# Unit U-B
# (All About)
# Forces and Moments

Readings:

CDL 1.5

16.001/002 -- *"Unified Engineering"*
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

# LEARNING OBJECTIVES FOR UNIT **U-B**

*Through participation in the lectures, recitations, and work associated with Unit U-B, it is intended that you will be able to………*

- ….**utilize** the basic concepts associated with forces and moments
- ….**determine** moments about any location as caused by forces
- ….**utilize** the concept of a couple

# Forces

## First, what is a force?

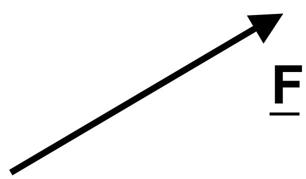
--> Definition: A force is a measure of the action of one body or media on another (push or pull)

Force has:

- magnitude
- direction
- point of application

and best represented by vectors…

*Figure U4.1* **A vector**




Note: Bar under F denotes it is a vector

- length proportional to magnitude
- arrow denotes direction
- tip is at point of application

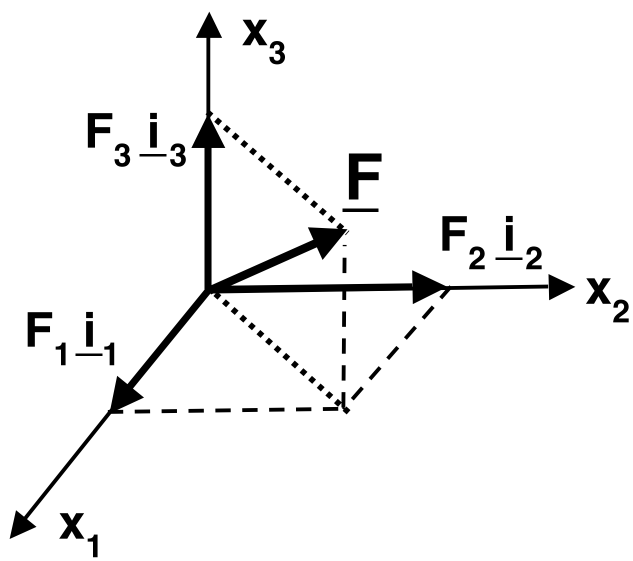
The Force can be expressed as:

$$\underline{F} = F_1 \underline{i}_1 + F_2 \underline{i}_2 + F_3 \underline{i}_3$$

where:

$F_i \left(F_1,\ F_2,\ F_3\right)$ are the components of the force in each of the three associated directions

*Figure U4.2* **Demonstration of the components of a force vector**

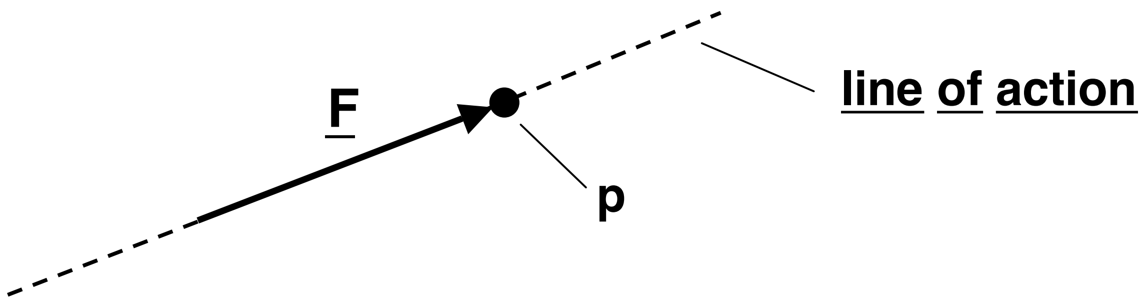



There are two important force relations:

## Transmissibility of Forces

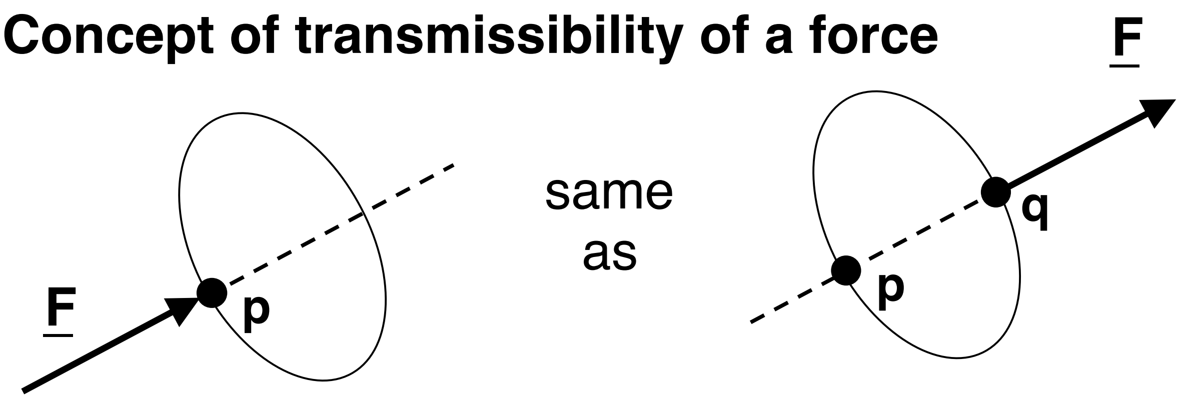
In addition to a magnitude, direction, and point of application, forces also have a line of action

***Figure U4.3*** **Line of action of a force**




But on rigid bodies, force can be "slid" along line of action without changing its influence (the force is "transmitted")

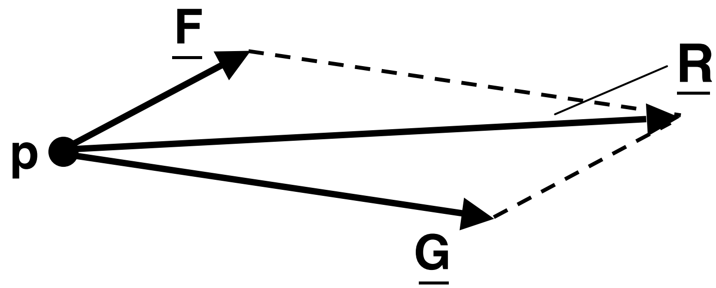
***Figure U4.4*** **Concept of transmissibility of a force**



## --> Summation of Forces - Parallelogram Rule:

To add two vectors graphically, construct a parallelogram to find “resultant”

***Figure U4.5*** **Demonstration of the parallelogram rule**




“easier” way is mathematically to add the components in each of the three directions of the rectangular cartesian system

$\underline{F}$ and $\underline{G}$ are two forces (vectors)

$$\underline{R} = \underline{F} + \underline{G}$$

$$\underline{F} = F_1 \underline{i}_1 + F_2 \underline{i}_2 + F_3 \underline{i}_3$$

$$\underline{G} = G_1 \underline{i}_1 + G_2 \underline{i}_2 + G_3 \underline{i}_3$$

$$\Rightarrow \underline{R} = (F_1 + G_1) \underline{i}_1 + (F_2 + G_2) \underline{i}_2 + (F_3 + G_3) \underline{i}_3$$

can also write as:

$$\underline{R} = \sum_{m=1}^{3} R_m \underline{i}_m$$

# Moments

We have looked at/considered forces.

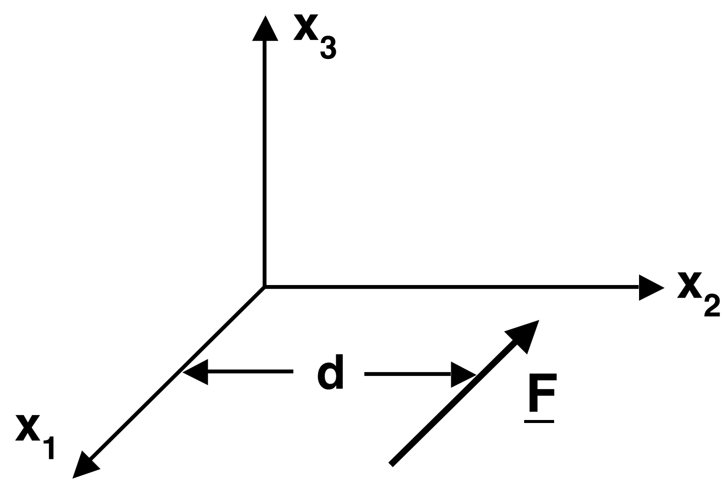
But a force actually results (or can result) in two things:

- line force
- moment about a point/force

Both are needed when we move on to consider equilibrium

Definition: A moment is a force about (or around) an axis

***Figure U4.6*** **Example: representation of force parallel to $x_1$ in $x_1$, $x_2$ plane**



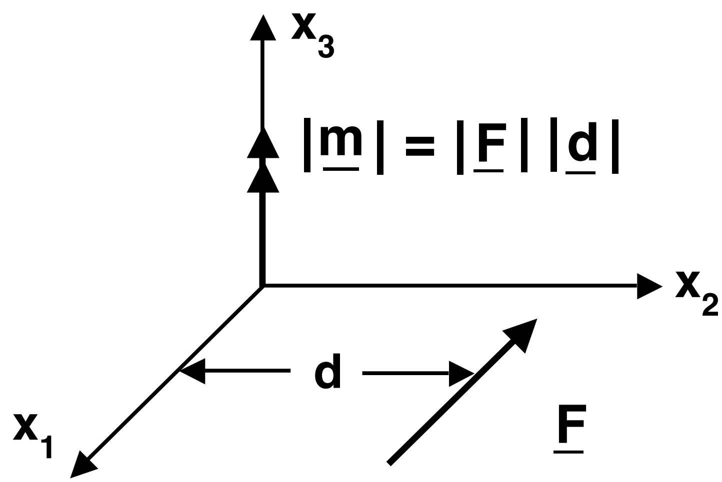
Magnitude of moment about $x_3$ is:

$$|\underline{F}| \cdot d = Fd \quad \text{(since sin 90° = 1)}$$

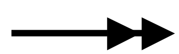
Direction of moment is given by Right Hand Rule

(vector to force) x (force vector) = (moment vector)

***Figure U4.7*** **Representation of moment due to force in $x_1$, $x_2$ plane**




Note:

 = Moment about that axis

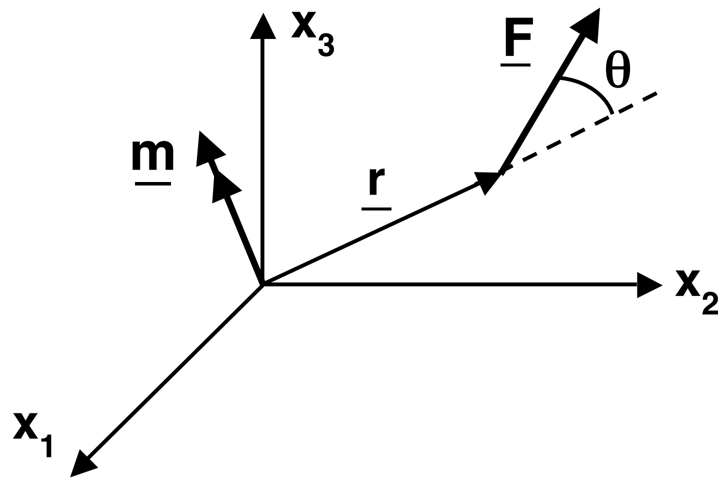
# Moment about origin

More generally, the moment of $\underline{F}$ about the origin is:

$$\boxed{\underline{m} = \underline{r} \times \underline{F}}$$

where $\underline{r}$ is (position) vector from origin to point of application of $\underline{F}$

*Figure U4.8* **Representation of moment about origin caused by general force**



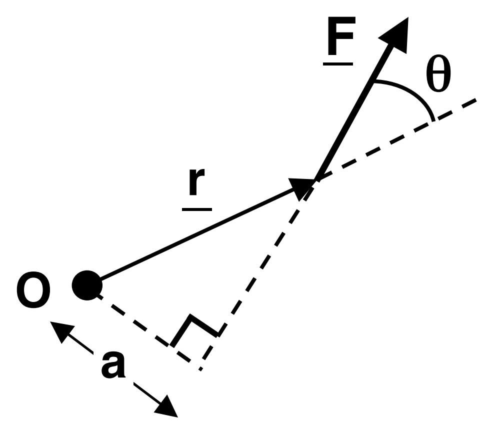

Consider a plane defined by $\underline{r}$ and $\underline{F}$:

$$|\underline{m}| = |\underline{r}| \; |\underline{F}| \sin \theta$$

can also look at as:

$$|\underline{m}| = [\underbrace{|\underline{r}| \sin \theta}_{a}]|\underline{F}|$$

*Figure U4.9* **Representation of plane of force and vector to force**



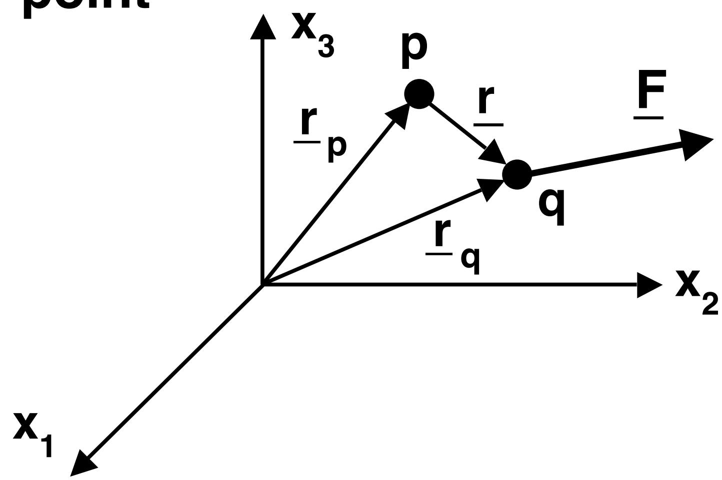

So: magnitude of moment is magnitude of force times perpendicular distance from origin to line of action of force

What about if we want to know the moment about some arbitrary point?

# Moment about arbitrary point

Consider force $\underline{F}$ at point $q$ and we want to know moment about $p$

***Figure U4.10*** **Representation of geometry to determine moment about arbitrary point**




$\underline{r}_p$ and $\underline{r}_q$ are position vectors from origin to points $p$ and $q$, respectively

$\underline{r}$ is vector from $p$ to $q$

So: $\underline{r}_q = \underline{r}_p + \underline{r} \Rightarrow \underline{r} = \underline{r}_q - \underline{r}_p$

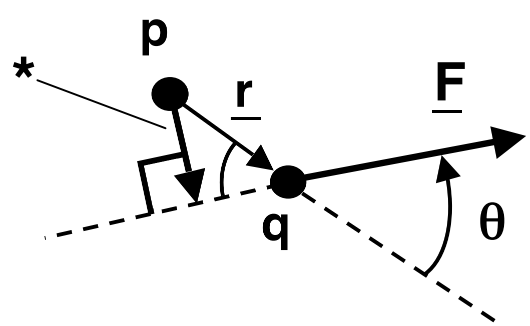
Moment about $p$ is:

$$\boxed{\underline{m} = \underline{r} \times \underline{F} = \left(\underline{r}_q - \underline{r}_p\right) \times \underline{F}}$$

Physically think of plane with $\underline{F}$, $q$, and $p$. Draw line from p perpendicular to line of action of $\underline{F}$. Length of this line is

$$\left| \underline{r}_q - \underline{r}_p \right| \sin \theta$$

***Figure U4.11*** **Representation of plane of force line of action and pointer vector**




We can take this one last step and find the

## Moment about arbitrary axis

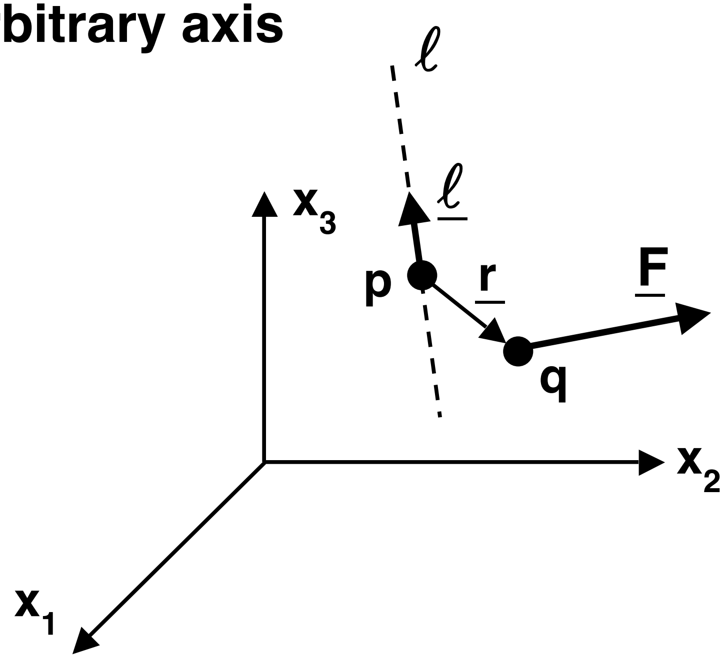
Consider force $\underline{F}$ at point $q$ and we want to know moment about line $\ell$

Define point $p$ on $\ell$ and a unit vector $\underline{\ell}$ from point $p$

$$\underline{\ell} = \frac{\underline{a}}{|\underline{a}|}$$ where $\underline{a}$ is any vector on $\ell$

arbitrary unit vector definition

*Figure U4.12* **Representation of geometry for determining moment about arbitrary axis**




We know that:

$\underline{m} = \underline{r} \times \underline{F}$

So component of $\underline{m}$ along $\ell$ can be found via dot product:

$$\boxed{m_\ell = \underline{\ell} \cdot \underline{m} = \underline{\ell} \cdot \left(\underline{r} \times \underline{F}\right)}$$

$m_\ell$ is a scalar

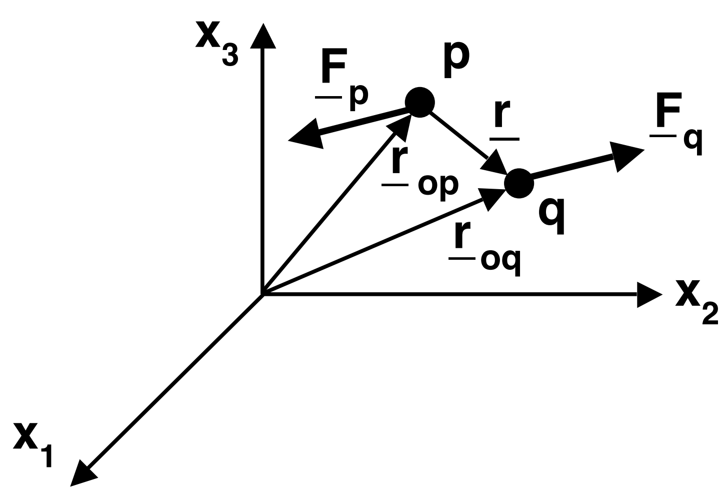
Physically, think of plane defined by $\ell$ and $\underline{F}$, then find angle between them and perpendicular distance

Often when we do analysis, we like to divide forces and moments. We therefore need to deal with the concept of a pure moment or a…..

## Couple

Results from 2 parallel (coplanar) forces of equal magnitude and opposite directions

***Figure U4.13*** **Representation of a force couple**




$\underline{F}_p$ and $\underline{F}_q$ are equal magnitude but opposite directions

Thus:

--> Net Force (resultant)

$$\underline{F} + (-\underline{F}) = 0 \qquad \underline{\text{None}}$$

--> Net Moment about origin (resultant)

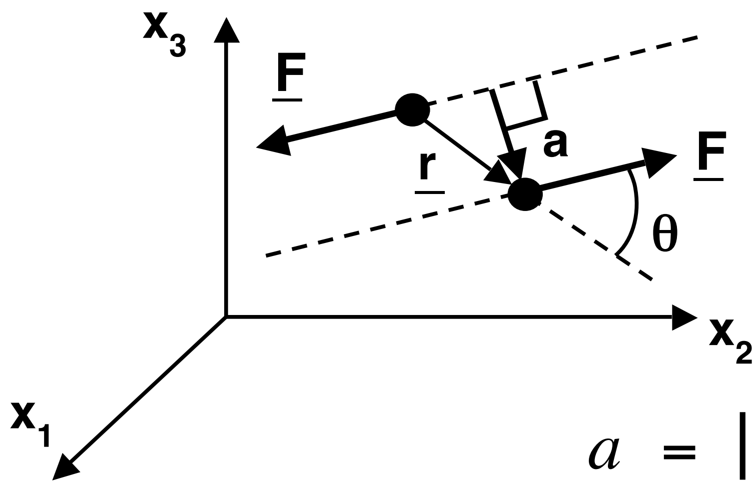
$$\underline{C}_o = \underline{r}_{op} \times (-\underline{F}) + \underline{r}_{oq} \times \underline{F}$$

$$= (\underline{r}_{oq} - \underline{r}_{op}) \times \underline{F} = \underline{r} \times \underline{F}$$

$$\boxed{\underline{C} = \underline{r} \times \underline{F}}$$

<u>Physically</u>, think of perpendicular distance between two forces

*Figure U4.14* **Representation of perpendicular distance between two forces**



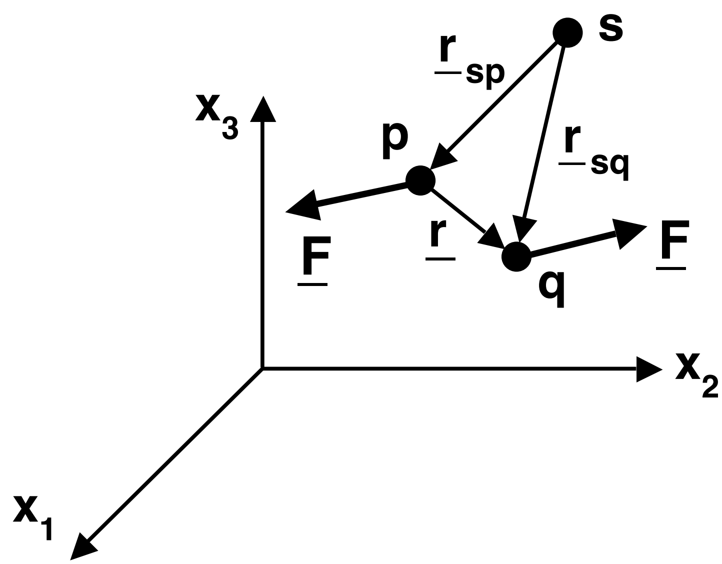

$$a = |r| \sin \theta$$

<u>Note</u>: Magnitude and direction of couple does not change as point about which couple is taken changes

Why? $a$ stays the same!

Demonstrate by considering points:

***Figure U4.15*** **Representation of considering couple about another point**



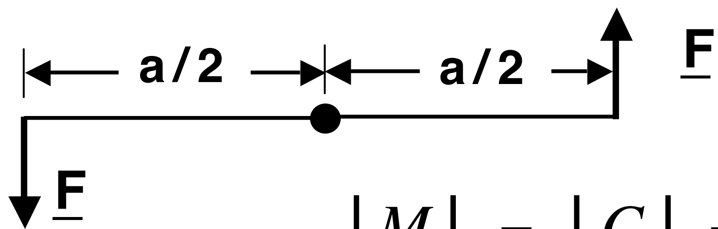

$$\underline{C}_s = \underline{r}_{sp} \times (-\underline{F}) + \underline{r}_{sq} \times \underline{F}$$

$$= (\underline{r}_{sq} - \underline{r}_{sp}) \times \underline{F}$$

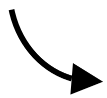
$$= \underline{r} \times \underline{F}$$ <u>as</u> <u>before</u>!

Think of couple as two equal forces pointed in opposite directions:

***Figure U4.16*** **Representation of alternate way to think of couple**




$$|M| = |C| = 2\,\frac{a}{2}\,F = aF$$

Use symbol  to represent this.

Gives sense/direction.

Need magnitude to go with it.

Our example:

***Figure U4.17*** **Representation of moment with moment symbol**


<u>Note</u>: Pure moments can "move around" anywhere since they have the same effect about any point!