16. Unified Fall 08 TS

a) Assumptions:
- steady level flight throughout } steady cruise
- no accelerations
- constant SFC throughout mission
- L/D constant throughout mission
- u_0 constant throughout mission

Clearly these assumptions don't hold for taxiing, climbing and descending so as long these flight legs are short compared to overall stage length, the Breguet range equation is valid. The assumptions break down for short-haul flights as the parameters change significantly over the short mission time

b) $\eta_0 = \frac{\text{what we get}}{\text{what we pay}} = \frac{\text{thrust power}}{\text{fuel power}} = \frac{T \cdot u_0}{\dot{m}_f \cdot h_f}$

Duck: $T = W \cdot \frac{1}{L/D}$ $\quad W = m \cdot g$ $\quad m \approx 1\,kg \rightarrow T \approx 1\,N$

$L/D \approx 10$

$u_0 \approx 10\,m/s$ so $\underline{T u_0 \approx 10\,W}$

$\dot{m}_f = 0.02/3600 \cdot m = 5.6 \cdot 10^{-6}\,kg/s$

$h_f \approx 35\,MJ/kg$ $\Big\} \rightarrow \underline{\dot{m}_f h_f \approx 200\,W}$

so estimate $\underline{\eta_0 \approx 0.05}$; assumed in class $\eta_0 = 0.1 \ldots 0.05 \rightarrow$ o.k. estimate!

c) Breguet Range eqn:

$$R = \frac{h_f}{g} \cdot \left(\frac{L}{D}\right) \cdot \eta_0 \cdot \ln\left(\frac{W_i}{W_f}\right)$$

B747-400 data: (see web page boeing.com)

$W_{MTOW} = 396{,}890\,kg$

$W_L = W_{MTOW} - W_F$

$W_F = V_{tank} \cdot \rho_f$; $V_{tank} = 216{,}840\,l = 216.84\,m^3$

$= 173{,}472\,kg$ $\quad \rho_f = 800\,kg/m^3$

so $W_L = 223{,}418\,kg$

$L/D_{max} \approx 17$ from charts , $h_{f_{JetA}} \approx 42.8\,MJ/kg$

$\eta_o \approx 0.32$ from charts for GE CF6-80C2 engines

Estimated range: $R = \frac{h_f}{g} \frac{L}{D_{max}} \eta_o \ln\left(\frac{W_{MTOW}}{W_L}\right) \approx 13{,}640\,km$

Boeing data: $R = 13{,}450\,km$ so rel error ~1.4%!

We overestimate the range slightly because neglected fuel reserves

d) All else equal:

$\rho_{LH} = 70\,kg/m^3$

$h_{LH} = 120\,MJ/kg$

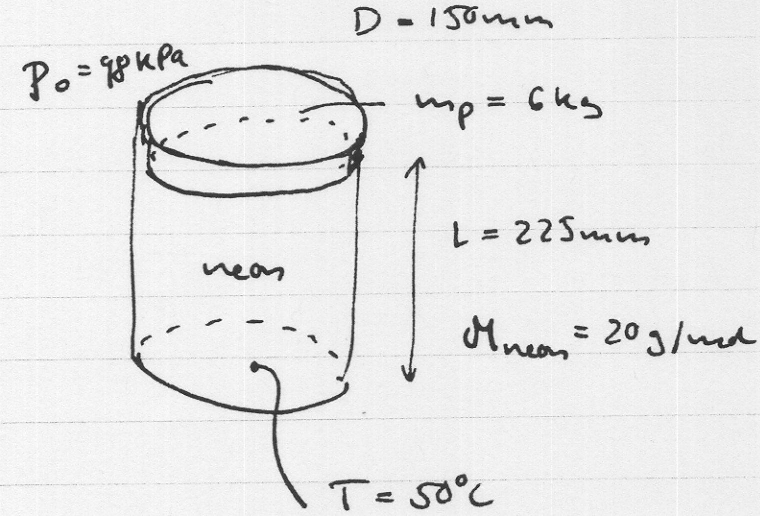
$$R_{LH} = \frac{h_{LH}}{g} \frac{L}{D_{max}} \eta_o \ln\left(\frac{W_L + V_{total} \cdot \rho_{LH}}{W_L}\right)$$

$R_{LH} = 4{,}374\,km$

→ range is about 1/3! Although heating value larger by factor 3 the low density reduces weight fraction more than energy content increases (log vs. linear)

T1

16.Unified Fall 08
ZS



Concepts:
- thermodynamic equil.
- equation of state
- ideal gas model

a) mech. equilibrium: $pA = p_0 A + m_p g$ $\qquad A = \left(\frac{D}{2}\right)^2 \pi$

$$p = p_0 + \frac{m_p g}{(D/2)^2 \pi} \qquad \underline{p = 1.013 \text{ bar}}$$

b) $R = \frac{\mathcal{R}}{\mathcal{M}} = \frac{8.31}{20} = \underline{415.5 \text{ J/kg-K}}$

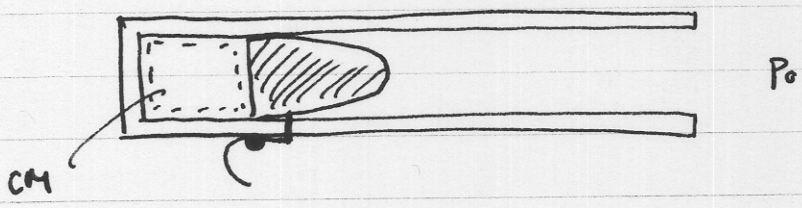
c) neon mass: $m = \rho V = \frac{pV}{RT}$; $V = \left(\frac{D}{2}\right)^2 \pi \cdot L$

$V = 0.00397 \text{ m}^3$ $\qquad \underline{m = 0.003 \text{ kg}}$

d) $p = \rho RT$, $\rho = \frac{p}{RT}$ $\rightarrow$ $\underline{\rho = 0.755 \text{ kg/m}^3}$

T2

16. Unified Fall 08 ZS



Assumptions:

- no friction, quasi-equil.
- isothermal expansion
- ideal gas $R = 287\ J/kg\text{-}K$
- $m_b = 0.02\ kg$

Concepts:
- equation of state
- 1st law of thermo
- work of compressible gas
- cons. of mass (CM)

Initially: $V_i = 1.6 \times 10^{-5}\ m^3$

$p_i = 1\ MPa$

$T_i = 27°C$

Finally: V_f

$p_f = p_o = 0.1\ MPa$

$T_f = T_i = 27°C$

a) equation of state: $p_i = \rho_i R T_i$, $m = \rho_i V_i$

$\rho_i = \frac{p_i}{RT_i}$; $\rho_i = 11.61\ kg/m^3$ $\underline{m = 1.85 \cdot 10^{-4}\ kg}$

$m = \rho_f V_f$ and $\rho_f = \frac{p_f}{RT_f} = 1.161\ kg/m^3$ so $\underline{V_f = 1.59 \cdot 10^{-4}\ m^3}$

b) Work done by system (air) : $W_{air} = \int_{V_i}^{V_f} p_s\, dV$ for quasi-equil.

$W_{air} = mRT_i \int_{V_i}^{V_f} \frac{dV}{V}$ isothermal exp. $W_{air} = mRT_i \ln\left(\frac{V_f}{V_i}\right)$

$\underline{W_{air} = 36.6\ J}$; Work done on atmosphere $\underline{W_{atm}} = p_o (V_f - V_i) = \underline{14.3\ J}$

$W_{atm} = \int_{V_i}^{V_f} p_o\, dV$

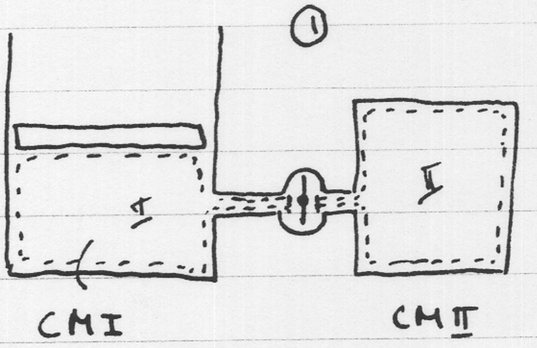
c) Work done on bullet: $W_{air} = W_{bullet} + W_{atm}$ $\rightarrow$ $\underline{W_{bullet}} = W_{air} - W_{atm} = \underline{22.3\ J}$

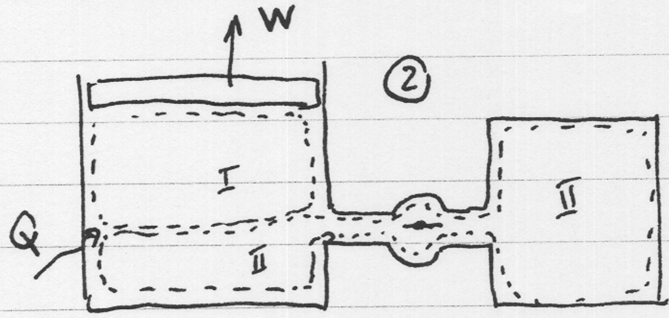
1st law: $\Delta E_{bullet} = \cancel{Q}^{0} - W_{bullet}$; $\Delta E_{bullet} = \cancel{\Delta U}^{0} + \cancel{\Delta E_p}^{0} + \Delta E_k = m\left(0 - \frac{c^2}{2}\right)$

$c = \sqrt{\frac{2 W_{bullet}}{m}}$ $\longrightarrow$ $\underline{c = 47.2\ m/s}$ rather low since matched pressure at exit and p_i only 10 atm

Concepts:

- thermodyn. equilibrium
- conservation of mass
- work compr. gas
- 1st law





①	②
$P_{I_1} = 1\,bar$	$P_2 = P_{I_2} = P_{II_2} = 1\,bar$
$T_{I_1} = 473\,K$	$T_2 = T_{I_2} = T_{II_2} = 323\,K$
$P_{II_1} = 3\,bar$	thermodyn. equilibrium
$T_{II_1} = 573\,K$	

State ①: $m_{II} = V_{II} \cdot \rho_{II_1}$, $\rho_{II_1} = \frac{P_{II_1}}{T_{II_1} R}$

$m_I = V_{I_1} \rho_{I_1}$, $\rho_{I_1} = \frac{P_{I_1}}{T_{I_1} R}$

$m_{tot} = \underbrace{m_{II}}_{0.183\,kg} + \underbrace{m_I}_{0.037\,kg}$

$m_{tot} = 0.22\,kg$

State ②: tank has fixed volume so mass remaining in tank is $m_{tank} = V_{II} \cdot \rho_{II_2}$ and $\rho_{II_2} = \frac{P_2}{RT_2}$

$m_{tank} = 0.11\,kg$

$\rightarrow m_{cyl} = m_{tot} - m_{tank} = 0.11\,kg$

$V_{cyl} = \frac{m_{cyl}}{\rho_2}$ and $\rho_2 = \frac{P_2}{RT_2}$ $\rightarrow V_{cyl} = 0.102\,m^3$

Piston work: quasi-equil. $\underline{W} = \int_{V_{I_1}}^{V_{cyl}} p\,dV = p_2 (V_{cyl} - V_{I_1}) = \underline{5.2\,kJ}$ done on surroundings

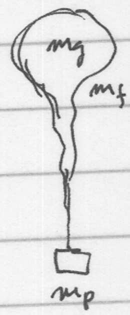
1st law: $U_2 - U_1 = Q - W$, $Q = U_2 - U_1 + W = m_{tot} c_v T_2 - (m_I c_v T_{I_1} + m_{II} c_v T_{II_1})$

$\underline{Q = -31.6\,kJ}$ heat rejected to surroundings

Solutions to F1

(20%) 1. The balloon volume increases as the helium gas expands due to a lower atmospheric pressure as it climbs. The assumption here is that the pressures inside and outside of the balloon are equal, as long as the fabric is not under tension.

2. what is m_g for neutral buoyancy?



For neutral buoyancy $\Rightarrow$ gravity force = buoyant force

therefore:

(30%) $$(m_g + m_f + m_p)\cancel{g} = \rho \cancel{g} V \qquad (\rho \text{ is air density})$$

at sea-level $\Rightarrow$ $$\rho = \rho_0 = \frac{P_0}{R_a T_0} \equiv \frac{101325\,Pa}{\left(\frac{8314 \frac{J}{kmol\cdot K}}{29\,kg/kmol}\right)(298\,°K)} = 1.18 \frac{kg}{m^3}$$

and since $$V = \frac{m_g}{\rho_g} \quad \text{and} \quad \rho_g = \frac{P_g}{R_g T_0} = \frac{P_0}{\left(\frac{8314 \frac{J}{kmol\cdot K}}{4\,kg/kmol}\right)(298\,°K)} = 0.163 \frac{kg}{m^3}$$

P_0 = 101325 Pa

we assume the temp. to be equal to air temp.

We can solve for m_g:

$$m_g\left(1 - \frac{\rho}{\rho_g}\right) = -m_f - m_p \quad \text{or} \quad \boxed{m_g = \frac{m_f + m_p}{\frac{\rho}{\rho_g} - 1}}$$

Substituting values:

$$\boxed{m_{g_{MIN}} = \frac{1\,kg + 0.250\,kg}{\frac{1.18\,kg/m^3}{0.163\,kg/m^3} - 1} = 0.2\,kg}$$ ← For neutral buoyancy.

(30%) 3. Assuming $m_g = 1.1\, m_{g_{MIN}} = 1.1(0.2) = 0.22\,kg$

we need this to have a climbing balloon, this is arbitrary, in general $m_g > m_{g_{MIN}}$

When the balloon becomes spherical, its volume is: $V = \frac{4}{3}\pi r^3$

Since the gas mass is always constant (as long as there are no leaks) we have: $$\boxed{\rho = \frac{m_g}{\frac{4}{3}\pi r^3}}$$ Helium density for a spherical balloon

3. cont'd

The question is, at what altitude h_s, the helium density is $\rho = \frac{m_g}{\frac{4}{3}\pi r^3}$?

$$\rho = \frac{P_g(h_s)}{R_g\, T_g(h_s)} \quad \text{and since} \quad P_g(h_s) = P_{air}(h_s) = \rho_{air}\, R_a\, T_a(h_s)$$

at h_s

we also assumed $\boxed{T_g = T_a}$

Therefore : $\rho(h_s) = \rho_a(h_s)\,\frac{R_a}{R_g} = \frac{m_g}{\frac{4}{3}\pi r^3}$

and $\frac{R_a}{R_g} = \frac{M_g}{M_a} = \frac{4\ \text{g/mol}}{29\ \text{g/mol}} = 0.138$

and we also have a model for the atmosphere density: $\rho_a = \rho_0 e^{-h/h_0}$

so that $\rho_a(h_s) = \rho_0 e^{-h_s/h_0}$

So finally $\Rightarrow$ $\rho_0 e^{-h_s/h_0}\,\frac{R_a}{R_g} = \frac{m_g}{\frac{4}{3}\pi r^3}$

or $$\boxed{h_s = -h_0 \ln\left[\frac{3 m_g}{4(M_g/M_a)\,\rho_0\,\pi r^3}\right]}$$

with our values:

$$h_s = -8000\text{m} \ln\left[\frac{3(0.22\text{ kg})}{4(0.138)(1.18\frac{\text{kg}}{\text{m}^3})\pi(5\text{m})^3}\right] = 47,678\text{ m}$$

(20%) 4. What is the maximum altitude?

The balloon becomes neutrally buoyant when:

$$(m_g + m_f + m_p)\cancel{g} = \rho \cancel{g} V \quad \text{but now} \quad V = \frac{4}{3}\pi r^3$$

and $\rho = \rho_0 e^{-h_{max}/h_0}$

$$\boxed{h_{max} = -h_0 \ln\left[\frac{m_g + m_f + m_p}{\rho_0 \frac{4}{3}\pi r^3}\right]}$$

with our values:

$$h_{max} = -8000\text{m} \ln\left[\frac{0.22\text{ kg} + 1\text{ kg} + 0.25\text{ kg}}{1.18\frac{\text{kg}}{\text{m}^3}\cdot\frac{4}{3}\pi(5)^3\text{ m}^3}\right] = 48,327\text{ m}$$