Unified Engineering

Homework SOLUTIONS
Week #3; Problem Set #2

M1 **(M3.1)** There are multiple design considerations for all structures and these inevitably involve tradeoffs. The important point here is to identify those considerations that are the most important (i.e. key) to the particular design and thus purpose of that structure. In this vein, there are no "right" answers here since the specific purposes of the structure were not defined. Thus, the first part to answering this question is to define, in your mind, what purpose(s) the structure serves.

It is also important to realize that true tradeoffs (i.e. *relative* importance) can only be done quantitatively when a clear objective is in mind and the factors associated with the different design factors can be quantified. Otherwise, only general statements about tradeoffs can be made. This should become clearer in the "answers" that follow. Also note that in all cases, cost enters into the consideration, but in different ways as presented.

(a) Business jet: With safety as generally the number one consideration, a key item is strength -- ability to carry loads without failure. This also means that there are deformation resistance considerations at various parts of the aircraft. Clearly the wing must hold its shape to a certain margin, otherwise there will be a potential loss in the aerodynamic characteristics and thus the forces on the wing, known as lift and drag. The airplane is designed to last a number of years, so corrosion resistance is important as are considerations of fatigue and general durability. Weight is a clear consideration since this involves a tradeoff with additional payload/fuel/range. Business jet buyers often look for comfort in their interior. Thus, the interior design enters into the consideration and can even drive items such as fuselage size. Finally, the buyers of the aircraft must be able to afford the aircraft, so cost of the airplane is a key (including items such as operating cost). However, those in this market are often more willing to pay additonal for comfort and effectiveness.

(b) Traffic Tunnel: A traffic tunnel is a pretty basic structure and the design considerations are also pretty straightforward. The structure must be strong enough to carry the loads without failure. In addition, the rigidity of the structure must be sufficient for the tunnel to retain its shape since deflection/deformation must be resisted. Change of shape may allow the external material (e.g. earth fill) to move and potentially allow water paths and other problems that eventually access the ground surface. Tunnels are also exposed to some level of the elements as brought in through their access points as well as that experienced at the interface with the fill material. Thus, corrosion resistance is also an important consideration. This is linked to the final item which is longevity. Tunnels are made and used for decades and thus must resist fatigue, corrosion, etc. and be able to be maintained/repaired with ease of inspectability and access of crews. These items end up being tradeoffs with the other major consideration -- cost. It is a question of the up-front cost of the structure versus cost "down the road" (so to speak) to maintain and repair. This should include consideration of the inconvenience caused to commuters, etc. when tunnels are closed or traffic restricted when major repairs are made (think about the current state with the Longfellow Bridge....).

(c) Step ladder: This is primarily a work tool, so performance characteristics will tend to top the list. Those who purchase this want ease of use and transport, ability to easily place and take down, and safety in use. So this brings in considerations of strength, rigidity, and longevity. Longevity is less important here since it is relatively easy to replace this product and one can tell when the material starts to corrode, degrade, etc. Stiffness and strength, however, are clearly important. There are also other considerations. One of the main ones is electrical conductivity. Metal step ladders have been primary products for many years. The problem with metal (generally aluminum) ladders is that if they touch a live wire, the person touching the ladder can be electrocuted (or at least shocked). Plastic/glass-reinforced ladders have been introduced in large part because of this. They keep down the weight, which is important because people have to be able to carry these things, but they are not conductive and provide that extra electrical safety. Finally, this often being aimed at the general market, cost considerations are a key as this may drive the decision of the consumer to purchase.

(d) Space Station: The structures of a space station serve a number of important purposes. The structure must maintain its shape and must thus be resistant to deformation. The parts of the space station with any human occupancy have particular key purposes. These must maintain the internal pressure and environment for the occupants, and thus the safety to do such is a primary consideration. This requires strength for these structures in addition to retention of shape via the needed stiffness. These parts of the structure along with other parts of the station need to be protected from items in space and must thus be resistant to meteorite impact. The station structure can be in operation for many years and thus must have longevity. In orbit, this means resistance to uv, atomic oxygen, fatigue, and other long-term considerations. In orbit, a structure goes in and out of earth's shadow and thus can undergo very large thermal gradients. Thus, dimensional stability is important from thermal considerations. Weight and cost are always factors, but it is hard to evaluate these without knowing the details of station launch and assembly, subsequent station-keeping, and other factors associated with operation of the station. Part of this includes the loading that pieces of the station undergo in being launched and in any assembly.

(e) 18-wheeler transport truck: The main considerations in a road vehicle are generally safety and cost with cost probably being the most important. If this were truly a consumer product, one would look to the concerns of the general consumer, and the general consumer is much less likely to care about technical innovations and performance capability. They want a product that will work, will get them there, won't break down, and will last. Thus, the next key design consideration is longevity. With care, especially bodies, this is inherently linked to corrosion resistance. Another key design consideration is energy absorption in impacts (also known as "crashes"). Much of the body and structure of a vehicle is designed to absorb the energy in such an incident in order to protect the occupant(s). However, a key design consideration in a transport truck is that this is a business vehicle used to transport items. Thus, its overall effectiveness in doing such will tend to be the ultimate consideration. This includes items such as amount that can be hauled, speed and ease of maneuverability, and fuel efficiency (measured in terms of weight/mass/value of material that can be transported). Cost in this case thus is a tradeoff with these key performance considerations.

(f) Glider: A glider is also an airplane, but it looks quite different from a business jet or commercial transport. Glider wings have a very large "aspect ratio" (length of the wing/"width" of the wing). Due to this great length, glider wings must be very stiff, so deformation resistance is a key consideration with strength being an important design consideration, but not as great a consideration as the stiffness/rigidity. Gliders are unpowered and rely on a tow to get to altitude and then on "thermals" to soar higher (Ever watch a hawk or eagle? They do the same thing!). Thus, weight is a much more critical factor with regard to gliders. How about cost? Clearly, cost is a consideration in any consumer product. However, in the sports industry, many people with the "means" are willing to pay extra for extra performance. So cost becomes quite a different factor here since performance becomes a much greater consideration for which people are willing to pay. Thus, cost/performance becomes a key tradeoff.

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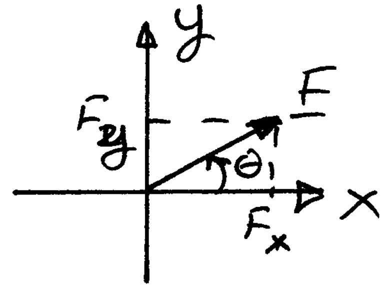
M2 (M3.1)

(a) To draw these forces, first resolve each force into x and y components with the use of $\hat{i}$ and $\hat{j}$ (*) vectors in order to simplify plotting and for describing each as a vector.

Note:



components are determined via:

$$\left.\begin{aligned} F_x &= |\underline{F}| \cos \theta \\ F_y &= |\underline{F}| \sin \theta \end{aligned}\right\} \text{units of [N]}$$

(*) NOTE: At times, unit vectors are noted via "hats" (^) rather than an underline or overbars as used for a general vector.

$\underline{\underline{So}}$: $F_i\,(x_i, y_i)$ → x and y locations in units of [m]

↑ force number

→ proceeding:

$\underline{F}_1(1,1) = (2\ N)\{\hat{i}\cos(0°) + \hat{j}\sin(0°)\}$
$= (2.0\ N)\,\hat{i}$

$\underline{F}_2(1,-4) = (5N)\{\hat{i}\cos(63.4°) + \hat{j}\sin(63.4°)\}$
$= (2.24N)\hat{i} + (4.47N)\hat{j}$

$\underline{F}_3(2,-3) = (5N)\{\hat{i}\cos(-116.6°) + \hat{j}\sin(-116.6°)\}$
$= (-2.24N)\hat{i} + (-4.47N)\hat{j}$

$\underline{F}_4(-5,-5) = (3N)\{\hat{i}\cos(45°) + \hat{j}\sin(45°)\}$
$= (2.12N)\hat{i} + (2.12N)\hat{j}$

$\underline{F}_5(2,4) = (3N)\{\hat{i}\cos(251.5°) + \hat{j}\sin(251.5°)\}$
$= (-0.95N)\hat{i} + (-2.84N)\hat{j}$

$\underline{F}_6(-5,5) = (4N)\{\hat{i}\cos(315°) + \hat{j}\sin(315°)\}$
$= (2.83N)\hat{i} + (-2.83N)\hat{j}$

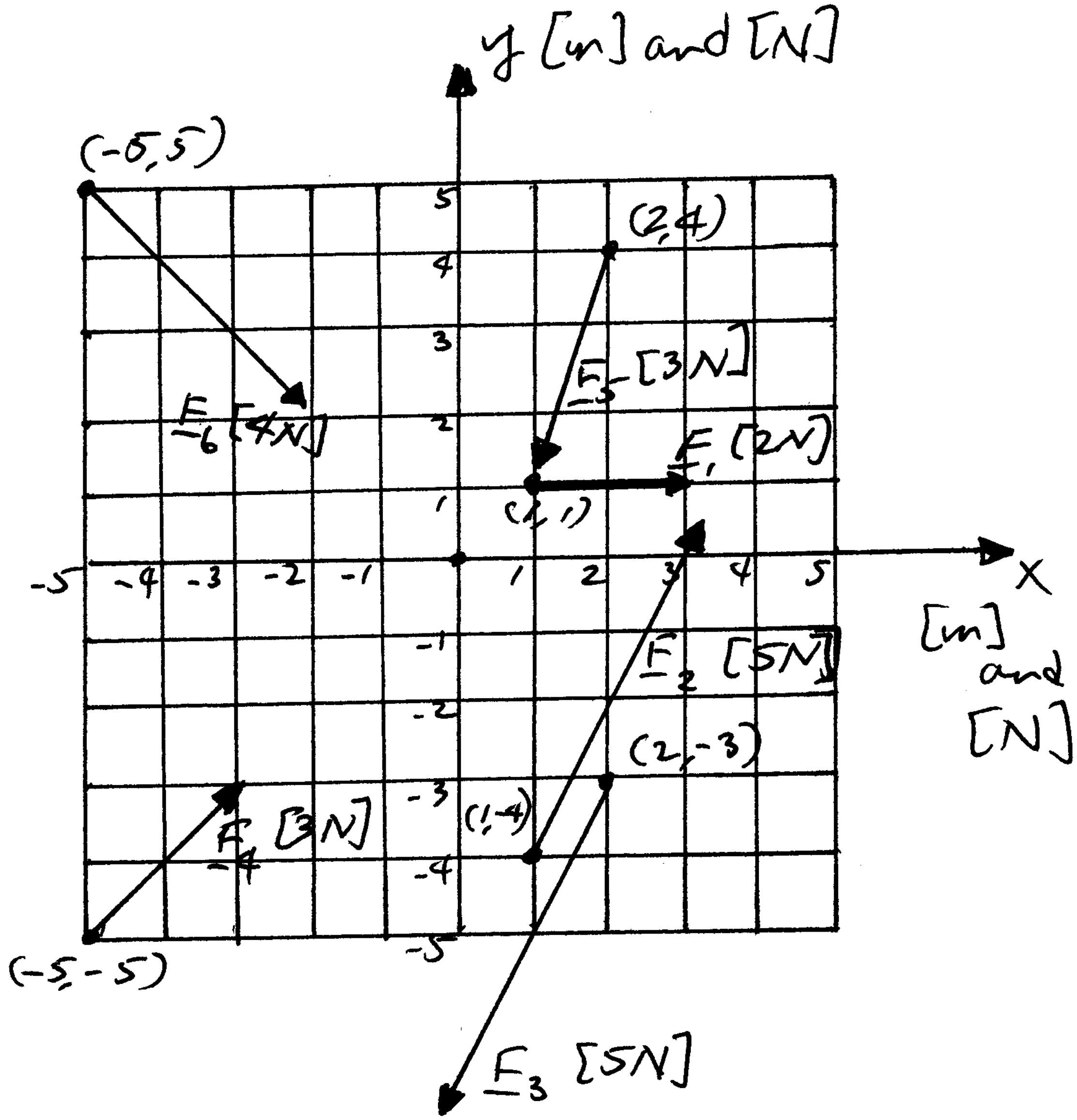
So the vector description is given by the magnitudes of the forces in each direction times the unit vectors, with indication of the (x, y) location from which the force vector acts.

Summarizing:

$$\underline{F}_1\,(1,1) = (2.0\,N)\,\hat{\imath}$$
$$\underline{F}_2\,(1,-4) = (2.24\,N)\,\hat{\imath} + (4.47\,N)\,\hat{\jmath}$$
$$\underline{F}_3\,(2,-3) = (-2.24\,N)\,\hat{\imath} + (-4.47\,N)\,\hat{\jmath}$$
$$\underline{F}_4\,(-5,-5) = (2.12\,N)\,\hat{\imath} + (2.12\,N)\,\hat{\jmath}$$
$$\underline{F}_5\,(2,4) = (-0.95\,N)\,\hat{\imath} + (-2.84\,N)\,\hat{\jmath}$$
$$\underline{F}_6\,(-5,5) = (2.83\,N)\,\hat{\imath} + (-2.83\,N)\,\hat{\jmath}$$

In order to plot these forces, one must choose a scale for the force magnitude as well as the x, y location. The x and y units are given in [m], so this is used. For convenience, a coincident grid where 1[N] equals 1[m] is used.

Draw a grid with the vectors on it:



(b) $\underline{F}_{total} = \sum \underline{F}_i$

It is easiest to sum components in the x and y directions (i.e. $\hat{i}$ and $\hat{j}$ components).

$$\Rightarrow \underline{F}_{total} = \hat{i}\,(2.0N + 2.24N - 2.24N + 2.12N - 0.95N + 2.83N) + \hat{j}\,(4.47N - 4.47N + 2.12N - 2.84N - 2.83N)$$

$$\Rightarrow \boxed{\underline{F}_{total\,(net)} = (6.0N)\hat{i} + (-3.55N)\hat{j}}$$

the overall magnitude is

$$|\underline{F}| = \sqrt{(F_x)^2 + (F_y)^2}$$

$$\Rightarrow |\underline{F}_{total}| = \sqrt{(6.0N)^2 + (-3.55N)^2}$$

So:

$$\boxed{|F_{total}| = 6.97N}$$

(c) The definition of a couple is it results from two parallel (coplanar) forces of equal magnitude and opposite directions such that there is a net moment, but no net force.

Thus, first look for forces of equal magnitude. There are two pairs: ($\underline{F}_2$ and $\underline{F}_3$) and ($\underline{F}_4$ and $\underline{F}_5$). The latter pair are not opposite in direction and thus do not qualify as they result in a net force. However, $\underline{F}_2$ and $\underline{F}_3$ are opposite in direction. They thus satisfy the definition of a couple.

So: $\boxed{\text{YES -- } \underline{F}_2 \text{ and } \underline{F}_3 \text{ are a couple}}$

→ The expression for a couple is:

$$\underline{C} = \underline{r} \times \underline{F}$$

where:

$\underline{F}$ is one of the vectors

$\underline{r}$ is the position vector from one vector to the other

To get $\underline{r}$, subtract one (x, y) position from the other, i.e. (from $\underline{F}_3$ to $\underline{F}_2$)

$$\Rightarrow \quad \underline{r} = (2-1)[m]\,\hat{i} + (-3-(-4))[m]\,\hat{j}$$

giving: $\underline{r} = (\hat{i} + \hat{j})\ [m]$

Thus:

$$\underline{C} = (\hat{i} + \hat{j})[m] \times ((-2.24N)\hat{i} + (-4.47N)\hat{j})$$

Note that:

$$\hat{i} \times \hat{j} = \hat{k} \quad (z\text{-direction})$$
$$\hat{j} \times \hat{i} = -\hat{k}$$
$$\hat{i} \times \hat{i} = 0$$
$$\hat{j} \times \hat{j} = 0$$

$$\Rightarrow \quad \underline{C} = (-4.47\ N\cdot m)\hat{k} + (-2.24\ N\cdot m)(-\hat{k})$$

Finally: $\boxed{\underline{C} = -2.23\ [N\cdot m]\ \hat{k}}$

(d) Determine the moment about the origin of each force vector $(\underline{F}_i) \rightarrow (\underline{M}_i)$ and sum these to determine the net moment:

$$\underline{M}_{net} = \sum_i (\underline{r}_i \times \underline{F}_i)$$

So:

$$\underline{M}_{Oi} = (x_i\,\hat{i} + y_i\,\hat{j}) \times (F_{x_i}\,\hat{i} + F_{y_i}\,\hat{j})$$

location vector ($\underline{r}_i$) for force $\underline{F}_i$

force vector $\underline{F}_i$

Proceeding:

$$\underline{M}_{O1} = (\hat{i}+\hat{j})\,[m] \times (2.0\,\hat{i})\,[N]$$
$$= -2.0\,[N\cdot m]\,\hat{k}$$

$$\underline{M}_{O2} = (\hat{i} - 4\hat{j})\,[m] \times \{(2.24\,\hat{i}) + (4.47\,\hat{j})\}\,[N]$$
$$= (4.47)\,[N\cdot m]\,\hat{k} + (8.96)\,[N\cdot m]\,\hat{k}$$
$$= 13.43\,[N\cdot m]\,\hat{k}$$

$$\underline{M}_{O3} = (2\hat{i} - 3\hat{j})\,[m] \times \{(-2.24\,\hat{i}) + (-4.47\,\hat{j})\}\,[N]$$
$$= (-8.94)\,[N\cdot m]\,\hat{k} + (-6.72)\,[N\cdot m]\,\hat{k}$$
$$= -15.66\,[N\cdot m]\,\hat{k}$$

$$\underline{M}_{O4} = (-5\hat{i} - 5\hat{j})\,[m] \times \{(2.12\,\hat{i}) + (2.12\,\hat{j})\}\,[N]$$
$$= (-10.6)\,[N\cdot m]\,\hat{k} + (10.6)\,[N\cdot m]\,\hat{k}$$
$$= 0\,\hat{k}$$

$$\underline{M}_{O5} = (2\,\hat{i} + 4\,\hat{j})\,[m] \times \{(-0.95\,\hat{i}) + (-2.84\,\hat{j})\}\,[N]$$
$$= (-5.68)\,[N\cdot m]\,\hat{k} + (3.81)\,[N\cdot m]\,\hat{k}$$
$$= -1.87\,[N\cdot m]\,\hat{k}$$

$$\underline{M}_{O6} = (-5\hat{i} + 5\hat{j})\,[m] \times \{(2.83\,\hat{i}) + (-2.83\,\hat{j})\}\,[N]$$
$$= (14.15)\,[N\cdot m]\,\hat{k} + (-14.15)\,[N\cdot m]\,\hat{k}$$
$$= 0\,\hat{k}$$

and then: $\underline{M}_{o_{net}} = \sum_i \underline{M}_{oi}$

$$\Rightarrow \boxed{\underline{M}_{o_{net}} = -6.10 \, [N \cdot m] \, \hat{k}}$$

(e) This is the same set of calculations as in part (d), except the position vector in each case, $\underline{r}_i$, is from the upper right hand corner of the grid (5m, 5m) to the vector (rather than the origin).

Thus: $\underline{r}_i = (x_i - 5[m])\hat{i} + (y_i - 5[m])\hat{j}$

$$\Rightarrow \underline{M}_1 = (-4\hat{i} - 4\hat{j})[m] \times (2.0\hat{i})[N]$$
$$= 8.0 \, [N \cdot m] \, \hat{k}$$

$$\underline{M}_2 = (-4\hat{i} - 9\hat{j})[m] \times \{(2.24\hat{i}) + (4.47\hat{j})\}[N]$$
$$= (-17.88)[N \cdot m]\hat{k} + (20.16)[N \cdot m]\hat{k}$$
$$= 2.28 \, [N \cdot m] \, \hat{k}$$

$$\underline{M}_3 = (-3\hat{i} - 8\hat{j})[m] \times \{(-2.24\hat{i}) + (-4.47\hat{j})\}[N]$$
$$= (13.41)[N \cdot m]\hat{k} + (-17.92)[N \cdot m]\hat{k}$$
$$= -4.51 \, [N \cdot m] \, \hat{k}$$

$$\underline{M}_4 = (-10\hat{i} - 10\hat{j})[m] \times \{(2.12\hat{i}) + (2.12\hat{j})\}[N]$$
$$= (-21.2)[N \cdot m]\hat{k} + (21.2 \, N \cdot m)\hat{k}$$
$$= 0 \, \hat{k}$$

$$\underline{M}_5 = (-3\,\hat{i} - 1\,\hat{j})[m] \times \{(-0.95\,\hat{i}) + (-2.84\,\hat{j})\}[N]$$
$$= (8.52)[N\cdot m]\,\hat{k} + (-0.95)[N\cdot m]\,\hat{k}$$
$$= 7.57\,[N\cdot m]\,\hat{k}$$

$$\underline{M}_6 = (-10\,\hat{i})[m] \times \{(2.83\,\hat{i}) + (-2.83\,\hat{j})\}[N]$$
$$= 28.3\,[N\cdot m]\,\hat{k}$$

Proceeding with:

$$\underline{M}_{net} = \sum M_i$$

$$\Rightarrow \boxed{\underline{M}_{net} = 41.6\ [N\cdot m]\,\hat{k} \quad \text{about } (5m, 5m)}$$

(f) By examination, one can see there are no components acting about the x-axis and y-axis since the only unit vector in the expression for the moments is $\hat{k}$ (z-direction)

More generally, one can find:

Moment about axis = (unit vector about axis) $\cdot$ ($\underline{r} \times \underline{F}$)

So:

M_x = Moment component about x-axis

$= \hat{i} \cdot \underline{M}_{net}$ with $\hat{i} \cdot \hat{k} = 0$

$\Rightarrow \boxed{M_x = 0}$

Similarly:

M_y = Moment component about y-axis

$= \hat{j} \cdot \underline{M}_{net}$ with $\hat{j} \cdot \hat{k} = 0$

$\Rightarrow \boxed{M_y = 0}$