Unit M1.3
Uses of Equilibrium

Readings:

CDL 1.7, 1.8

16.001/002 -- *"Unified Engineering"*
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

LEARNING OBJECTIVES FOR UNIT **M1.3**

Through participation in the lectures, recitations, and work associated with Unit M1.3, it is intended that you will be able to………

- ….**represent** the boundary conditions of a body via the use of the idealizations of supports
- ….**model** a body/system and forces and moments acting on such through the use of a Free Body Diagram
- ….**classify** mechanical systems into three categories
- ….**calculate** the reaction forces in a statically determinate system

Thus far, we have talked about external forces and the requirements for equilibrium. But there are another set of forces (besides internal forces) on bodies/structures.

In statics, these bodies are restrained/supported and thus we need to consider the forces created at such supports.

We must first consider

Types of Support (and Their Reactions)

Supports can also be thought of as a restraint or constraint.

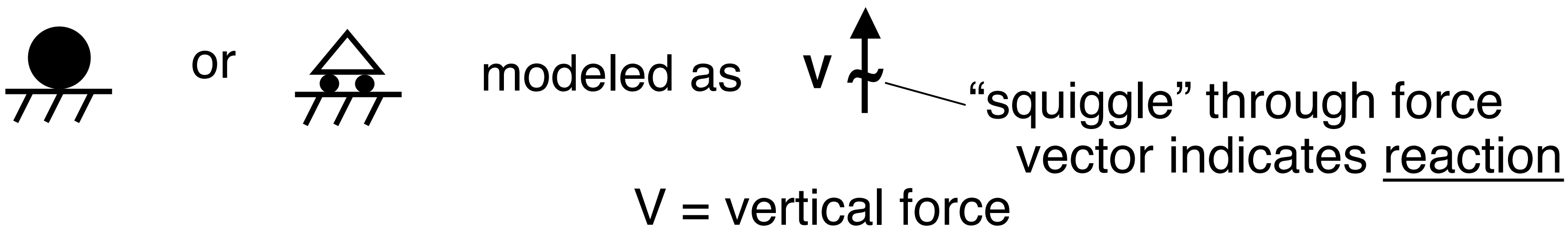
--> we draw idealized versions to represent reality.

IMPORTANT: Realize limitation of model. Model is not reality.

--> Typical idealizations:

1. Roller

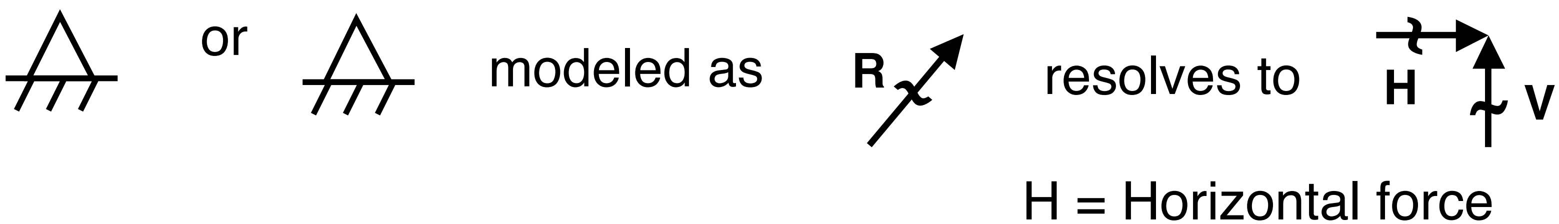
Figure M1.3-1 **Representation of a roller**



--> Supporting surface is frictionless. Therefore, support reaction is only perpendicular to surface of support (allows sliding) e.g., (point support)

2. Pin

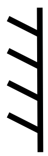
Figure M1.3-2 **Representation of a pin**



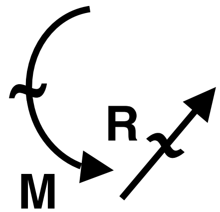
Attached but allows swiveling. Thus reactions in perpendicular and parallel directions to surface, but no moment (actually in any direction, but resolve to two components, e.g., hinge).

3. <u>Clamp</u>

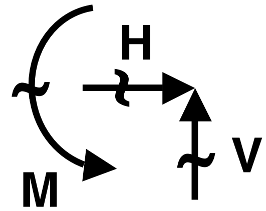
Figure M1.3-3 **Representation of a clamp**



modeled as



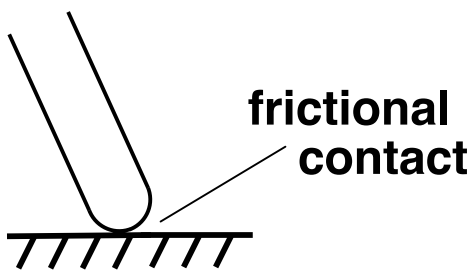
resolves to



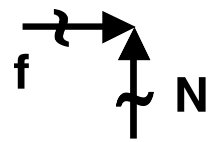
M = Moment

4. <u>Friction Surface</u>

Figure M1.3-4 **Representation of a friction surface**



modeled as



N = Normal force

f = frictional force

With:

$$f \leq \mu N$$

where: μ = coefficient of friction

f is

- parallel to plane of support
- proportional to normal force, N
- acting in opposite direction to motion which would occur if $f = 0$

We can now use these with the external forces on bodies to draw…

Free Body Diagrams

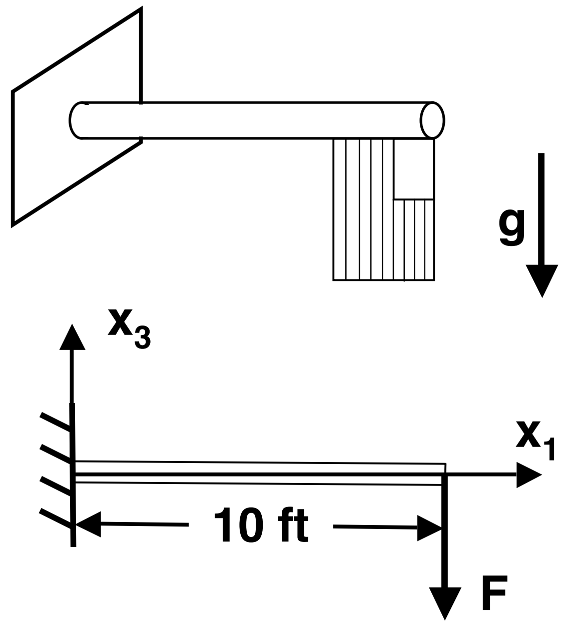
A free body diagram isolates a body and identifies the system of forces (external and reactions) acting on it

Similar concept in other disciplines…

--> There are two basic steps in drawing the free body diagram:

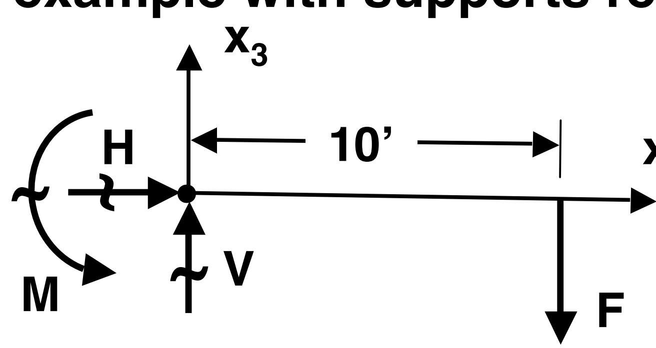
1. Make a <u>neat</u> diagram of the body with applied loads and idealized supports and important dimensions and axis system

Figure M1.3-5 **<u>Example</u>: 10-foot flagpole in a wall with weight (flag) at end**



2. Replace supports by reactions which model them in ideal case

Figure M1.3-6 **Flagpole example with supports replaced by reactions (models)**



Once we have a free body diagram, what can we do with it?

Determine reaction forces, internal forces, etc. But the results lump into

Three Problems/Classes/Categories

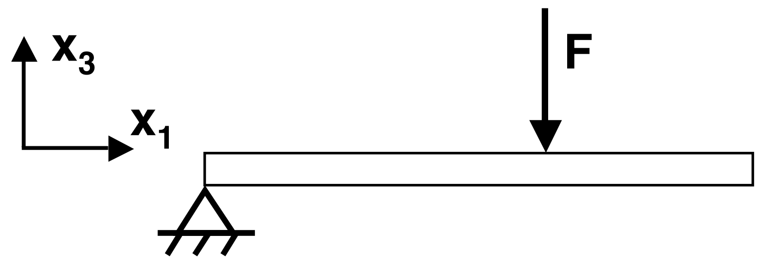
(illustrate in 2-D, can generalize to 3-D)

1. Dynamic

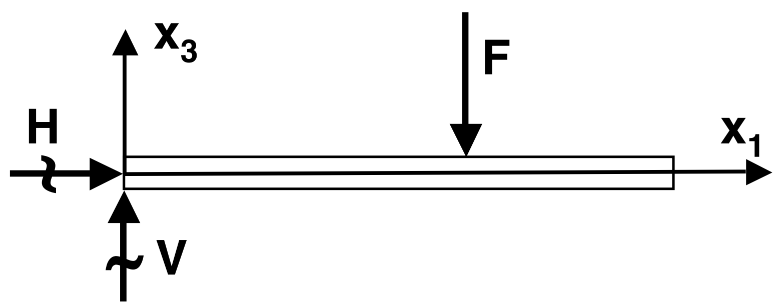
(Number of rigid body mode $\underbrace{\text{degrees of freedom}}_{\text{d.o.f.}}$) **>** (Number of reactions)

=> Body moves

Figure M1.3-7 **Example of Dynamic Problem**



--> Draw FBD (Free Body Diagram):



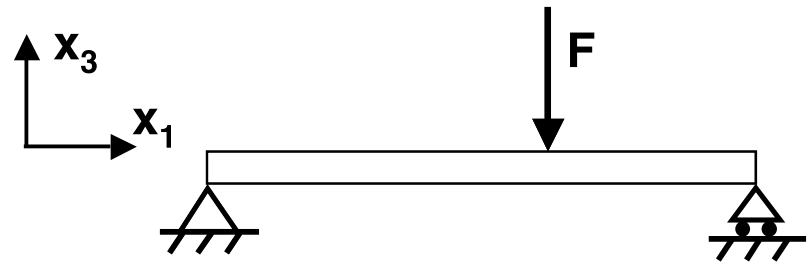
2 reactions
3 dof (lateral in x_1, lateral in x_3, rotation in x_1 - x_3 plane)

=> *Dynamics!*

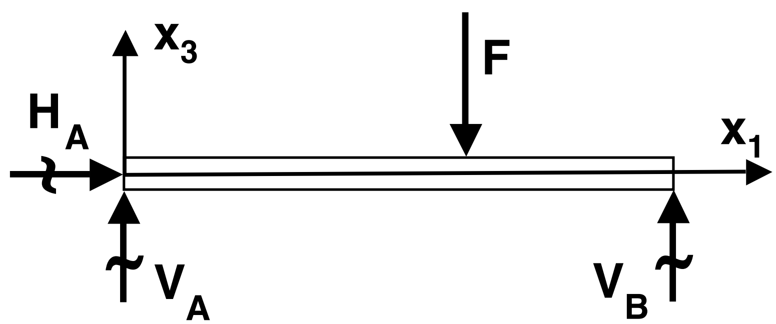
2. Statically Determinate

(Number of rigid body mode degrees of freedom) **=** (Number of reactions)

Figure M1.3-8 **Example of Statically Determinate Problem**



--> Draw FBD (Free Body Diagram):



3 reactions

3 dof (lateral in x_1, lateral in x_3, rotation in x_1 - x_3 plane)

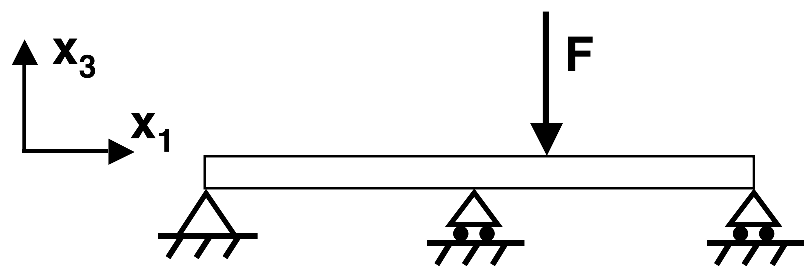
--> *Implication: Can determine reactions and internal forces purely from equilibrium considerations.*

=> Does not matter what the material is with regard to reactions and internal forces!

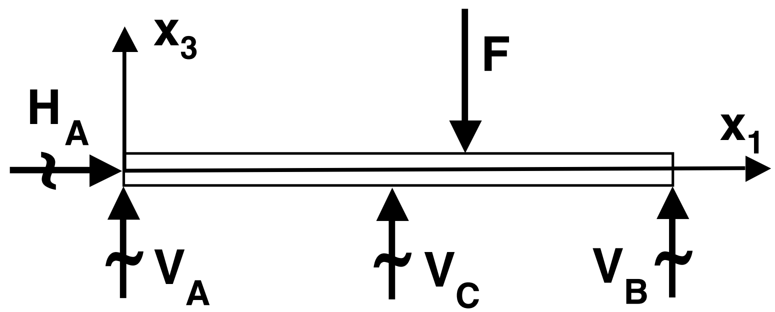
3. Statically Indeterminate

(Number of rigid body mode degrees of freedom) **<** (Number of reactions)

Figure M1.3-9 **Example of Statically Indeterminate Problem**



--> Draw <u>FBD</u> (Free Body Diagram):



4 reactions
3 dof (lateral in x_1, lateral in x_3, rotation in x_1 - x_3 plane)

--> *<u>Implication</u>: Can <u>not</u> determine reactions and internal forces from equilibrium but also need constitutive relations (deformation of structure affects reactions and internal forces.*

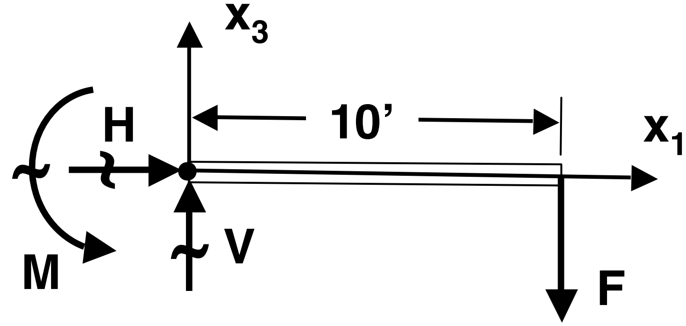
=> Material does make a difference for internal forces and reactions

Thus far we have learned how to apply equilibrium. Let's, therefore, concentrate on category 2…..

Statically Determinate Systems

a) Can determine force distribution by applying equations of equilibrium
  --> Use flagpole as example (Planar System, but again can generalize to 3-D)

Figure M1.3-10 **Free body diagram of 10-foot flagpole (massless) with flag at end**



$$\left.\begin{array}{l}\text{3 reactions}\\ \text{3 dof}\end{array}\right\} \Rightarrow \text{Statically Determinate}$$

$$\sum F \left(x_1 - \text{direction} \right) = 0 \quad \text{(generic form)}$$

apply to specific case

$$\sum F_H = 0 \quad \xrightarrow{+} \quad \Rightarrow H + \underbrace{0}_{\text{no applied forces}} = 0 \Rightarrow \boxed{H = 0}$$

indicate positive direction

$$\sum F \left(x_3 - \text{direction} \right) = 0$$

$$\Rightarrow \sum F_V = 0 \quad \uparrow + \quad \Rightarrow V - F = 0$$

$$\Rightarrow \boxed{V = F}$$

Finally… $\sum M \left(\text{about } x_2 \right) = 0$

$$\Rightarrow \sum M_0 = 0 \quad \circlearrowleft +$$

Choose convenient point about which to take moments.

Origin is often a selection.

$$\Rightarrow \; M - F(10\,ft) = 0$$

$$\Rightarrow \boxed{M = F(10\,ft)}$$

All reactions determined, so….

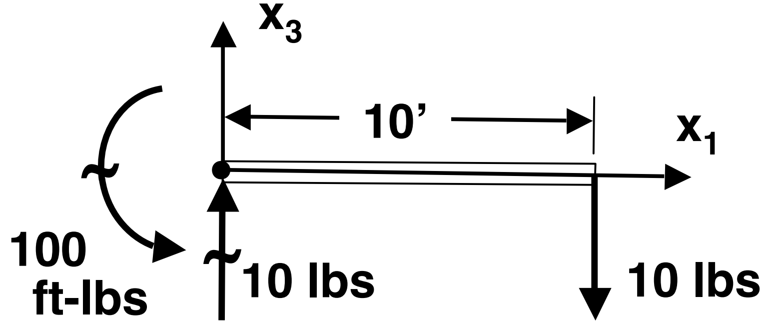
If F = 10 lbs, then V = 10 lbs.

M = 100 ft. lbs.

b) Redraw Free Body Diagram (FBD) using results:

Figure M1.3-10 **Free body diagram of (massless) flagpole redrawn with 10-pound flag and values of reactions**

Note: *H = 0* , so don't draw it



Now let's see how we can investigate the distribution of forces <u>in</u> a structure once we know the reactions in a statically determinate case (still using only the equations of equilibrium)