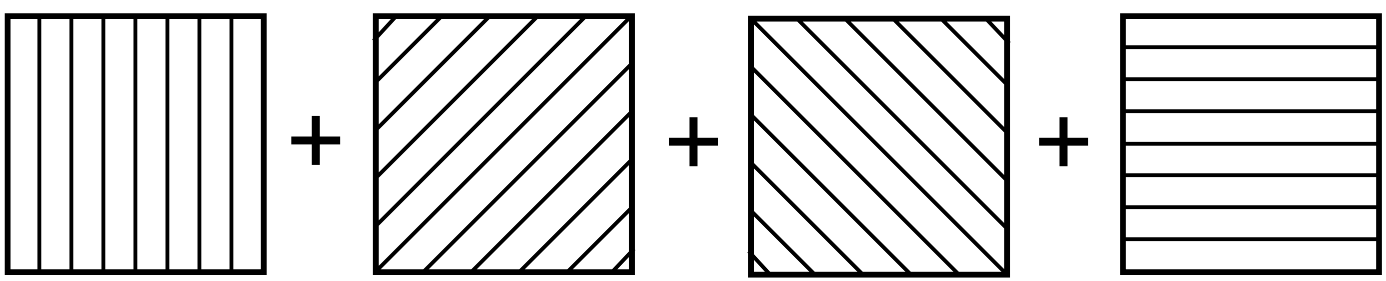
"laminate axes" "ply axes"

$$\tilde{E}_{ijkl} = \ell_{\tilde{i}} \, \ell_{\tilde{j}s} \, \ell_{\tilde{k}t} \, \ell_{\tilde{l}u} \, E_{rstu}$$

2. Treat plies as parallel springs ("combined action" ⇒ strain the same in each) and add up effect

$$\left(E_{ijkl}\right)_{total} = \sum_{\substack{\text{\# of} \\ \text{plies}}} v_{\mathbf{f}ply} \left(E_{ijkl}\right)_{ply}$$

Figure M3.4-4 **Adding up contributors of individual plies of a laminate**



0° **+45°** **-45°** **90°**

- Make a structure out of it and characterize overall behavior of structure 10^{-3} m to ? m

“Effective” Modulus (Behavior) refers to a characteristic length and associated model of behavior