PAL
11/13/08

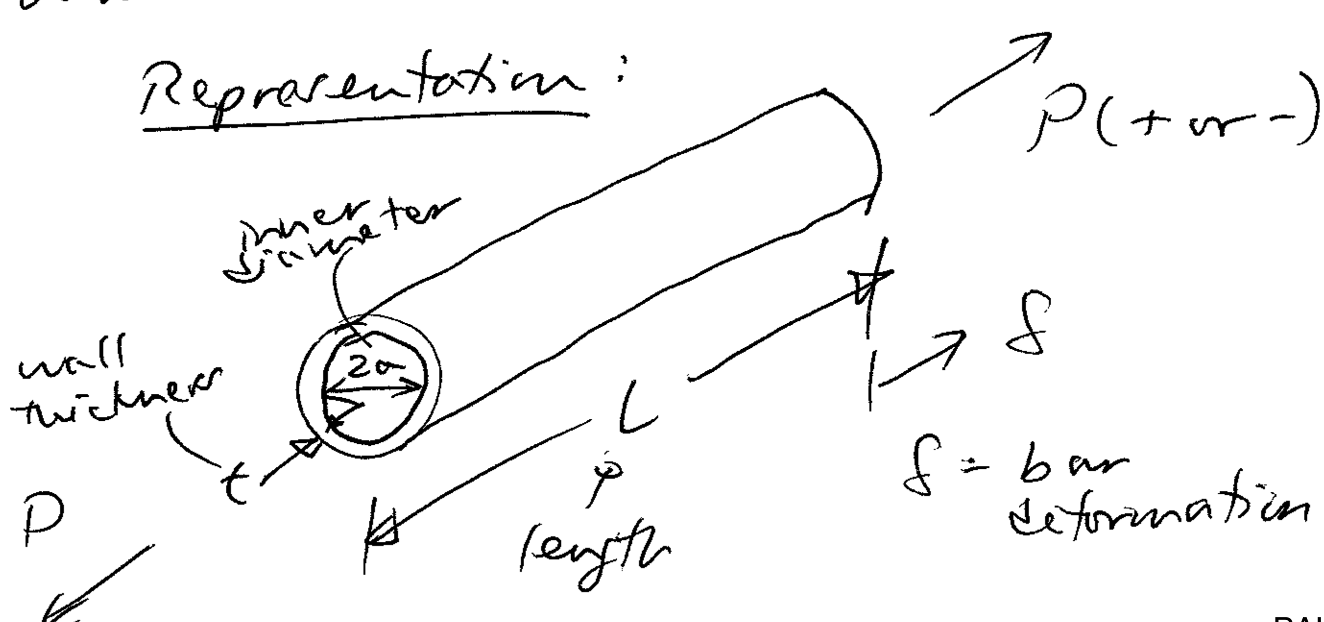
Unified Engineering

Week 12 – Problem Set 11
Fall, 2008
SOLUTIONS

M21 (M12.1)

Bar is of a given length with a circular tube cross-section, and the bar must carry a constant axial load, in tension or compression, of no greater magnitude than P. The cross-section has a given (and constant) inner radius of a. The wall thickness of the cross-section is significantly smaller than the radius.

Representation:



(a) List the constants: P, L, a

Requirement: carry load of a magnitude no more than P

Needs: Deform as little as possible and weigh as little as possible

Design variables: Bar wall thickness (t) and material used

→ List items to be considered for minimization, etc:

mass/weight (m)
deformation (δ)
cost (c)

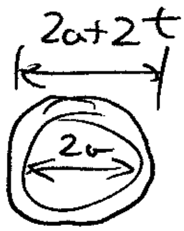
→ List key equations:

stress-strain: $\sigma = E\epsilon$ (1)

Strain – displacement (for bar): $\epsilon = \delta/L$ (2)

stress – load (for bar): $\sigma = P/A$ (3)

and for a circular tube with walls of thickness t:



the outer radius is (a+t). Thus subtracting the inner area from the outer area gives the tube material area:

$$\pi(a+t)^2 - \pi a^2 = A$$

$$\Rightarrow A = \pi a^2 + 2\pi a t + \pi t^2 - \pi a^2$$

simp: $A = \pi(2at + t^2)$

$$\text{if } a \gg t \Rightarrow A \approx 2\pi a t$$

This is the same as looking at the inner circumference ($2\pi a$) and multiplying by the wall thickness (t). So the operative equation is:

$$A = 2\pi a t \quad (4)$$

... proceeding ...

mass-density (weight): $M = \rho A L$ (5) (for bar)

cost: $\left(\frac{\text{cost}}{\text{weight}}\right)(\text{weight}) = \text{cost}$ (6)

→ List other variables, parameters:

E = modulus
t = thickness
$\mathcal{I}$
A = Area
ρ = density

The figures of merit are based on the overall items to be considered and expressing these in terms of geometrical and material parameters/properties.

→ First consider the deformation:

- from (2): $\delta = \epsilon L$
- use (1) to give: $\epsilon = \sigma / E$

  and thus: $\delta = \dfrac{\sigma L}{E}$

- Now use (3) in this last equation:

$$\Rightarrow \delta = \frac{PL}{AE}$$

- Finally, use (4) to get this in terms of load, length, wall thickness, and modulus along with inner radius:

$$\boxed{\delta = \frac{PL}{2\pi E a t}}$$

First
Figure of Merit
(*1)

→ Now consider mass/weight:

- from (5): weight $= \rho A L$

ρ ← weight density

- Use (4) to get this in terms of load, length, inner radius, wall thickness, and density:

$$\boxed{\text{weight} = 2\pi \rho a t L}$$

Second
Figure of Merit
(*2)

→ Finally consider cost:

- From (6):

Cost $= c$ (weight)

c ← cost/weight

- Using the second Figure of Merit gets this in terms of key parameters:

$$\boxed{\text{Cost} = 2\pi C_\rho a t L}$$

Third
Figure of Merit
(* 3)

(b) We have three equations that allow us to explore the possibilities in terms of the key items (deformation, weight cost).

However, separately considering any one item and its minimization is generally insufficient (the specific minimization are called out in this problem statement). For example, one can decrease deformation by continuing to increase bar wall thickness. This results in increased weight. The key is to consider the ability of any choice for other fixed considerations. This leads to considering ...

<u>tradeoffs</u>!

If one considers the ability (for example) to provide a specified minimum deformation (call it δ_0) and first consider mass (weight), the first and second figures of merit can be combined for this consideration:

from (*1): $\delta_0 = \dfrac{PL}{2\pi E a t}$

$$\Rightarrow t = \frac{PL}{2\pi E a \delta_0}$$

and place this in (*2):

$$\text{weight} = (2\pi \rho a)\left(\frac{PL}{2\pi E a \delta_0}\right)L$$

$$\Rightarrow \text{weight} = \frac{\rho P L^2}{E\delta_0}$$

Here $\dfrac{PL^2}{\delta_0}$ is a constant of the problem so we assess the material possibilities via the factor: ρ/E

Material	ρ/E $\left[\text{lb/in}^3\right] / \left[\frac{10^6\,\text{lb}}{\text{in}^2}\right] = \left[\frac{1}{10^6\,\text{in}}\right]$
$\boxed{\text{Silicon Carbide}}$	0.0018
Aluminum	0.0096
Wood	0.0122
Steel	0.0098
Carbon fiber Composite	0.0022
Titanium	0.0100

best choice to minimize mass/weight for a given deformation

→ Now consider cost by using (*3)

$$\Rightarrow \text{Cost} = 2\pi C \rho a \left(\frac{PL}{2\pi E a \delta_0}\right) L$$

giving.... $\text{Cost} = \dfrac{C P L^2 \rho}{E \delta_0}$

Here, $\frac{PL^2}{\delta_0}$ again is a constant of the problem. So we assess the material possibilities via the factor: $C\rho/E$

Material	$C\rho/E$ [$/10^6 in·lb]
Silicon Carbide	0.292
Aluminum	0.066
$\boxed{\text{Wood}}$	0.012
Steel	0.016*
Carbon fiber Composite	0.194
Titanium	0.245

best choice to minimize cost for a given deformation

* = a close second

→ Finally, think about minimizing deformation for a fixed cross-section using (*1):

$$\delta = \frac{PL}{2\pi E a t}$$

Here, $\frac{PL}{2\pi a t}$ is given as a constant by the problem, so we assess the material possibilities via the factor: $1/E$

Material	$1/E$ $\left[\frac{in^2}{10^6\ lb}\right]$
$\boxed{\text{Silicon Carbide}}$	0.017
Aluminum	0.095
Wood	0.552
Steel	0.034
Carbon fiber Composite	0.041
Titanium	0.063

best choice to minimize deformation for a given area

→ Final note: There is no final answer without further clarification of the objectives and the relative values of the various aspects/criteria. It depends on decisions with regard to the tradeoffs.

M22 (M12.2)

Compliance tensor

(a) Start with:

$$\epsilon_{mn} = S_{mnpq}\sigma_{pq}$$

This has the same form as the elasticity equations:

$$\sigma_{mn} = E_{mnpq}\epsilon_{pq}$$

and since the same symmetries exist for the compliance tensor as for the elasticity tensor, the full anisotropic equations have the same form.

Thus:

$$\boxed{\begin{aligned}
\epsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1133}\sigma_{33} \\
&\quad + 2S_{1123}\sigma_{23} + 2S_{1113}\sigma_{13} + 2S_{1112}\sigma_{12} \\
\epsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33} \\
&\quad + 2S_{2223}\sigma_{23} + 2S_{2213}\sigma_{13} + 2S_{2212}\sigma_{12} \\
\epsilon_{33} &= S_{1133}\sigma_{11} + S_{2233}\sigma_{22} + S_{3333}\sigma_{33} \\
&\quad + 2S_{3323}\sigma_{23} + 2S_{3313}\sigma_{13} + 2S_{3312}\sigma_{12}
\end{aligned}}$$

(cont.)

(cont. from previous page)

$$\boxed{\begin{aligned}
\epsilon_{23} &= S_{1123}\sigma_{11} + S_{2223}\sigma_{22} + S_{3323}\sigma_{33} \\
&\quad + 2S_{2323}\sigma_{23} + 2S_{2313}\sigma_{13} + 2S_{2312}\sigma_{12} \\
\epsilon_{13} &= S_{1113}\sigma_{11} + S_{2213}\sigma_{22} + S_{3313}\sigma_{33} \\
&\quad + 2S_{1323}\sigma_{23} + 2S_{1313}\sigma_{13} + 2S_{1312}\sigma_{12} \\
\epsilon_{12} &= S_{1112}\sigma_{11} + S_{2212}\sigma_{22} + S_{3312}\sigma_{33} \\
&\quad + 2S_{1223}\sigma_{23} + 2S_{1213}\sigma_{13} + 2S_{1212}\sigma_{12}
\end{aligned}}$$

Likewise, these can be grouped into the 3 groups similar to those for the elasticity tensor, E_{mnpq}:

<u>extensional</u> <u>stresses</u>
to
<u>extensional</u> <u>strains</u>

$$\left\{\begin{array}{ll} S_{1111} & S_{1122} \\ S_{2222} & S_{1133} \\ S_{3333} & S_{2233} \end{array}\right.$$

<u>shear</u> <u>stresses</u>
to
<u>shear</u> <u>strains</u>

$$\left\{\begin{array}{ll} S_{1212} & S_{1213} \\ S_{1313} & S_{1323} \\ S_{2323} & S_{1223} \end{array}\right.$$

COUPLING TERMS

extensional stresses to shear strains

or

shear stresses to extensional strains

$$\left\{ \begin{array}{lll} S_{1112} & S_{2212} & S_{3312} \\ S_{1113} & S_{2213} & S_{3313} \\ S_{1123} & S_{2223} & S_{3323} \end{array} \right.$$

(b) For the orthotropic case, all COUPLING TERMS are zero (in the principal axes of the material). Thus:

$$\begin{array}{lll} S_{1112} = 0 & S_{2212} = 0 & S_{3312} = 0 \\ S_{1113} = 0 & S_{2213} = 0 & S_{3313} = 0 \\ S_{1123} = 0 & S_{2223} = 0 & S_{3323} = 0 \end{array}$$

In addition, shear stresses (strains) in one plane do not cause shear strains (stresses) in another plane. Thus three of the shear stress to shear strain terms become zero:

$$S_{1213} = 0$$

$$S_{1323} = 0$$

$$S_{1223} = 0$$

→ All other terms are nonzero and independent

⇒ This gives 9 independent compliance components as for the elasticity tensor case. The resulting equations are:

$$\boxed{\begin{aligned}
\epsilon_{11} &= S_{1111}\sigma_{11} + S_{1122}\sigma_{22} + S_{1133}\sigma_{33}\\
\epsilon_{22} &= S_{1122}\sigma_{11} + S_{2222}\sigma_{22} + S_{2233}\sigma_{33}\\
\epsilon_{33} &= S_{1133}\sigma_{11} + S_{2233}\sigma_{22} + S_{3333}\sigma_{33}\\
\epsilon_{23} &= 2S_{2323}\sigma_{23}\\
\epsilon_{13} &= 2S_{1313}\sigma_{13}\\
\epsilon_{12} &= 2S_{1212}\sigma_{12}
\end{aligned}}$$

Note Title 11/23/2008

Series - Parallel combination of Inductors

Series Connection

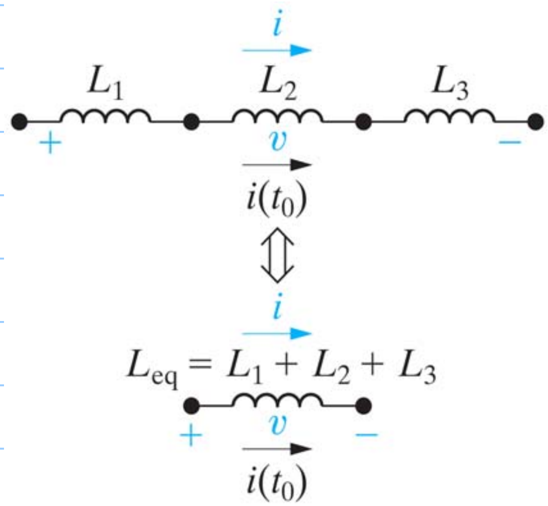


Figure: 06-14

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$$v_1 = L_1 \frac{di}{dt} \qquad (1)$$

$$v_2 = L_2 \frac{di}{dt} \qquad (2)$$

$$v_3 = L_3 \frac{di}{dt} \qquad (3)$$

$$v = L_{eq} \frac{di}{dt} \qquad (4)$$

KVL $\Rightarrow$ $$v = v_1 + v_2 + v_3 \qquad (5)$$

Substituting (1) – (3) into (5) gives

$$v = (L_1 + L_2 + L_3)\frac{di}{dt} \qquad (6)$$

comparing (4) and (6) gives

$$L_{eq} = L_1 + L_2 + L_3$$

In general:

$$\boxed{L_{eq} = L_1 + L_2 + \cdots + L_n}$$

Parallel connection

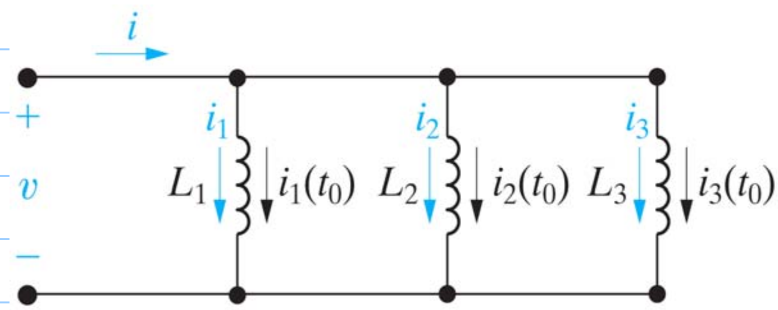

Figure: 06-15

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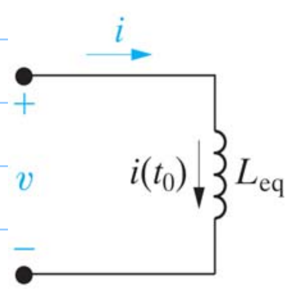

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Figure: 06-16

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$$i_1(t) = \frac{1}{L_1}\int_{t_0}^{t} v \, d\tau + i_1(t_0) \qquad (1)$$

$$i_2(t) = \frac{1}{L_2}\int_{t_0}^{t} v \, d\tau + i_2(t_0) \qquad (2)$$

$$i_3(t) = \frac{1}{L_3}\int_{t_0}^{t} v \, d\tau + i_3(t_0) \qquad (3)$$

$$i(t) = \frac{1}{L_{eq}} \int_{t_0}^{t} v \, d\tau + i(t_0) \qquad (4)$$

KCL $\Rightarrow$ $i(t) = i_1(t) + i_2(t) + i_3(t)$ (5)

substituting (1) – (3) into (5) gives

$$i(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int_{t_0}^{t} v \, d\tau$$

$$+ \; i_1(t_0) + i_2(t_0) + i_3(t_0) \qquad (6)$$

comparing (4) and (6) gives

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

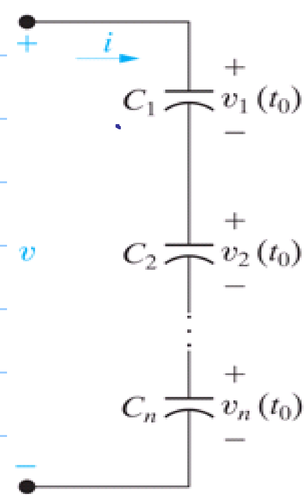
$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

In general

$$\boxed{\begin{gathered}\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \\ i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0)\end{gathered}}$$

Series - Parallel Combination of Capacitors

Series connection



$$v_1(t) = \frac{1}{C_1}\int_{t_0}^{t} i\,d\tau + v_1(t_0) \qquad (1)$$

$$v_2(t) = \frac{1}{C_2}\int_{t_0}^{t} i\,d\tau + v_2(t_0) \qquad (2)$$

$$v_3(t) = \frac{1}{C_3}\int_{t_0}^{t} i\,d\tau + v_3(t_0) \qquad (3)$$

$$v(t) = \frac{1}{C_{eq}}\int_{t_0}^{t} i\,d\tau + v(t_0) \qquad (4)$$

KVL $\Rightarrow$ $v(t) = v_1(t) + v_2(t) + v_3(t)$ (5)

Substituting (1) – (3) into (5) gives

$$v(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \int_{t_0}^{t} i \, d\tau$$

$$+ v_1(t_0) + v_2(t_0) + v_3(t_0) \qquad (6)$$

Comparing (4) and (6) yields

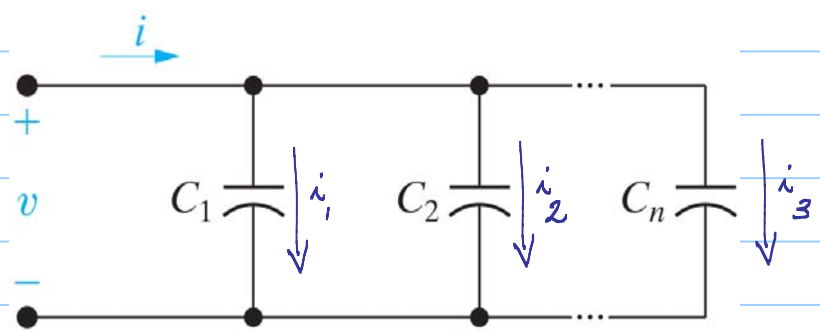
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + v_3(t_0)$$

In general

$$\boxed{\begin{aligned} \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \\ v(t_0) &= v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0) \end{aligned}}$$

Parallel connection



$$i_1 = C_1 \frac{dv}{dt} \qquad (1)$$

$$i_2 = C_2 \frac{dv}{dt} \qquad (2)$$

$$i_3 = C_3 \frac{dv}{dt} \qquad (3)$$

$$i = C_{eq} \frac{dv}{dt} \qquad (4)$$

KCL $\Rightarrow$ $i = i_1 + i_2 + i_3 \qquad (5)$

Substituting (1) – (3) into (5) gives

$$i = (C_1 + C_2 + C_3) \frac{dv}{dt} \qquad (6)$$

comparing (4) and (6) yields.

$$C_{eq} = C_1 + C_2 + C_3$$

In general

$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_N}$$

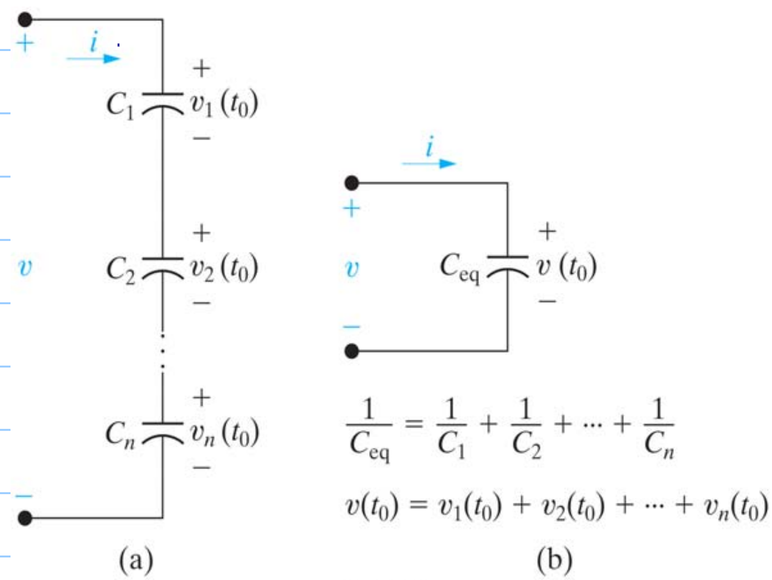


Figure: 06-17a,b

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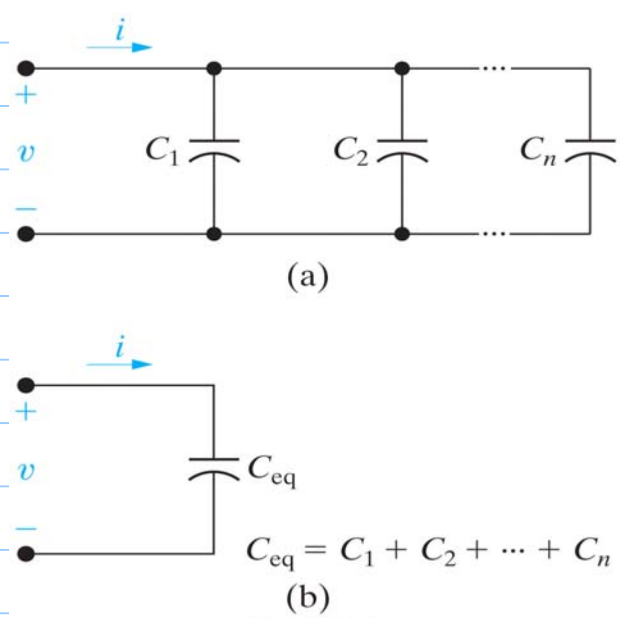

Figure: 06-18a,b

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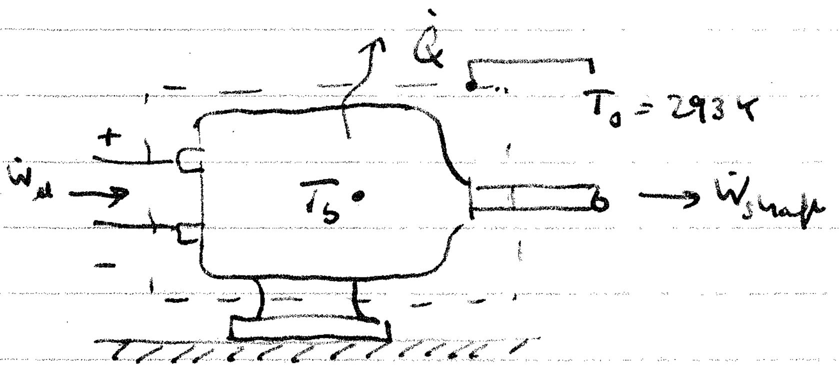
T23

16. Unified Fall 08
ZS



Assume:
- convection heat x-fer from motor to surroundings
- steady-state operation for 100 seconds

a) 1st law : $0 = \dot{W}_{el} - \dot{W}_{shaft} - \dot{Q}$

convection: $\dot{Q} = hA\,(T_S - T_0)$

shaft power: $\dot{W}_{shaft} = T \cdot \Omega = 1.67\,kW$, El. Power $\dot{W}_{el} = V \cdot I = 2.2\,kW$

find $T_S = T_0 + \frac{1}{hA}(\dot{W}_{el} - \dot{W}_{shaft})$ $\quad \underline{T_S = 320\,K}$

b) Entropy is a state variable : → steady operation so T_S = const so entropy remains the same

→ $\underline{\Delta S_{motor} = 0}$ (Note: entropy is constantly generated in motor but then leaves with $\dot{Q}$ → $\frac{dS_{motor}}{dt} = 0$)

c) Surroundings receive heat : $dS = \frac{dQ}{T}$, $\Delta S_{surr} = \frac{Q}{T_0}$

$Q = \dot{Q} \cdot \Delta t = hA(T_S - T_0) \cdot \Delta t$
$= 52.65\,kJ$ → $\underline{\Delta S_{surr} = 179.7\,J/K}$

d) $\Delta S_{total} = \Delta S_{motor} + \Delta S_{surr} = \underline{179.7\,J/K} > 0$ irreversible process

e) $W_{lost} = T_0\,\Delta S_{total} = T_0\,\Delta S_{surr} = Q$

$\underline{W_{lost} = 52.65\,kJ}$ amount of work dissipated and (heat) not converted into shaft work