Unit U-A
The Language of Engineering

Readings:

CDL 1.4

16.001/002 -- *"Unified Engineering"*
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

LEARNING OBJECTIVES FOR UNIT **U-A**

Through participation in the lectures, recitations, and work associated with Unit U-A, it is intended that you will be able to………

- ….**apply** the language of engineering systems (units, dimensions, coordinates)
- ….**describe** systems in different coordinate systems **using** transformations and other concepts

Need a number of “things” in order to describe the concepts and items associated with (Unified) Engineering.

The first of these is…

Dimensions and Units

--> A dimension is a physical quantity which can be directly measured

Fundamental dimensions are:

Dimensions of other quantities are "derived"

<u>example</u>: Force … use Newton's law

$$F = Ma = \left[M \frac{L}{T^2}\right]$$

<u>Note</u>: [ ]
bracket means "has the dimensions of"

--> A <u>unit</u> is used to quantify dimensions.

There are many systems of units (e.g., furlongs/fortnights) but the two used in engineering are

- Metric or SI (standard international)
  --> predominant everywhere except US
- British system (English)

- Fundamental Dimensions (examples)

Quantity	Dimension	SI Unit	English Unit	Conversion
Length	L	m	ft (in)	0.305 m/ft
Time	S	s(ec)	sec	-
Mass	M	kg	slug	14.6 kg/slug

- Derived Dimensions (examples)

Quantity	Dimension	SI Unit	English Unit	Conversion
Force	$M \frac{L}{T^2}$	N	lb	4.45 N/lb
Pressure	$M \frac{L}{T^2 \quad L^2}$	Pa	psi	6900 Pa/psi

Notes: Newton = 1 kg m/sec^2
Pound = slug ft/sec^2
Pascal = N/m^2
psi = lb/in^2

*Be careful of weight/mass confusion

--> Use units consistently

Question: If you weigh 200 lbs., what is your mass?

Can use dimensions/units as a check:

$$\underset{\text{energy}}{e} = \underset{\text{mass}}{m}\,\underset{\text{speed of light}}{c}^{2}$$

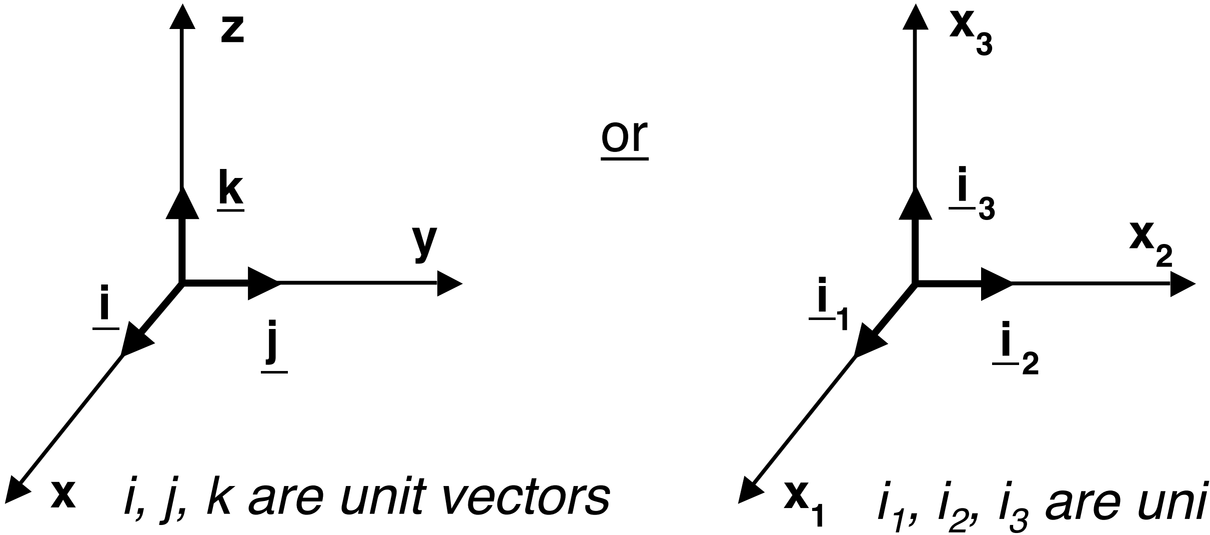
Start of "Dimensional Analysis"

Coordinate Systems

In order to describe items which are characterized by vectors (e.g., force, velocity, acceleration), need a coordinate system.

Most commonly will use rectangular cartesian

Figure U3.1 **Representation of rectangular cartesian system**



i, j, k are unit vectors
In the direction of x, y, z

i_1, i_2, i_3 *are unit vectors*
In the direction of x_1, x_2, x_3

Note: $x \text{ --> } x_1$
$y \text{ --> } x_2$
$z \text{ --> } x_3$

Other coordinate systems are used when it makes sense to describe a body/form in that manner.

Examples?

Important Concept:

The physical quantity does not change because it is described in a different coordinate system or different unit. It remains the same, only the description changes (like a different language).

Transformation of Coordinates

Oftentimes still want to use a rectangular cartesian system, but one that is oriented differently from the original set of axes.

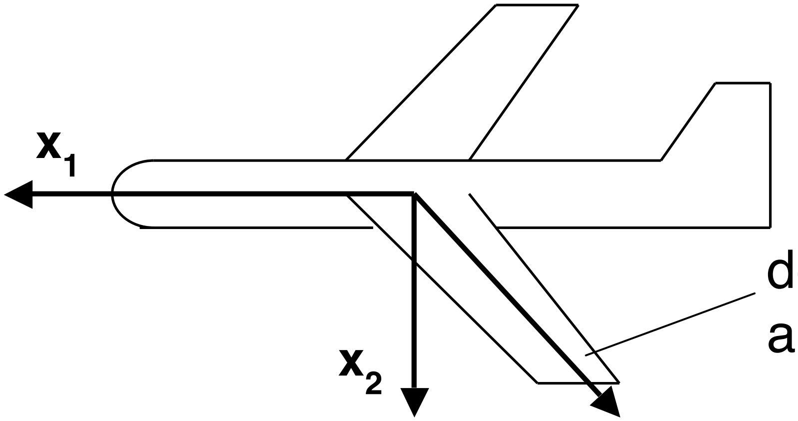
Why?

- structural axes (natural form)
- loading axes

Examples

--> consider a swept wing:

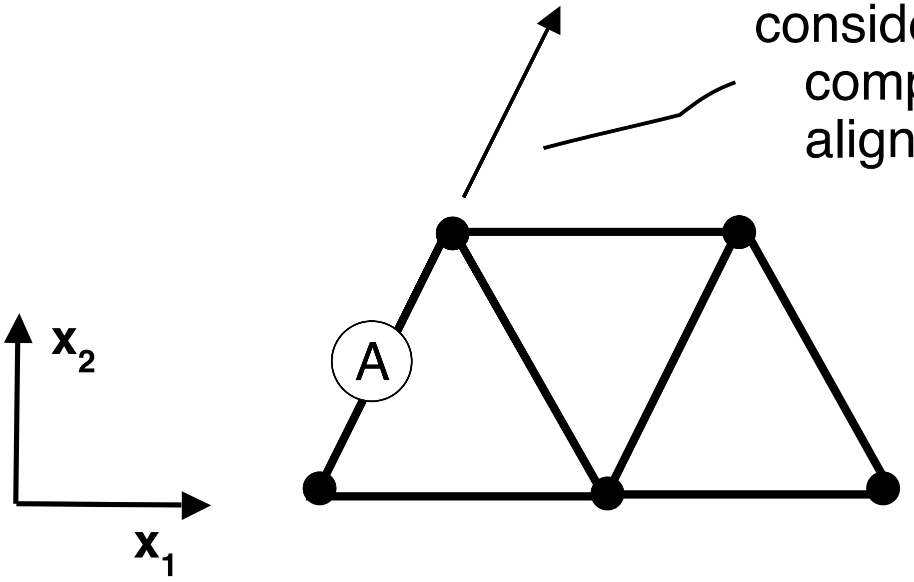
Figure U3.2a



describe wing behavior along axis of wing

--> consider a space truss

Figure U3.2b

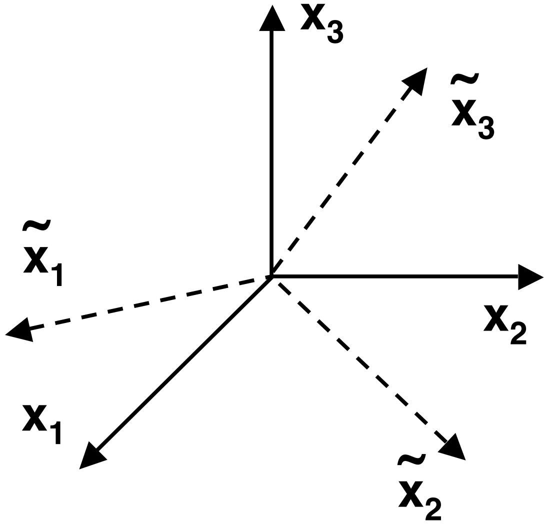


consider behavior of
component A along axis
aligned with its direction

--> other examples?

--> consider this <u>formally</u> via the mathematics:

Figure U3.3 Two rectangular cartesian coordinate systems with the same origin



~ = “tilde” => rotated coordinate system

point p is located by the vector $\underline{r}$ in both systems:

$$\underline{r} = x_1 \underline{i}_1 + x_2 \underline{i}_2 + x_3 \underline{i}_3 = \sum_{m=1}^{3} x_m \, \underline{i}_m$$

$\underline{i}_m$ is unit vector in x_m

and

$$\underline{r} = \tilde{x}_1 \tilde{\underline{i}}_1 + \tilde{x}_2 \tilde{\underline{i}}_2 + \tilde{x}_3 \tilde{\underline{i}}_3 = \sum_{n=1}^{3} \tilde{x}_n \, \tilde{\underline{i}}_n$$

$\tilde{\underline{i}}_n$ is unit vector in $\tilde{x}_n$

Thus: $$\underline{r} = \sum_{m=1}^{3} x_m \, \underline{i}_m = \sum_{n=1}^{3} \tilde{x}_n \, \underline{\tilde{i}}_n$$

--> To relate x_m to $\tilde{x}_n$, let's take the dot product of both sides with $\underline{i}_1$:

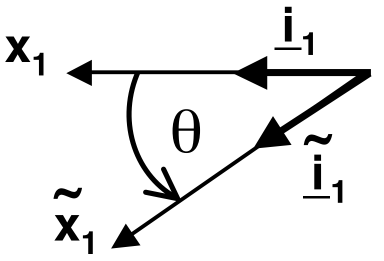
$$\underline{i}_1 \cdot \sum_{m=1}^{3} x_m \, \underline{i}_m = \underline{i}_1 \cdot \sum_{n=1}^{3} \tilde{x}_n \, \underline{\tilde{i}}_n$$

Only non zero term on left hand side is $\underline{i}_1 \cdot \underline{i}_1$

$$x_1 = \tilde{x}_1 \underline{i}_1 \cdot \underline{\tilde{i}}_1 + \tilde{x}_2 \underline{i}_1 \cdot \underline{\tilde{i}}_2 + \tilde{x}_3 \underline{i}_1 \cdot \underline{\tilde{i}}_3 \qquad (*)$$

Recall definition of dot product:

$$\underline{i}_1 \cdot \underline{\tilde{i}}_1 = \underset{1''}{\left|\underline{i}_1\right|} \underset{1''}{\left|\underline{\tilde{i}}_1\right|} \cos\left(\widehat{x_1 \, \tilde{x}_1}\right)$$



$$= \cos \widehat{x_1 \, \tilde{x}_1}$$

angle from x_1 axis to $\tilde{x}_1$ axis

Generalizing:

$$\underline{i}_m \cdot \tilde{\underline{i}}_n = \cos \widehat{x_m \tilde{x}_n} \equiv \ell_{m\tilde{n}}$$

goes with $\tilde{x}_n$

goes with x_m

Definition:

$$\boxed{\ell_{m\tilde{n}} = \cos \widehat{x_m \, \tilde{x}_n} = \underline{i}_m \cdot \tilde{\underline{i}}_n}$$

= Direction Cosine

Returning to equation $(*)$ and using the direction cosines:

$$x_1 = \ell_{1\tilde{1}} \tilde{x}_1 + \ell_{1\tilde{2}} \tilde{x}_2 + \ell_{1\tilde{3}} \tilde{x}_3$$

Note repeated index, so can write:

$$x_1 = \sum_{n=1}^{3} \ell_{1\tilde{n}} \, \tilde{x}_n$$

For the other two components (x_2 and x_3) similar work gives:

$$\left(\underline{i}_2 \cdot \underline{r}\right) \rightarrow x_2 = \sum_{n=1}^{3} \ell_{2\tilde{n}} \, \tilde{x}_n$$

$$\left(\underline{i}_3 \cdot \underline{r}\right) \rightarrow x_3 = \sum_{n=1}^{3} \ell_{3\tilde{n}} \, \tilde{x}_n$$

So have 3 equations that can be expressed as:

$$\boxed{x_m = \sum_{n=1}^{3} \ell_{m\tilde{n}} \, \tilde{x}_n} \qquad (m = 1, 2, 3)$$

--> can also show the reverse

$$\boxed{\tilde{x}_n = \sum_{m=1}^{3} \ell_{\tilde{n}m} \, x_m} \qquad (n = 1, 2, 3)$$

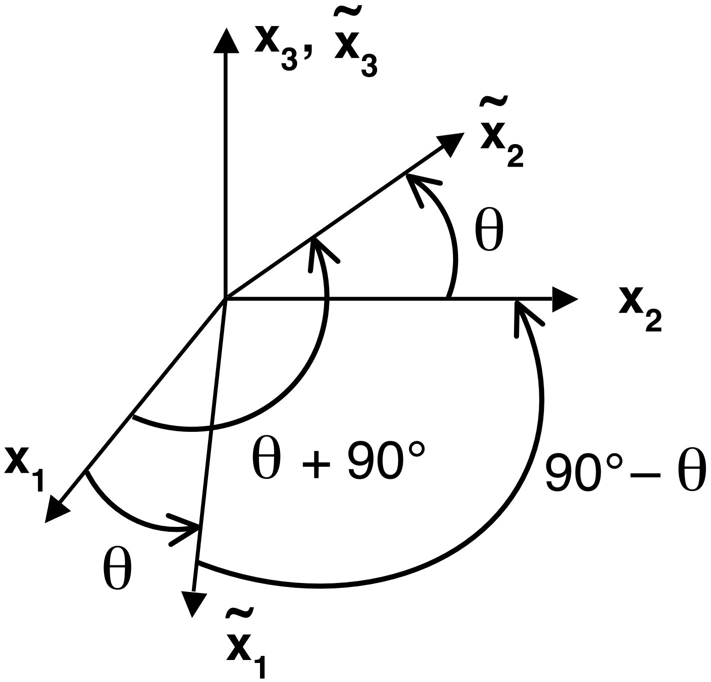
In addition, can transform forces (velocity, acceleration, etc.,) the same way

$$\tilde{F}_n = \sum_{m=1}^{3} \ell_{\tilde{n}m} F_m \qquad (n = 1, 2, 3)$$

Notes:

- Angle is measured Positive Counterclockwise (+CCW)
- $\ell_{\tilde{m}n} = \ell_{n\tilde{m}}$ since cos is an even function: $\cos(\theta) = \cos(-\theta)$
- But $\ell_{\tilde{m}n} \neq \ell_{\tilde{n}m}$ since angle from $\tilde{y}_m$ to y_n differs by 90° from that from $\tilde{y}_n$ to y_m

Figure U3.4 --> Demonstrate in a 2-D Rotation



Make a table of direction cosines:

	x_1	x_2	x_3
$\tilde{x}_1$			
$\tilde{x}_2$			
$\tilde{x}_3$			

Remember IMPORTANT CONCEPT:

Axis system in which we describe a quantity does not change the quantity, only its description