**Massachusetts Institute of Technology**
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# 16.001/16.002 Unified Engineering I, II
## Fall 2008

Problem Set 12

Name: ______________________

Due Date: 12/2/2008

| | Time Spent (min) |
|---|---|
| **F8** | |
| **F9** | |
| **S10** | |
| **S11** | |
| **S12** | |
| **S13** | |
| **M23** | |
| **M24** | |
| **Study Time** | |

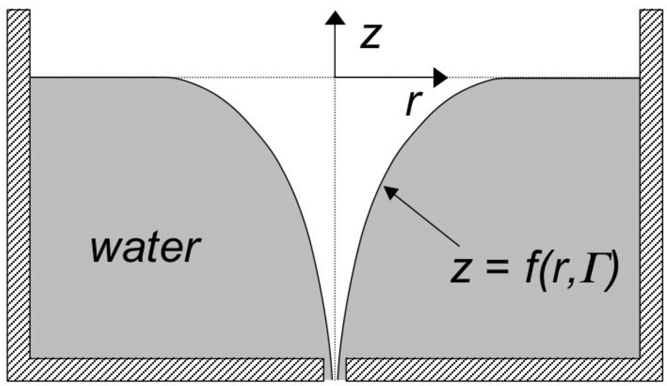
Announcements:

Please note the due date (December 2nd) is not a Friday, it's the Tuesday after Thanksgiving.

**Unified Engineering** Fall 2008
**Fluids** P. Lozano

## Problem F8

Consider a situation in which water flows out of a large container through a hole as shown in the diagram below. The relative sizes of the container and the hole are such that the water level does not change significantly over a considerable amount of time. A vortex is formed around the hole with a velocity potential and stream function respectively given by $\phi = A\theta$ and $\psi = -A\ln r$. Flow is inviscid.




(a) Using $\phi$ and $\psi$, find an expression for the fluid velocity vector.
(b) Is this flow irrotational? Explain.
(c) Using Bernoulli's equation, find an expression for the liquid surface shape in terms of the circulation.
(d) Draw a sketch of the profile found in (c) with A = 1 and comment on the validity of this expression near $r = 0$.

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## Problem F9

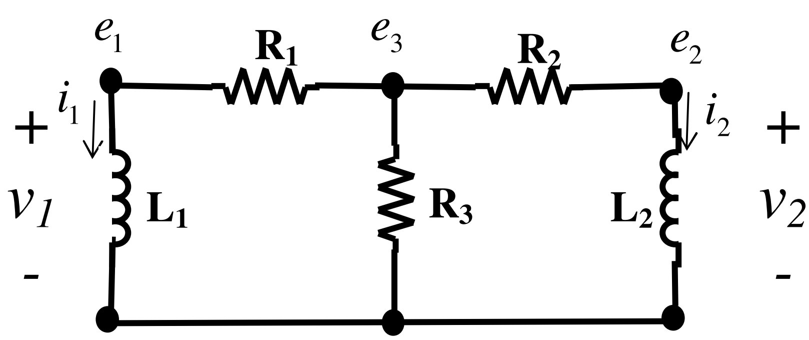
The following velocity potential corresponds to a spiral vortex:

$$\phi = A\theta - B\ln r,$$

where $A$ and $B$ are constants.

(a) Find an expression for the fluid velocity vector.
(b) Show analytically that the angle $\beta$ between the velocity vector and the radial direction is constant.
(c) Is this flow irrotational? Explain.
(d) Is this flow compressible? Explain.
(e) Find an equation for the streamlines.
(f) Draw two streamlines ($A$=$B$ and $A$=4$B$) that go through the point ($r$ = 1, $\theta$ = 0) and identify the angle $\beta$ at two different locations on each streamline (include the numerical value of $\beta$ in all cases).

Problem S10 (Signals and Systems)




Consider the circuit above with

$R_1 = R_2 = R_3 = 1\ \Omega$ and $L_1 = L_2 = 1\text{H}$

The initial conditions on the inductor currents are,

$i_1(0) = 5\text{A}$, $i_2(0) = -5\text{A}$

Find $i_1(t)$ and $i_2(t)$ using the node voltage method (hint: after your write the node equations, you should try to eliminate $e_3$, so that you get just two first order differential equations).

Problem S11 (Signals and Systems)

For the same circuit in S10, with the same initial conditions. Find $i_1(t)$ and $i_2(t)$ using the mesh current method. Which method did you find easier?

Problem S12 (Signals and Systems)

For the same circuit in S10, with the same initial conditions. Let the state of the system be,

$$x(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

a) Find the state-space equations that describe the evolution of the circuit, in the form: $\dot{x} = Ax$

b) Suppose that the output of the system is the voltage at the middle node ($e_3$); write the system output equations, and draw the block diagram representation for the complete system.

Problem S13 (Signals and Systems)

For the matrix,

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix}$$

a) Find the characteristic polynomial of A
b) Find the eigen-values and eigen-vectors of A

**Unified Engineering Problem Set 12** **Lectures: M23, M24**
**Week 13 Fall, 2008** **Units: M3.2**

**M23 (M13.1)** ***(10 points)*** *[Continuation of M22 (M12.2) ]*

Let's continue the exploration of the ties of the three-dimensional compliance tensor to the overall set of stress-strain relationships:

(a) For the orthotropic case, relate the engineering constants to the components of the compliance tensor.

(b) For the orthotropic case, **show** how to relate the engineering constants back to the components of the elasticity tensor, using the results of M22 (M12.2) and part (a) of this problem, as appropriate. (**Do not** get final component-by-component relations.)

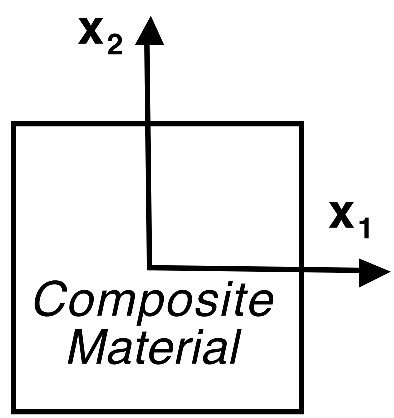
**M24 (M13.2)** ***(15 points)*** A composite material is made with a polymer matrix reinforced with fibers in multiple directions along with nanofibers in the through-thickness direction. A set of three experiments are performed on this material configuration. The stresses applied in each case are noted and various strains measured. Note that the strain gage in the 2-direction broke during Experiment C and no readings were obtained. The stresses and strains for the three experiments are:

Experiment A

$\sigma_{22} = 50$ ksi
$\varepsilon_{11} = -1100\ \mu$strain
$\varepsilon_{22} = +63000\ \mu$strain

Experiment B

$\sigma_{12} = 30$ ksi
$\varepsilon_{12} = +4400\ \mu$strain



<u>Experiment C</u>

$\sigma_{11} = 30$ ksi
$\sigma_{22} = 15$ ksi
$\varepsilon_{11} = +1800$ $\mu$strain

<u>NOTE</u>: **Any stresses or in-plane strains not specified are equal to zero, except that related to gage failure in Experiment C. Also, all strains are tensorial.**

(a) Determine the in-plane engineering constants (all possible) and characterize the in-plane stress-strain behavior of the material.

(b) If possible, determine what the broken strain gage along the 2-direction in Experiment C should have read. If not possible, explain why it is not possible.

(c) Determine as many components of the compliance tensor as possible and put this in matrix form.