UNIFIED HANDOUT

MATERIALS AND STRUCTURES - #M-5

Fall, 2008

Concept Review Sheet

for Unified Q(TM)3: Units M1.5, M2.1-2.2

STATICS AND EQUILIBRIUM

(key from earlier Units)

- The Three Great Principles of Solid Mechanics are the Concept of Equilibrium, the Compatibility of Displacement, and Constitutive Relations

- For static equilibrium, forces and moments must sum to zero (in all directions)
- There are a total of six equations of equilibrium in three dimensions (three force, three moment)
- There are a total of three equations of equilibrium in two dimensions (two force, one moment)
- The equilibrium of a structural system requires the sum of external forces and reaction forces, and their resulting moments, to be zero
- Modeling of "cutting" a structural element is accomplished by replacing the cut on each side by a force (convention is to draw a tensile force -- pointing outwards from cut surface)
- Equilibrium of a system or subsystem must be fulfilled
- A *Free Body Diagram* isolates a body and identifies the system of forces (external and reactions) acting on it
- There are three classes/categories of problems: *dynamics* (number of dof > number of reactions); *statically determinate* (number of dof = number of reactions); *statically indeterminate* (number of dof < number of reactions)
- The force distribution can be determined just by applying the equations of equilibrium in a statically determinate system

(new from specified Units)

- Constitutive relations and compatibility of displacement must be applied to determine the reactions in a statically indeterminate system (simultaneously determine displacements)
- *Constitutive relations* are between a force applied and the resulting displacement (or vice versa)
- Constitutive relations depend on the material (and certain properties) and the shape of the part (geometry)

- The general form of a constitutive relation is: (general force) = (constitutive factor)(general displacement)
- The *Compatibility of Displacement* indicates that configurations which are attached must have displacements consistent with the attachments
- If a structure is geometrically symmetric and is loaded symmetrically, then the internal forces must also be symmetric
- If the response of materials/structures are linear (elastic materials) and undergo small (i.e., linear) deflections, then the effects of different loading can be superposed

INDICIAL NOTATION AND TRANSFORMATIONS....

- Latin subscripts (m, n, p, q,….) take on the values 1, 2, 3 and represent 3-D; Greek subscripts (α, β, γ,......) take on the values 1, 2 and represent 2-D.
- Repeated subscripts within one term are called *dummy/repeated* indices and are summed on; subscripts which appear only once on the left side of the equation within one term are called *free* indices and represent separate equations
- The *order* of a tensor is denoted by the number of subscripts it has
- A tensor is transformed by using a direction cosine for each order of the tensor

STRESS

- ***Stress*** is a measure of intensity of force acting at a point (Force/Area as Area → 0) and has magnitude and direction
- There are two types of stress -- normal/extensional and shear
- The stress tensor is symmetric -- this is due to (moment) equilibrium
- There are other notations by which stress is sometimes represented (involving x, y, z, τ, etc.). Change in the notation does not change what the stress is, only how it is represented.

- Stress acts on a face (positive associated with a positive face normal) in a direction
- The stress tensor, σ_{mn}, indicates the face (x_m) on which the stress acts and its direction (x_n)
- All bodies are in equilibrium and this can be represented on a pointwise basis via the three equations of stress equilibrium
- The case where stresses in only two dimensions are important is known as *plane stress*