Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2008

Problem Set 13

Name: ______________________

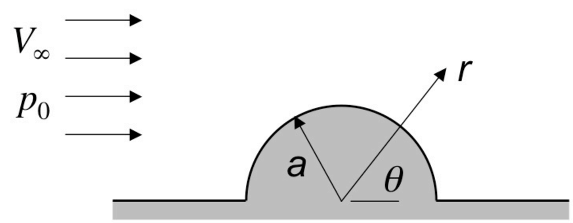
Due Date: 12/10/2008

	Time Spent (min)
F10	
F11	
S14	
S15	
S16	
S17	
Study Time	

Announcements:

Problem F10

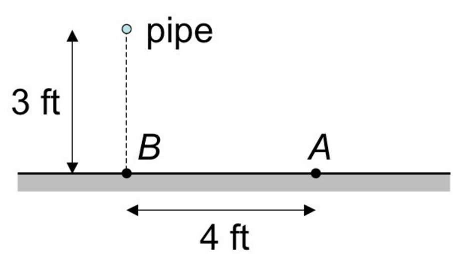
A steady, incompressible fluid flows past an infinitely long semicircular hump as shown in the figure. Far upstream from the hump the velocity field is uniform, and the pressure is p_0.



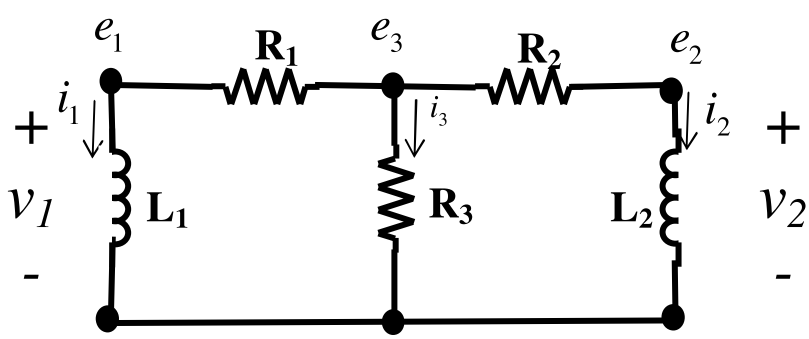
(a) Find expressions for the maximum and minimum values of the pressure along the hump in terms of V_∞, ρ and p_0, and indicate their location.
(b) If the solid surface is the $\psi = 0$ streamline, determine the equation of the streamline passing through the point $\theta = \pi/2$, $r = 2a$.

Problem F11

A long porous pipe runs parallel to a horizontal plane surface as shown in the figure. The longitudinal axis of the pipe is perpendicular to the plane of the paper. Water flows radially from the pipe at a rate of 0.5π ft^3/s per foot of pipe. Determine the difference in pressure (in lb/ft^2) between point B and point A. You may approximate the flow from the pipe by a two dimensional source.



Problem S14 (Signals and Systems)



Consider the circuit above with

$R_1 = R_2 = R_3 = 1\ \Omega$ and $L_1 = L_2 = 1$ H

The initial conditions on the inductor currents are,

$i_1(0) = 10$ A, $i_2(0) = 0$ A

Let the state of the system be,

$$x(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

a) Find the state equations that describe the evolution of the circuit, in the form: $\dot{x} = Ax$

b) Find the characteristic equation for the system

c) Solve for the complete solution for $i_1(t)$ and $i_2(t)$ using the initial values above

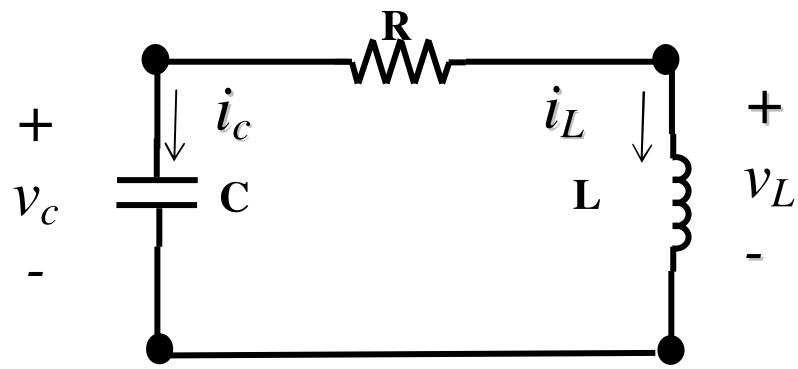
Problem S15 (Signals and Systems)

For the same circuit as in problem S14, with the same initial conditions. Let the state of the system now be

$$\tilde{x}(t) = \begin{bmatrix} i_1(t) \\ i_3(t) \end{bmatrix}$$

a) Find the state transformation matrix T for transforming the original states x to $\tilde{x}$
b) For the transformed system, find the state equations of the form $\dot{\tilde{x}} = A\tilde{x}$
c) Using the state equations derived above and the initial conditions, solve for the complete solution for $i_1(t)$ and $i_3(t)$ (Hint, you must first derive the initial conditions on $i_3(0)$).
d) Check that your answer is consistent with what you obtained for problem S14.

Problem S16 (Signals and Systems)



Consider the above circuit, consisting of a resistor with resistance R, capacitor with capacitance C, and inductor with inductance L. Let the state variables for the circuit be the capacitor voltage (V_c) and the inductor current (i_L).

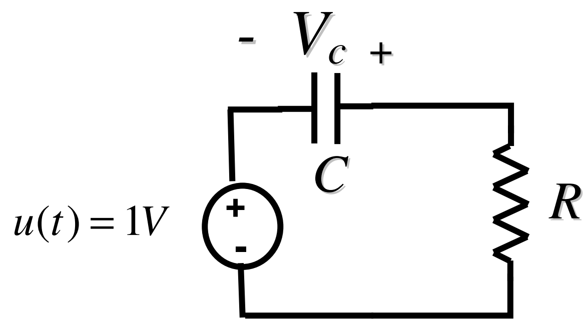
a) Obtain the state equations in standard form, and solve for the characteristic values of the circuit in terms of R, L and C.

b) Now let R = 0, L = 2 and C = 2 and solve for the complete solution for the system with initial voltage $V_c(0) = 10$ V and initial current $i_L(0) = 0$ A. (for this part you can leave your solution in complex form).

c) Use Euler's rule to express the current and voltage as real values (sinusoids).

d) What do you think is happening in this circuit? What would the effect of a positive (non-zero) resistance value be on the response of this circuit?

Problem S17 (Signals and Systems)



Consider the circuit above with constant voltage source u(t) = 1 V, and initial voltage across the capacitor, $V_c(0) = 0$ V.

Solve for the complete solution for $V_c(t)$, for $t > 0$.

Note: This problem involves the solution to differential equations with (non-zero) inputs. I will cover the material for this problem on Monday, December 8. You can read-ahead on the complete solution to a system with inputs in Signals and Systems (Oppenheim, Willskey and Nawab), Section 2.4.1.