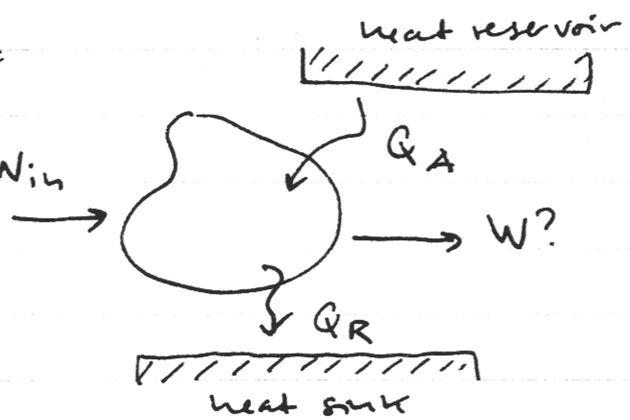
T4

16. Unified Fall 08 ZS

1)



Concepts:
- 1st law
- heat and work exchange
- cycles ($\Delta U = 0$)

1st law: $\Delta U = Q - W$ Q: heat to system

W: work done by system

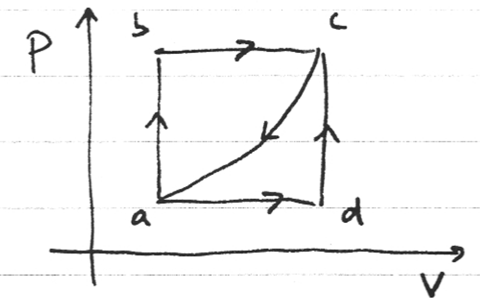
for cycle: $\Delta U_{cycle} = 0$

$\rightarrow \quad 0 = Q_A - Q_R - (-W_{in} + W)$; $W = Q_A - Q_R + W_{in} = 55\,kJ$

done by system

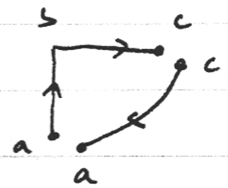
net work: $W_{net} = W - W_{in} = 40\,kJ$ done by system

2)



Concepts:
- 1st law
- heat and work exchange
- cycles ($\Delta U = 0$)

a)



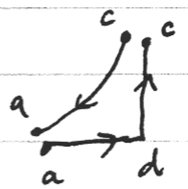
$\Delta U_{abc} = Q_{abc} - W_{abc}$ $Q_{abc} = 80\,kJ$, $W_{abc} = 35\,kJ$

$\Delta U_{ca} = Q_{ca} - W_{ca}$ $Q_{ca} = -60\,kJ$, $W_{ca} = ?$

$\Delta U_{abca} = 0 \rightarrow \quad 0 = Q_{abc} + Q_{ca} - W_{abc} - W_{ca} \rightarrow W_{ca} = -15\,kJ$

done on system

b)



$\Delta U_{adc} = Q_{adc} - W_{adc}$

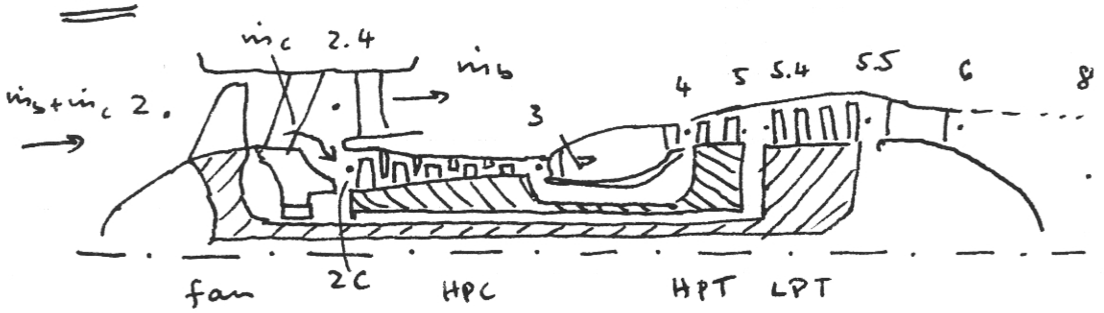
$Q_{adc} = 70\,kJ$, $W_{adc} = ?$

$\Delta U_{ca} = Q_{ca} - W_{ca}$

$\Delta U_{adca} = 0 \rightarrow \quad 0 = Q_{adc} + Q_{ca} - W_{ca} - W_{adc} \rightarrow W_{adc} = 25\,kJ$

done by system

<u>T5</u> 16. Unified Fall 08 ZS



Charts: $T_2 = 300\,K$

$T_{2c} = T_{2.4} = 366\,K$

$T_3 = 773\,K$

$T_4 = 1533\,K$

$T_{5.4} = 1033\,K$

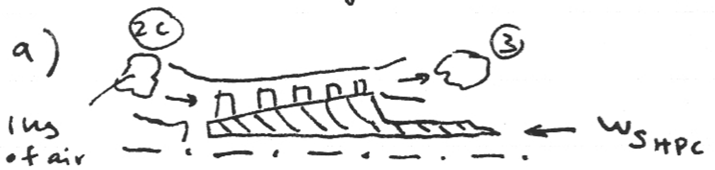
$T_{5.5} = 866\,K$

Given: $\dot m_c = 30\,kg/s$; $c_{pc} = 1000\,J/kgK$, $c_{pT} = 1200\,J/kgK$

Concepts: - 1st law, flow work, shaft work, shaft power balance

Assume: ideal gas, adiabatic compr. and turbine, steady flow, neglect ΔKE and ΔPE, neglect fuel mass flow

a)



CM: 1 kg of air

work exchange: flow and shaft work

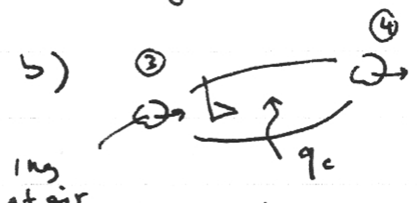
1st law: $u_3 - u_{2c} = -(-p_{2c}v_{2c} - w_{sHPC} + p_3 v_3)$ → enthalpy

$h_3 - h_{2c} = w_{sHPC}$, $\dot W_{sHPC} = \dot m_c \cdot w_{sHPC} = \dot m_c c_{pc}(T_3 - T_{2c})$

ideal gas: $dh = c_{pc}\,dT$

$\underline{\dot W_{sHPC} = 12.2\ MW}$

b)



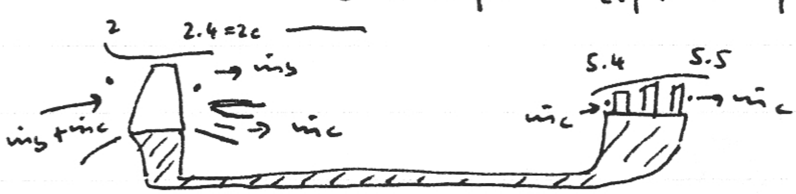
1st law: $u_4 - u_3 = q_c - (-p_3 v_3 + p_4 v_4)$

$h_4 - h_3 = q_c = c_{pT}\left(T_4 - T_3 \frac{c_{pc}}{c_{pT}}\right)$

Note: h_3 → air, h_4 → comb. gas

$\dot Q_c = \dot m_c q_c = \dot m_c (c_{pT} T_4 - c_{pc} T_3) = \underline{32\ MW}$

c) Bypass ratio $\beta = \frac{\dot m_b}{\dot m_c}$



Shaft power balance:

$(\dot m_b + \dot m_c)\, w_{sfan} = \dot m_c\, w_{sLPT}$; from 1st law: $w_{sfan} = h_{2.4} - h_2$; $w_{sLPT} = h_{5.4} - h_{5.5}$

so $(\beta+1)(h_{2.4} - h_2) = h_{5.4} - h_{5.5}$ → $\beta = \frac{h_{5.4} - h_{5.5}}{h_{2.4} - h_2} - 1$; $\beta = \frac{c_{pT}}{c_{pc}} \frac{T_{5.4} - T_{5.5}}{T_{2.4} - T_2} - 1$

$\underline{\beta = 2}$

d) from c: $\dot W_{sfan} = \dot m_{b+c} w_{sfan} = (\beta+1)\dot m_c c_{pc}(T_{2.4} - T_2)$, $\underline{\dot W_{sfan} = 5.9\ MW}$

Solutions (Fluids) 9/18/08

Problem F2

For a droplet of radius R and net charge Q, the electric field is:

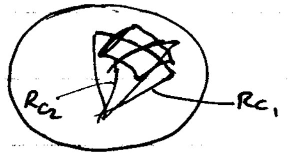
$$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2}\frac{\bar{R}}{|\bar{R}|}$$

This field points radially, normal to the liquid surface. The presence of this field produces an electric force per unit area on the liquid interface, or electric pressure: $P_E = \frac{1}{2}\epsilon_0 E^2$, which is counteracted by surface tension: $P - P_\infty = \kappa\gamma$.

In a sphere, the curvature is constant and is given by:

$$\kappa = \frac{1}{R_{c_1}} + \frac{1}{R_{c_2}} = \frac{2}{R}$$

$R_{c_1} = R_{c_2} = R$



therefore $\Rightarrow$ $P = P_\infty + \frac{2\gamma}{R}$ In vacuum, $P_\infty = 0$

For the droplet to be stable, we require $\boxed{P > P_E}$

In the limit of "explosion", $P = P_E$ or $\frac{2\gamma}{R} = \frac{1}{2}\epsilon_0 E^2$

then we have: $\frac{2\gamma}{R} = \frac{1}{2}\epsilon_0 \frac{Q^2}{4^2\pi^2\epsilon_0^2 R^4}$ $\xrightarrow{\text{solve for } Q}$ $\boxed{Q = 8\pi\sqrt{\gamma\epsilon_0 R^3}}$

that means that:

50% 1) The maximum # of elementary charges is N, where $Ne = Q$

therefore $\boxed{N = \frac{8\pi\sqrt{\gamma\epsilon_0 R^3}}{e}}$

With our values: $N = \frac{8\pi\sqrt{(0.05\frac{N}{m})(8.854\times10^{-12}\frac{F}{m})(20\times10^{-9})^3 m^3}}{1.6\times10^{-19}C} = 295.6$

$N = 296$

It only takes 296 elementary charges to make this droplet explode.

50% 2) The charge per unit mass is given by: $\frac{Q}{M} = \frac{8\pi\sqrt{\gamma\epsilon_0 R^3}}{\rho_l \frac{4}{3}\pi R^3}$

huge!

therefore $\frac{Q}{M} = \frac{6}{\rho_l}\sqrt{\frac{\gamma\epsilon_0}{R^3}} = \frac{6}{1200\frac{kg}{m^3}}\sqrt{\frac{(0.05\,N/m)(8.854\times10^{-12}\,F/m)}{(20\times10^{-9})^3 m^3}} = 1176\,\frac{C}{kg}$

0% [Corollary: if electrostatically accelerated using 1000V, these droplets will move with a velocity $v = \sqrt{2\frac{Q}{M}V} = \sqrt{2(1176)(1000)} = 1533\,m/s$ not bad.]

Problem F3

9/18/08

(1) Functional forms for the airfoil and pressure distributions: $y_u(x) = Ax(1+Bx)$

20%

At $x=0 \rightarrow y=0$ (automatic)

At $x=c \rightarrow y=0 \Rightarrow B = -1/c$

At $x=c/2 \rightarrow y=w \Rightarrow A = 4w/c$

$$\boxed{\begin{aligned} y_u(x) &= \frac{4w}{c}\, x\left(1-\frac{x}{c}\right) \\ y_l(x) &= 0 \equiv \text{const} \end{aligned}}$$

Similarly for $P_l - P_u$: $P_l - P_u = \frac{4}{c}(P_l-P_u)_{max}\, x\left(1-\frac{x}{c}\right)$

$$P_l - P_u = \frac{4}{c}\left(2\rho V_\infty^2\left(\frac{w}{c}\right)^2\right) x\left(1-\frac{x}{c}\right)$$

using $q_\infty = \frac{1}{2}\rho V_\infty^2$

or $\boxed{P_l - P_u = 16\, q_\infty \left(\frac{w}{c}\right)^2 \frac{x}{c}\left(1-\frac{x}{c}\right)}$

(2) For $c_l' = \frac{L'}{q_\infty c}$ $\Rightarrow$ $L' = \underbrace{\cos\alpha \int_0^c (P_l - P_u)\,dx}_{L_1'} + \underbrace{\sin\alpha \int_0^c \left(P_l \frac{dy_l}{dx} - P_u \frac{dy_u}{dx}\right)dx}_{L_2'}$

50%

$\frac{c^2}{2} - \frac{c^2}{3} = \frac{c^2}{6}$

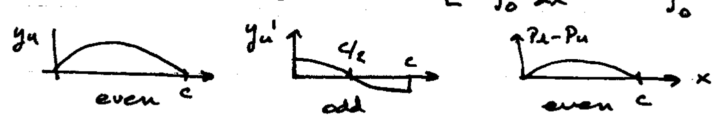
for $L_1' = \frac{16}{c} q_\infty \left(\frac{w}{c}\right)^2 \cos\alpha \int_0^c x(1-x/c)\,dx = \frac{16}{c} q_\infty \left(\frac{w}{c}\right)^2 \cos\alpha \left[\frac{x^2}{2} - \frac{x^3}{3c}\right]_0^c$

therefore $\boxed{L_1' = \frac{8}{3} q_\infty c \left(\frac{w}{c}\right)^2 \cos\alpha}$

for $L_2' \Rightarrow$ We first note that $\frac{dy_l}{dx} = 0$ (since $y_l = \text{const} = 0$), we are then left with the integral: $L_2' = -\sin\alpha \int_0^c P_u \frac{dy_u}{dx}\,dx$

and $P_u = P_l - (P_l - P_u)$ then $L_2' = -\sin\alpha \left[\overset{\text{const}}{P_l} \int_0^c \frac{dy_u}{dx}\,dx - \int_0^c (P_l - P_u)\frac{dy_u}{dx}\,dx\right]$

By inspection:



$y_u' = \frac{dy_u}{dx}$ is an odd function in $[0,c] \Rightarrow \int_0^c y_u'\,dx = 0$

$(P_l - P_u)\,y_u'$ is an even × odd = odd function in $[0,c] \Rightarrow \int_0^c (P_l - P_u)\,y_u'\,dx = 0$

$\Rightarrow L_2' = 0$

For our symmetrical functions we get: $\boxed{c_l' = \frac{8}{3}\left(\frac{w}{c}\right)^2 \cos\alpha}$

(2) cont'd...

for the c/4 moment: $c_{m_{c/4}} = \dfrac{M'_{c/4}}{q_\infty C^2}$

and $M'_{c/4} = \int_0^c \left[p_u\left(x - \frac{c}{4} + \frac{dy_u}{dx} y_u\right) - p_l\left(x - \frac{c}{4} + \frac{dy_l}{dx} y_l\right)\right] dx$

$$= \int_0^c -(p_l - p_u)(x - \tfrac{c}{4})\,dx + \underbrace{\int_0^c p_u \frac{dy_u}{dx} y_u\,dx}_{\int_0^c [p_l - (p_l - p_u)] \frac{dy_u}{dx} y_u\,dx} - \int_0^c p_l \cancel{\frac{dy_l}{dx}}^{\,0} y_l\,dx \qquad y_l = \text{const} = 0$$

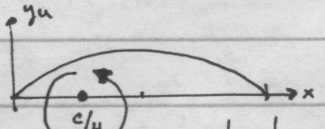
$$\underbrace{\int_0^c p_l \frac{dy_u}{dx} y_u\,dx}_{\text{const} \times \text{odd} \times \text{even} = \text{odd}} - \underbrace{\int_0^c (p_l - p_u) \frac{dy_u}{dx} y_u\,dx}_{\text{even} \times \text{odd} \times \text{even} = \text{odd}} = 0$$

therefore $\boxed{M'_{c/4} = \int_0^c -(p_l - p_u)(x - c/4)\,dx}$ only!

and $M'_{c/4} = \int_0^c -16 q_\infty \left(\frac{w}{c}\right)^2 \frac{x}{c}(1 - x/c)(x - c/4)\,dx = -\frac{16}{c} q_\infty \left(\frac{w}{c}\right)^2 \int_0^c \left[x^2 - \frac{xc}{4} - \frac{x^3}{c} + \frac{x^2}{c}\frac{c}{4}\right] dx$

$$= -\frac{16}{c} q_\infty \left(\frac{w}{c}\right)^2 \left[\frac{5}{12}x^3 - \frac{x^2 c}{8} - \frac{x^4}{4c}\right]_0^c = -\frac{2}{3} q_\infty c^2 \left(\frac{w}{c}\right)^2$$

So finally: $\boxed{c_{m_{c/4}} = -\frac{2}{3}\left(\frac{w}{c}\right)^2}$ independent of α, of course.



← which makes sense as the pressure dist. is symmetric around c/2

(3) the expression for $c'_{m_{c/4}}$ is identical.

30% the expression for c'_l is very similar:

$$\boxed{c'_l \simeq \frac{8}{3}\left(\frac{w}{c}\right)^2}$$

α	w/c	c'_l	c'_l $\alpha = y' = 0$	$c_{m_{c/4}}$
	0.01	0.0003	0.0003	−0.001
0	0.1	0.0267	0.0267	−0.0067
	0.5	0.667	0.667	−0.1667
	0.01	0.0003	↓	↓
5	0.1	0.0266		
	0.5	0.6641		
	0.01	0.0003	↓	↓
10	0.1	0.0263		
	0.5	0.6565		

The effect of α is very small.

In this symmetric configuration there is no change in the moment.

But remember that, in general, airfoils are not symmetric.