UNIFIED HANDOUT

MATERIALS AND STRUCTURES - #M-7

Fall, 2008

Concept Review Sheet

for Unified Q(TMS)4: Units M2.1-2.4

INDICIAL NOTATION AND TRANSFORMATIONS....

- Latin subscripts (m, n, p, q,....) take on the values 1, 2, 3 and represent 3-D; Greek subscripts (α, β, γ,......) take on the values 1, 2 and represent 2-D.
- Repeated subscripts within one term are called *dummy/repeated* indices and are summed on; subscripts which appear only once on the left side of the equation within one term are called *free* indices and represent separate equations
- The *order* of a tensor is denoted by the number of subscripts it has
- A tensor is transformed by using a direction cosine for each order of the tensor

STRESS AND STRAIN....

- ***Stress*** is a measure of intensity of force acting at a point (Force/Area as Area $\rightarrow$ 0) and has magnitude and direction
- ***Strain*** is the deformation of the continuum at a point (or the percentage deformation of an infinitesimal element)
- There are two types of stress and strain -- normal/extensional and shear
- The stress tensor is symmetric -- this is due to (moment) equilibrium
- The strain tensor is symmetric -- this is due to geometrical considerations
- There are other notations by which stress and strain are sometimes represented (involving x, y, z, τ, γ, etc.). Change in the notation does not change what the stress or strain is, only how it is represented.

- Stress acts on a face (positive associated with a positive face normal) in a direction
- The stress tensor, σ_{mn}, indicates the face (x_m) on which the stress acts and its direction (x_n)
- All bodies are in equilibrium and this can be represented on a pointwise basis via the three equations of stress equilibrium
- The case where stresses in only two dimensions are important is known as *plane stress*

- Overall displacement has contributions from 4 basic parts: pure translation, pure rotation, elongation, shear

- For "small" strains, there is no coupling between extensional and angular deformation, and strain and displacement are directly related via geometry in the strain-displacement relations involving derivatives of displacement
- Shear strain represents angular change
- There is a factor of 2 difference between *engineering* shear strain (total angular change) and *tensorial* shear strain (half the angular change)
- Due to geometrical considerations and the need for *compatibility* in a continuum, there are three conditions of compatibility that relate the six strains

- Stress and strain are second-order tensors and require two direction cosines for transformation
- Stress transformation is based on equilibrium; strain transformation is based on geometry
- There is a set of axes into which any state of stress (or strain) can be resolved such that there are no shear stresses (or strains). These are known as the principal axes of stress (or strain) and the resolved set of stresses (or strain) are known as the *principal stresses* (or *strains*)
- There are *principal directions* associated with these principal stresses (or strains)
- There are planes along which the values of the shear stresses (or strains) are maximized and these are oriented at 45° to the principal axes of stresses (or strains)