

Massachusetts Institute of Technology Department of Aeronautics and Astronautics
Cambridge, MA 02139
16.001/16.002 Unified Engineering I, II

Fall 2008

## Problem Set 3

Name: $\qquad$

Due Date: 9/26/2008

|  | Time Spent <br> (min) |
| :--- | :---: |
| T6 |  |
| F4 |  |
| M3 |  |
| Study <br> Time |  |

Announcements:

## Unified Engineering <br> Thermodynamics \& Propulsion

Fall 2008
(Add a short summary of the concepts you are using to solve the problem)

## Problem T6

An industrial plant uses a two-stage compressor. The working fluid is air. The first stage compressor takes in air at $p_{1}$ and $T_{1}$. It compresses the air to the pressure $p_{2}$. Between the two compressors, the air is cooled at constant pressure from $T_{2}$ to $T_{1}$. It is then compressed by the second compressor to the pressure $p_{3}$. Both compressors are ideal and can be assumed adiabatic. Neglect kinetic and potential energy effects. Air can be modeled as an ideal gas with $\gamma=1.4$ and $R=287 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$.
a) Make a sketch of the situation and label all stations.
b) Draw the overall process in $p-v$ coordinates and label all states. Mark the isotherms at each of the states.
c) Determine the shaft work per unit mass to run the two-stage compressor. Express your answers as a function of $p_{1}, p_{2}, p_{3}, T_{1}, \gamma$ and $c_{p}$.
d) Determine the pressure, $p_{2}$, which minimizes the two-stage compressor shaft work.

## Problem F4

Consider that you have available a measuring device, a velocimeter, that is capable of reading fluid velocity values along the $x$ direction only, but can be moved to any point in a $x$ - $y$ map. As you scan a two-dimensional (no dependence along the $z$-axis) fluid flow with this instrument, you discover that the velocity has a value $V_{0}$ at the origin, and increases by a factor $1+\left(x / x_{0}\right)^{2}$ when the detector is moved along $x$ (for any $y$ ), while it decreases by a factor $\exp \left(-y / y_{0}\right)$ when the detector is moved along $y$ (for any $x$ ).


1. From these measurements, write down the complete expression for the velocity vector for an incompressible fluid in steady state, in the non-dimensional form:

$$
\frac{\vec{V}}{V_{0}}=f_{x} \hat{i}+f_{y} \hat{j}
$$

2. Assuming $x_{0}=y_{0}$, draw velocity vector arrows in the 2 D range delimited by $\left[0, x_{0}\right]$ and $\left[0, y_{0}\right]$, the length of the arrow should be proportional to the speed. You can do this by hand, although graphics software is recommended. For example, you could use the function quiver in Matlab.
3. Evaluate the mass flow integral $\oint \rho(\vec{V} \cdot \hat{n}) d A$ for the control volume delimited by $\left[0, x_{0}\right]$ and $\left[0, y_{0}\right]$. Is this consistent with part 1 ?

## M3 (M4.1) (10 M-points) (Use U-B and M1.2 notes, CDL 1.6 with 1.4,

## 1.5 in review)

Consider a system of nine masses located in the $x_{1}-x_{2}$ plane. Each of the masses is located at the intersection of a square array of interconnecting rods. Each rod is considered to be rigid and massless, and each measures 2 feet in length. The mass at the system origin (center) is of 1.0 slugs. At each of the four corners of the system, there is a mass of 0.2 slugs. The other four masses are of 0.1 slugs. One force of 20 pounds acts in the negative direction parallel to the $\mathrm{x}_{2}$-direction on the mass at the upper right-hand corner, $\left(+\mathrm{x}_{1},+\mathrm{x}_{2}\right)$, of the system. A second force of 15 pounds acts in the positive direction parallel to the $x_{1}$-direction on the mass at the upper left-hand corner, $\left(-x_{1},+x_{2}\right)$, of the system.
(a) Neatly draw this configuration.
(b) This system is not in equilibrium, describe its initial motion.

For the following cases, carefully give your reasoning and express any forces and moments as vectors, as appropriate.
(c) Can equilibrium be achieved via the application of a force on the mass at the origin? If so, what is the force?
(d) Can equilibrium be achieved via the application of a moment on the mass at the origin? If so, what is the moment?
(e) Can equilibrium be achieved via the application of a force and moment at the origin? If so, what are the force and moment?
(f) Can equilibrium be achieved via the application of a couple anywhere (including along the rods)? If so, what is the couple and where must it be applied?
(g) Can equilibrium be achieved via the application of a force anywhere (including along the rods)? If so, what is the force and where must it be applied?

