Unit M1.4
(All About)
Trusses

Readings:

CDL 1.9

16.001/002 -- *"Unified Engineering"*
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LEARNING OBJECTIVES FOR UNIT **M1.4**

Through participation in the lectures, recitations, and work associated with Unit M1.4, it is intended that you will be able to………

- ….**model** a truss structure through the use of a Free Body Diagram

- ….**calculate** the reaction forces for a statically determinate truss structure

- ….**determine** the loads carried in each bar of a truss through the **use** of the Method of Joints and the Method of Sections

A truss is a very useful structural configuration in which <u>bars</u> are connected at <u>joints</u> and the overall configuration carries load through <u>axial</u> <u>force</u> in the bars.

Generally trusses are three-dimensional (3-D) although, they can be reduced to two-dimensional (2-D) form as we shall see…..

<u>Uses of trusses</u>

Figure M1.4-1 **Bridges**

Figure M1.4-2 **Buildings**
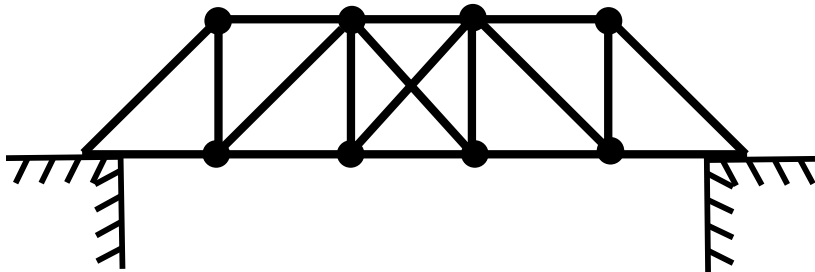
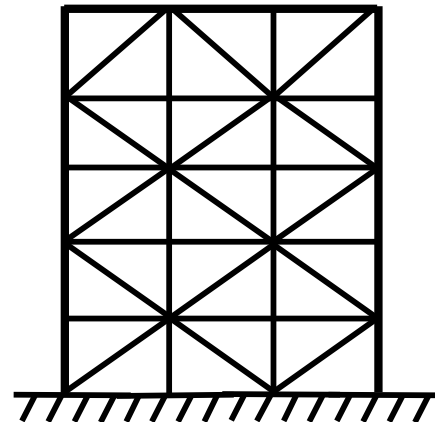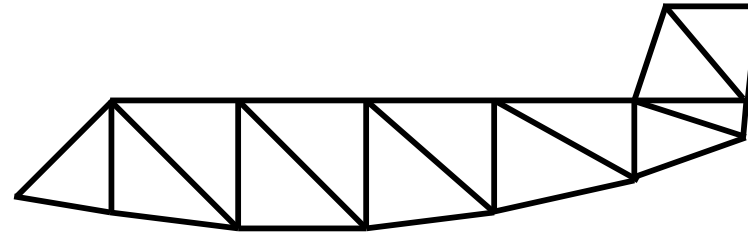- **Cranes**
- **Others?**

Figure M1.4-3 **Early days of aircraft**

main load carrying members covered by light skin

Now: semi-monocoque (egg shell)

--> load-carrying members with load-carrying skin

• Space station, other space structure

As previously noted, these are generally 3-D, but let's consider the

Idealized Planar Truss

(concepts and techniques developed here can be extended/applied to the 3-D case)
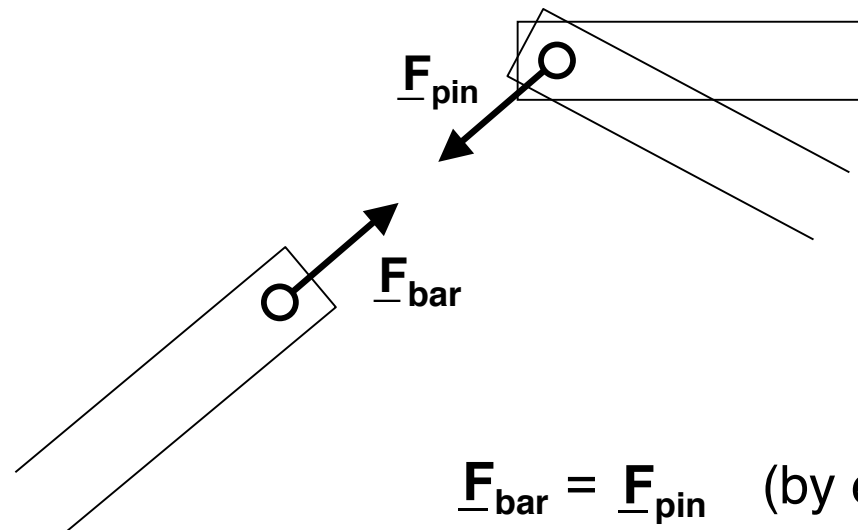
Let's first **define** an idealized planar truss (this is a ***model***)

1.   All bars are straight
2.   Bar joints are frictionless pins
3.   Bars are massless and perfectly rigid (for loading analysis)
4.   All loads and reactions are applied at the joints
5.   Loads in members are colinear (axial -- aligned with long axis of bar)

<u>Thus</u>:

Bars carry <u>only</u> axial forces

Figure M1.4-4 **Consideration of load transfer at pin**

$\underline{F}_{pin}$

$\underline{F}_{bar}$

$\underline{F}_{bar} = \underline{F}_{pin}$ (by equilibrium)

Pin bears or pulls on bar and only axial force can result

So we now "have" an idealized planar truss.

The first step in the analysis is.....
   (purpose of analysis: determine reaction forces and the internal
      load/forces in bars)
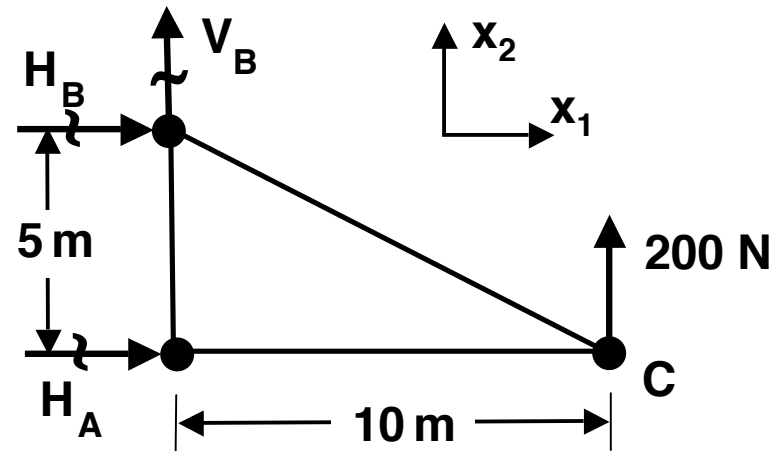
Determination of reactions

The fact that the structural body is a truss does not change the procedure:

   - Draw Force Body Diagram

   - Is it Statically Determinate (?)

   - If so, proceed, if not…(wait til future units!)

   - Apply planar equations of equilibrium

Figure M1.4-5 **Example of a "simplest" truss (3-member)**

B

5 m

200 N

A

C

10 m

--> Draw FBD

H_B

V_B

x_2

x_1

5 m

200 N

H_A

C

10 m
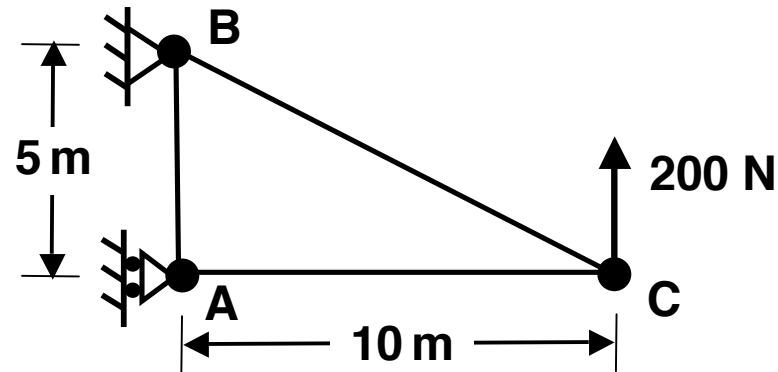
- Is this statically determinate?

YES

③ reactions = ③ degrees of freedom

(lateral in x_1, lateral in x_2, rotation about x_3)

=> Proceed

- Apply planar equations of equilibrium:

$\underline{\underline{3}}$ ($\underline{\underline{3}}$ degrees of freedom = $\underline{\underline{3}}$ reactions)

$$\sum F_1 = 0 \quad \xrightarrow{+} \quad \Rightarrow H_A + H_B = 0$$

$$\sum F_2 = 0 \quad \uparrow + \quad \Rightarrow V_B + 200N = 0$$

$$\Rightarrow V_B = -200N$$

$$\sum M_{3(A)} \quad \left(\begin{array}{c}+\\ \end{array}\right. \quad \Rightarrow (200N)(10\ m) - H_B(5\ m) = 0$$

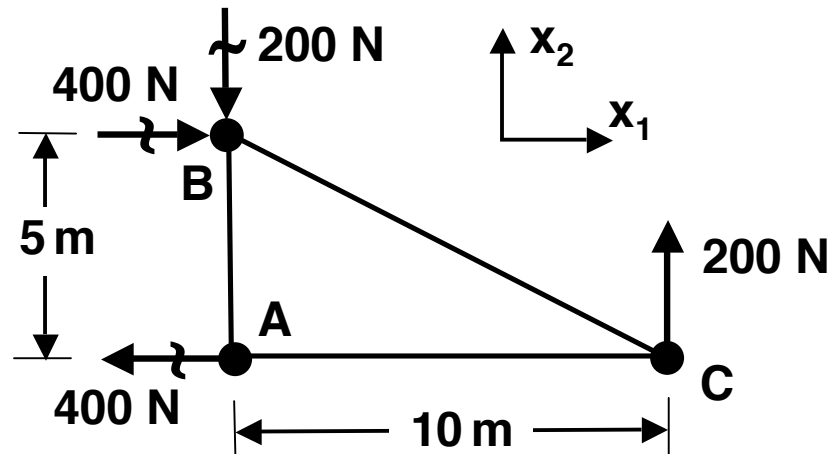*about point A* $\qquad \Rightarrow H_B = 400N$

And using $\sum F_1$ gives: $H_A = -400N$

Summarizing: $\qquad H_A = -400N$

$$H_B = 400\ N$$

$$V_B = -200N$$

Figure M1.4-6 **Redrawing Free Body Diagram with reactions determined**

check by taking $\sum M_{3(B)}$ ↻+ ...must also be zero

$$ -\left(400N\right)\left(5\ m\right) \ + \ \left(200N\right)\left(10\ m\right) \ = \ 0 \quad \sqrt{}\ \textbf{checks} $$

Once the reactions are determined, we move on to determining the internal forces in the bars.

These are two methods:

- Method of Joints
- Method of Sections

Let's first explore the…

Method of Joints

Basically isolate each joint and draw a free body diagram and analyze it. Work progressively along the truss.

So once reactions are known, the procedure is:
- isolate a joint by "cutting" bars
- "replace" "cut" bars by tensile internal forces pulling away from joint coincident with bar
- calculate and show orthogonal components of force for each bar (use geometry)
- apply equations of planar equilibrium
- positive forces are tensile; negative forces are compressive

--> do this at joints progressively from end of truss

This is best illustrated through an example…

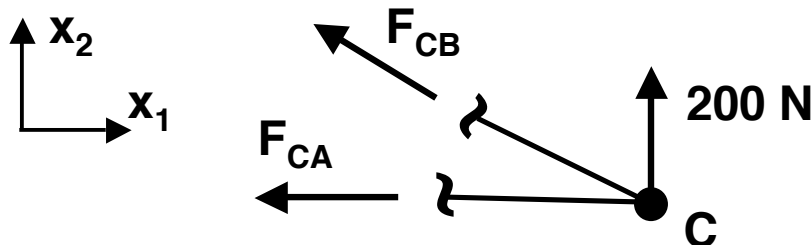*Figure M1.4-7* **Example of 3-bar truss (from before)**

Recall:

200 N

400 N

x_2

x_1

B

5 m

200 N

A

400 N

10 m
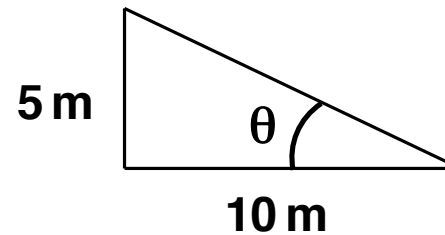
C
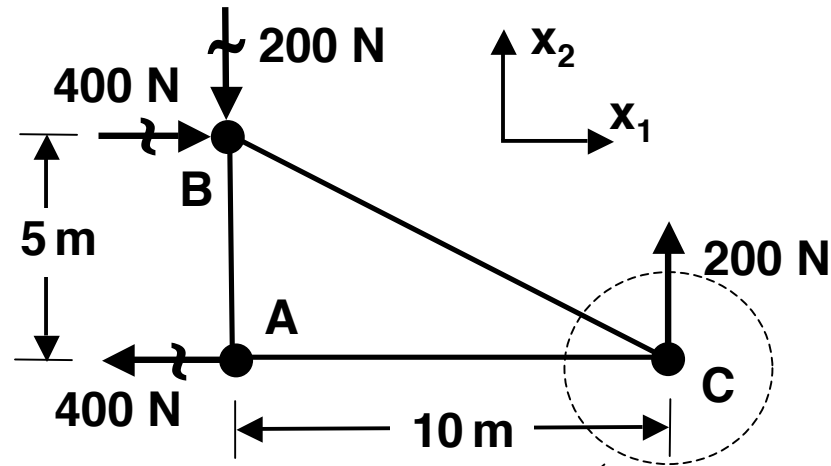
Start by "isolating" joint C

ʓ on a bar represents a "cut"

So we "cut" bars CB and CA and "replace" them by their associated internal forces F_{CB} and F_{CA}.

Figure M1.4-8 **Isolation of joint C**

x_2

x_1

F_{CB}

F_{CA}

200 N

C

F_{CB} is at an angle. Thus, need to find components along x_1 and x_2
Using geometry:

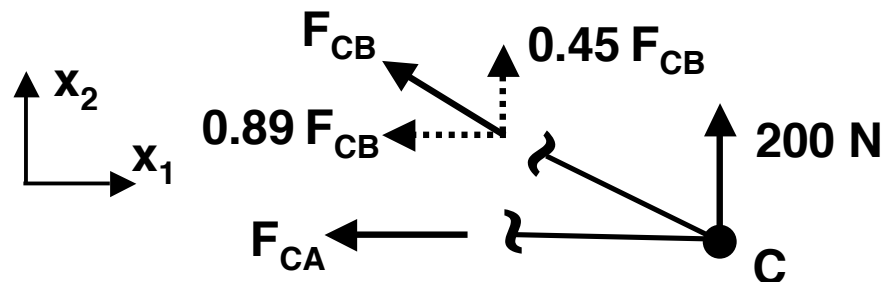**Figure M1.4-9** **Geometry of angle of bars (at joint C)**

5 m 10 m θ

$$\theta = \tan^{-1} \frac{5m}{10m}$$

$$\Rightarrow \tan^{-1} \frac{1}{2} \approx 27°$$

$$\Rightarrow x_1 - \text{component of } F_{CB} = F_{CB} \cos 27° = 0.89 F_{CB}$$

$$\Rightarrow x_2 - \text{component of } F_{CB} = F_{CB} \sin 27° = 0.45 F_{CB}$$

Figure M1.4-10 **Redrawing of joint C with components of F_{CB} illustrated**

F_{CB} 0.45 F_{CB} x_2 0.89 F_{CB} x_1 200 N F_{CA} C

--> Now apply planar equilibrium

$$\sum F_1 \; \xrightarrow{+} \quad \Rightarrow \quad -0.89 \, F_{CB} - F_{CA} = 0$$

$$\sum F_2 \; \uparrow + \quad \Rightarrow \quad 0.45 \, F_{CB} + 200N = 0$$

$$\Rightarrow \quad F_{CB} = -444N$$

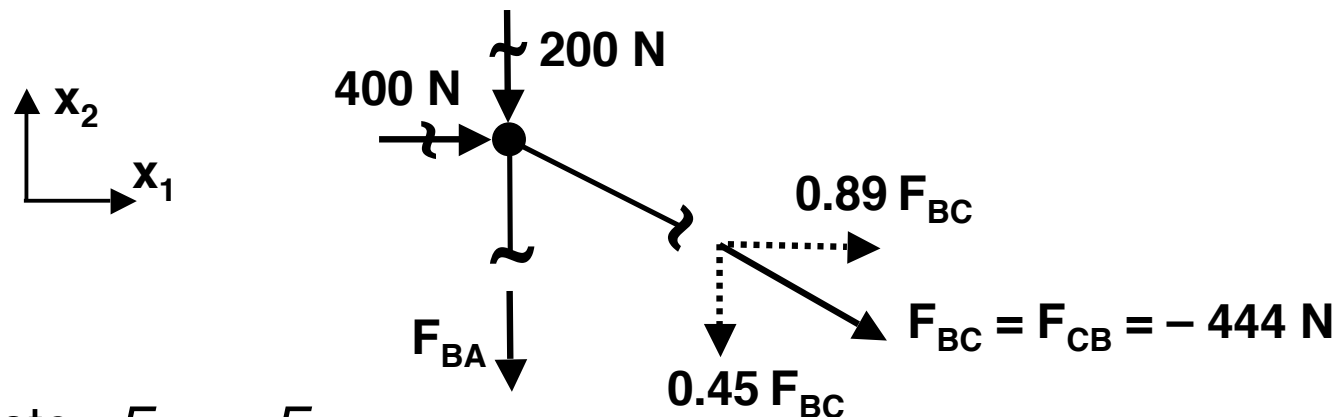$$\sum M_3 \; \curvearrowright + \quad \textbf{Nothing creates moment about joints!}$$

Using F_{CB} in $\sum F_1$ gives: $F_{CA} = 395N$

Figure M1.4-11 Redrawing of joint C with values of force

compression since arrow points <u>in</u>

444 N

200 N   *or*

395 N

− 444 N

200 N

395 N

C

C

x_2

x_1
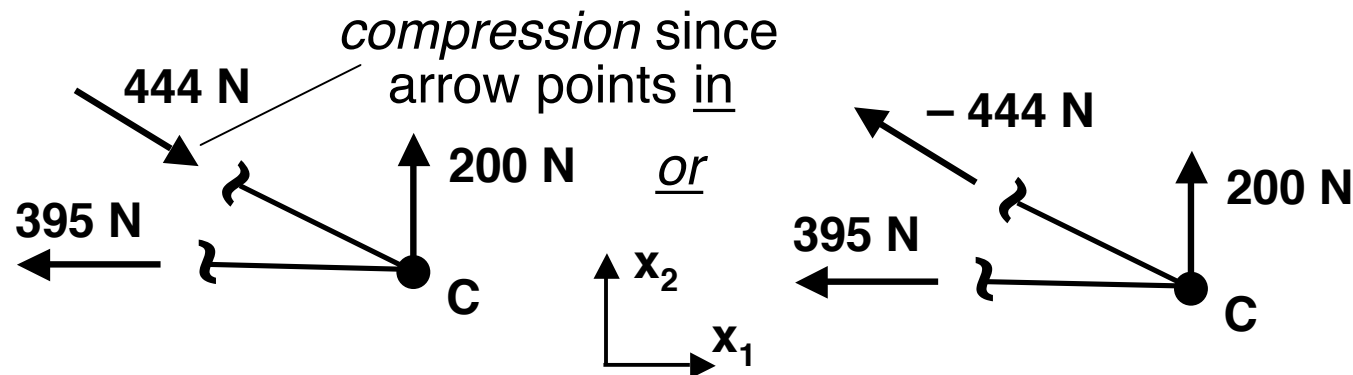
--> Now move on to next joint (joint B)

Figure M1.4-12 Isolation of joint B

200 N

400 N

x_2

x_1

0.89 F_{BC}

F_{BA}

0.45 F_{BC}

$F_{BC} = F_{CB} = -444$ N

Note: $F_{CB} = F_{BC}$

Note: Sense of F_{CB} "turns around" at other end of bar

--> Apply equilibrium

$$\sum F_2 = 0 \quad \uparrow_+ \quad \Rightarrow \quad -200N - F_{BA} - 0.45\,F_{CB} = 0$$

$$\Rightarrow F_{BA} = -200N - 0.45\,(-444N)$$

$$\Rightarrow F_{BA} = 0$$

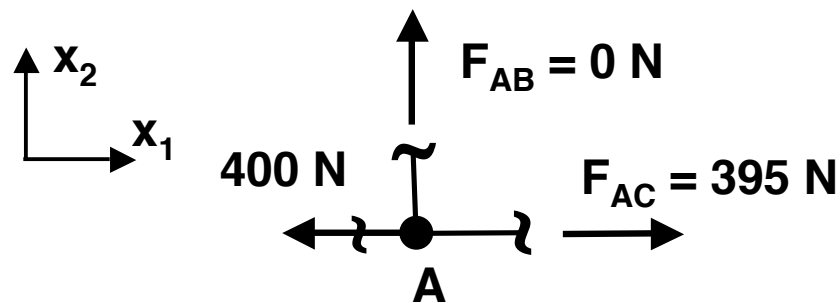Check using $\sum F_1 = 0 \xrightarrow{+}$

$$\Rightarrow 400N + 0.89 \left(F_{BC} \right) \stackrel{?}{=} 0$$

$$\Rightarrow 400N + 0.89 \left(-444N \right) \stackrel{?}{=} 0$$

Have all bar loads, but use final joint **as a check**

$$\Rightarrow 400N - 390N \stackrel{?}{=} 0$$

Figure M1.4-13 **Isolation of joint A**

By inspection, is this in equilibrium?

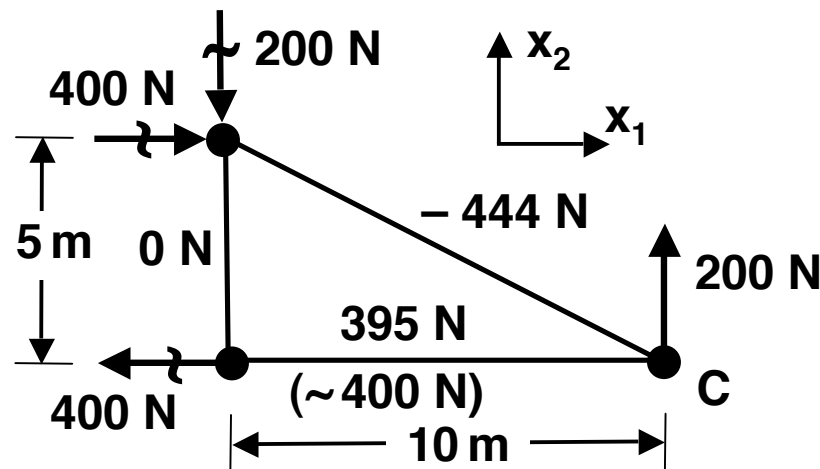$$\sum F_1 \text{ gives: } 395N - 400N = -5N$$

Why?

Finally, draw truss with bar forces written above corresponding bar

(+) tension

(–) compression

Figure M1.4-14 **Representation of truss with all bar loads**

--> Also <u>notice</u> that bar forces are much like the $\underline{R}_{ij}$ forces we used in Unit U4 when considering a group of particles

This worked quite well for a truss with only a few bars or if we want the load in each bar, then we march progressively through the truss.

<u>But</u>, what if we have a bigger truss (one with more members/"bays") and we want only one of few bar loads?   Go to the….

<u>Method of Sections</u>

Uses equilibrium of a <u>section</u> of the truss which contains two or more joints.

Again, begin by determining reactions, then isolate a section by….

- "cutting" the truss into sections (take a cut through the truss)
- (<u>again</u>) "replace" "cut" bars by tensile internal forces pulling away from joint coincident with bar
- (<u>again</u>) calculate and show orthogonal components of force for each bar (use geometry)
- (<u>again</u>) apply equations of planar equilibrium
- (<u>again</u>) positive forces are tensile; negative forces are compressive

--> Do this at any point of interest

Notes:

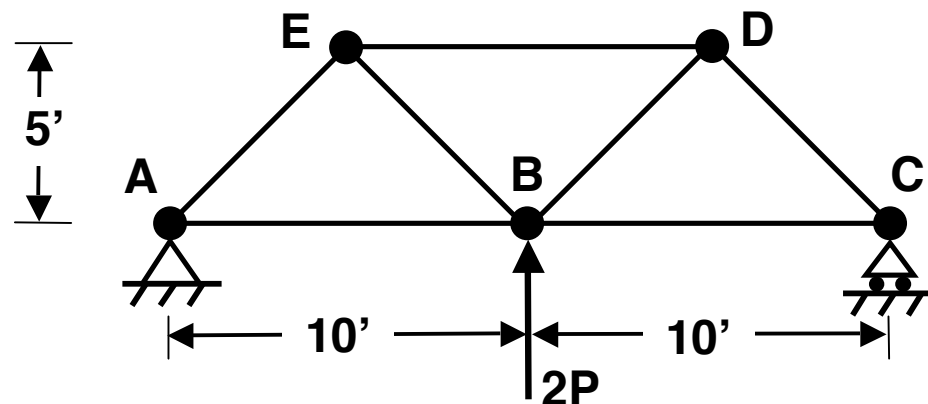- Moment equilibrium equation will be of use here

(Why? Not considering joint)
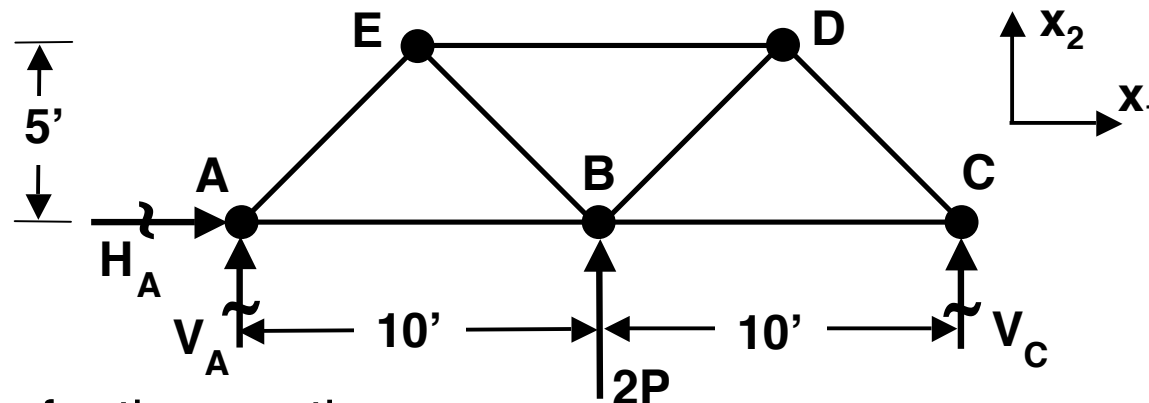
- can only have 3 unknown bar forces at a time

(Why? Only 3 equations of equilibrium)

Again, this is best illustrated by an

Figure M1.4-15a **Example of analysis of truss using method of sections**

First draw Free Body Diagram

Solving for the reactions:

$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow H_A = 0$$

$$\sum F_2 = 0 \uparrow + \Rightarrow V_A + V_C + 2P = 0$$

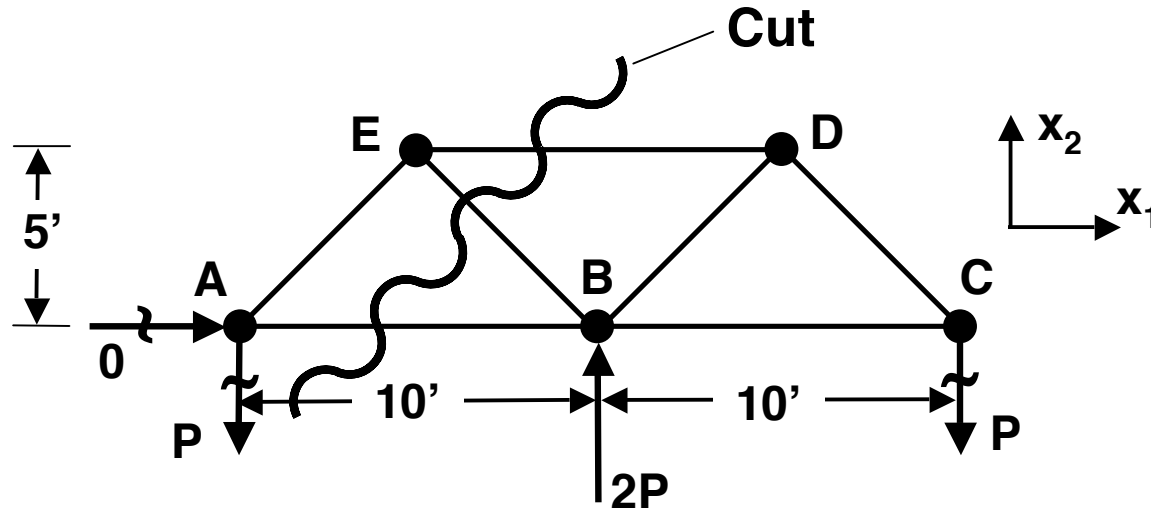$$\sum M_{3(A)} = 0 \,\circlearrowright + \Rightarrow 2P(10') + V_C(20') = 0$$

"about point A"

$$\Rightarrow V_C = -P$$
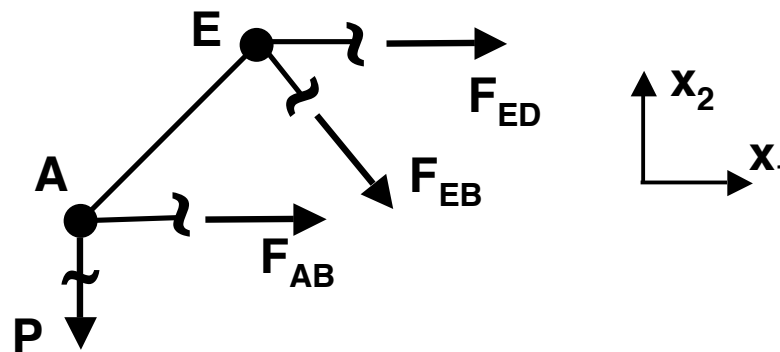
From $\sum F_2$ $V_A = -P$

We are interested in bar EB, so we redraw the Free Body Diagram and take an appropriate "cut"

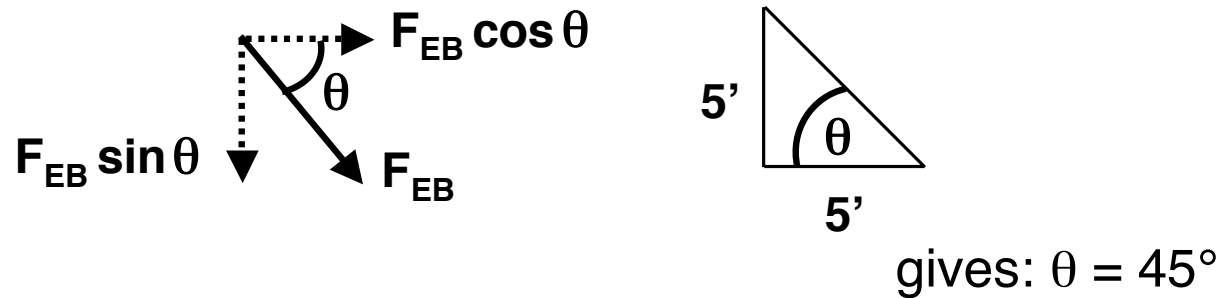*Figure M1.4-16* **Truss with "cut" through section of interest**

Now redraw the "cut" section:

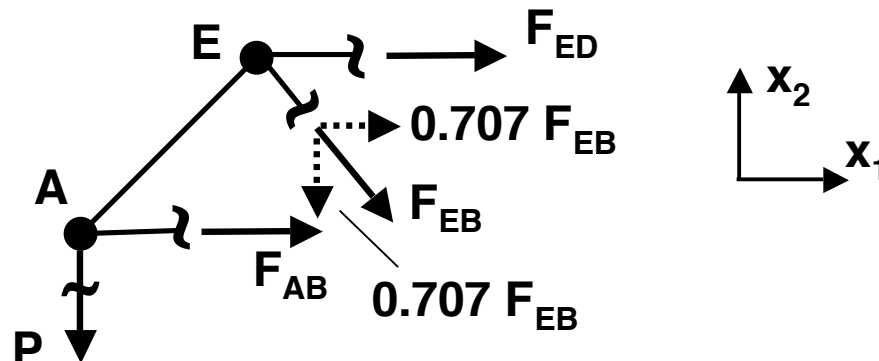*Figure M1.4-17* **Free Body Diagram of "cut" section**

Find the components of F_{EB}:

Figure M1.4-18 **Use of geometry to determine components of F_{EB}**

$F_{EB} \cos \theta$

$F_{EB} \sin \theta$

θ

F_{EB}

5'

5'

θ

gives: $\theta = 45°$

So:

Figure M1.4-19 **Free Body Diagram of "cut" section with resolution of F_{EB}**

E

F_{ED}

0.707 F_{EB}

F_{EB}

A

F_{AB}

0.707 F_{EB}

P

x_2

x_1

Now use the equations of equilibrium:

$$\sum F_1 = 0 \xrightarrow{\;+\;} \Rightarrow F_{AB} + 0.707\, F_{EB} + F_{ED} = 0$$

$$\sum F_2 = 0 \uparrow + \Rightarrow -P - 0.707\, F_{EB} = 0$$

$$\Rightarrow F_{EB} = -1.414\, P$$

<u>and</u>

$$\sum M_{3(E)} = 0 \;\circlearrowright\!+ \Rightarrow F_{AB}(5') + P(5') = 0$$

chose E to isolate F_{EB}

$$\Rightarrow F_{AB} = -P$$

Using these results in $\sum F_1$ gives:

$$-P - P + F_{ED} = 0$$

$$\Rightarrow F_{ED} = +2P$$

So can show:

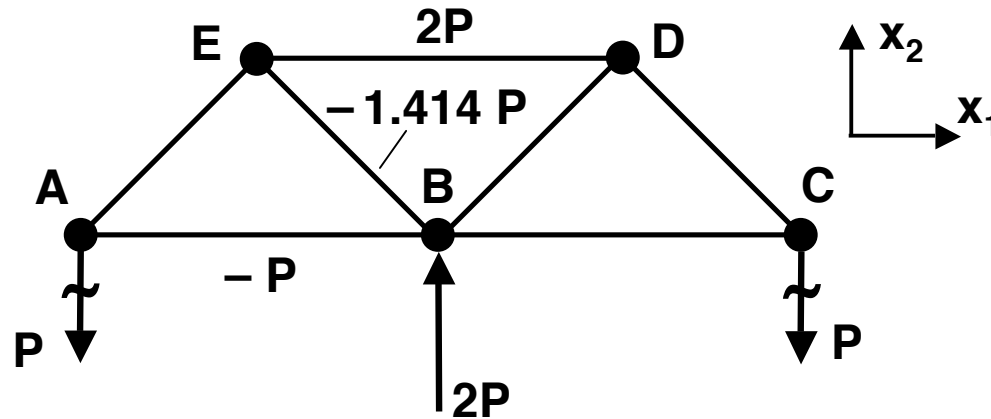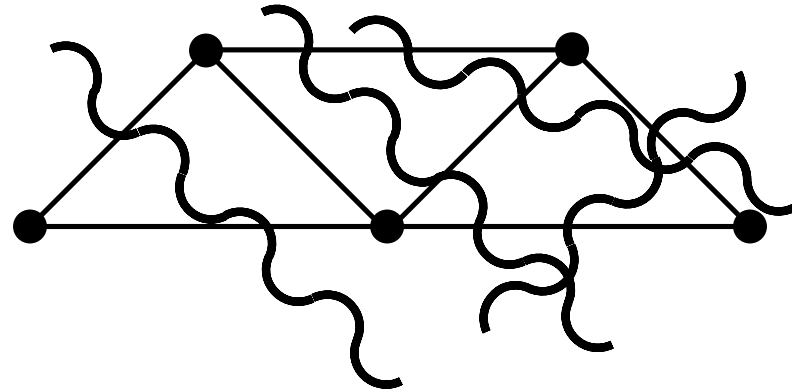**Figure M1.4-20** **Truss redrawn with loads for bars of interest**

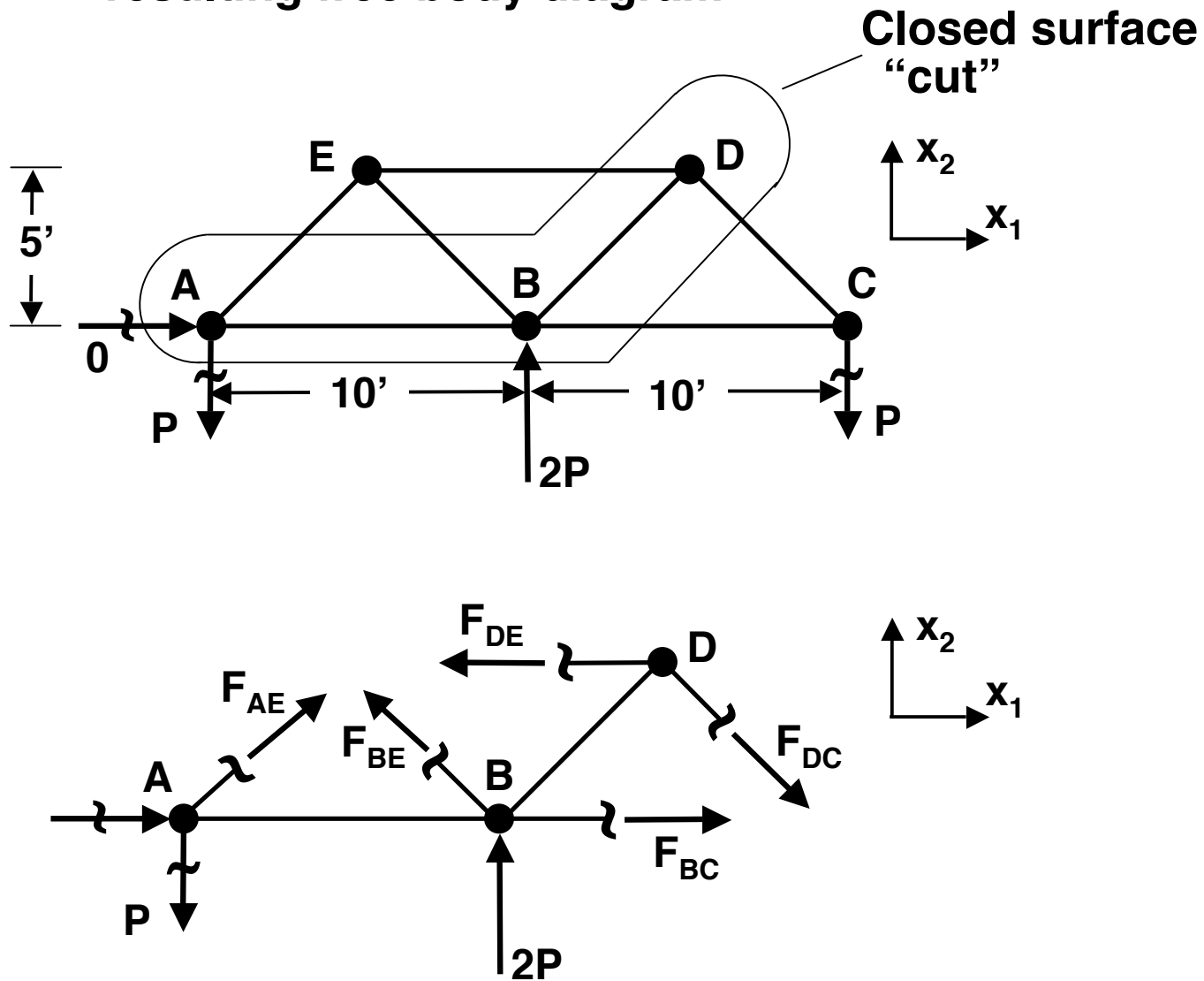**Figure M1.4-21** **Many possible "cuts" through truss depending upon section/bar(s) of interest**

Can go on with more sections as desired.

etc.

More generally can draw closed surface and consider all forces that cross surface to get information…..

Figure M1.4-22 **Illustration of closed surface drawn through truss and resulting free body diagram**


Closed surface "cut"

--> Both the Method of Joints and the Method of Sections are approaches to determining <u>internal</u> <u>equilibrium</u>

As a final note, this was a model (idealized truss). Consider some…

Joint Realities

There are naturally no such things as frictionless pins. Joints are generally more restrained.

Figure M1.4-23 **Some possible bar joints**

weld

solid piece (e.g. wrapped composite)

gusset plate
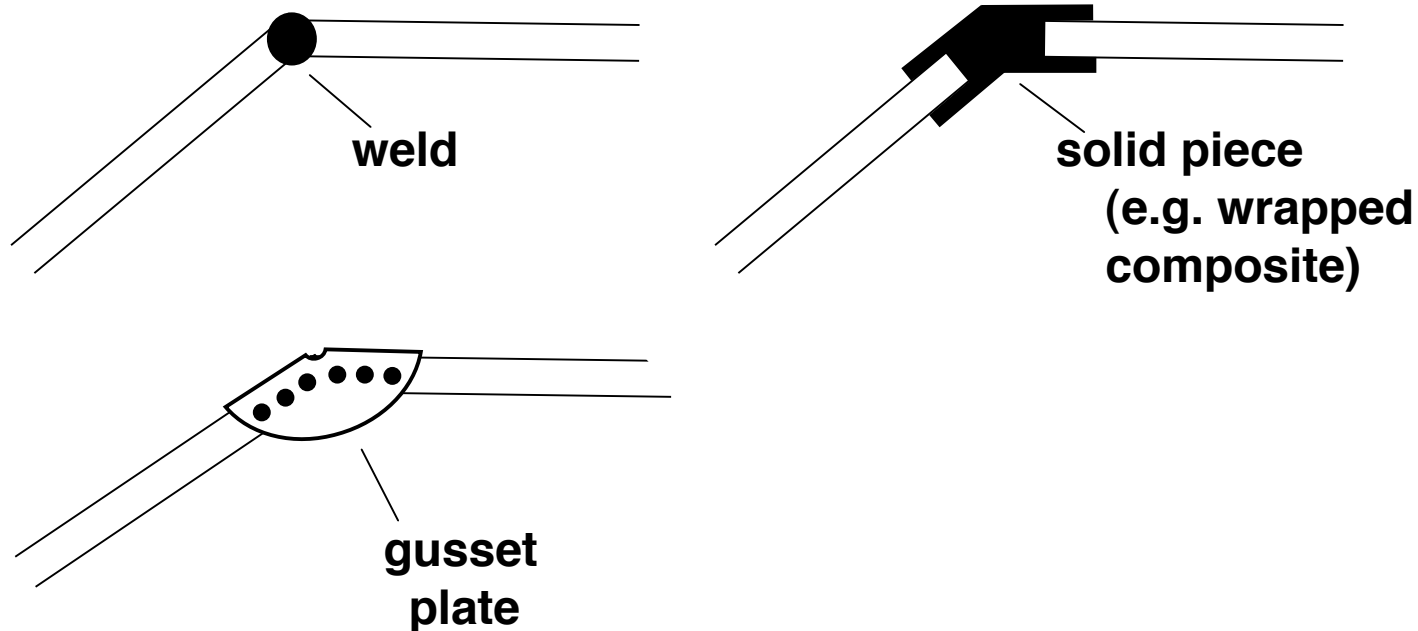
Some space joints are closer to pinned in effect because bars (components) are long and slender

When the ends are constrained, moments can be taken and we must also consider <u>beam</u> behavior (more next term).

Joints are a key <u>limitation</u> of this idealized truss analysis

We've done a lot with equilibrium, but let's now explore what happens when we need more than equilibrium and deal with statically indeterminate sections.