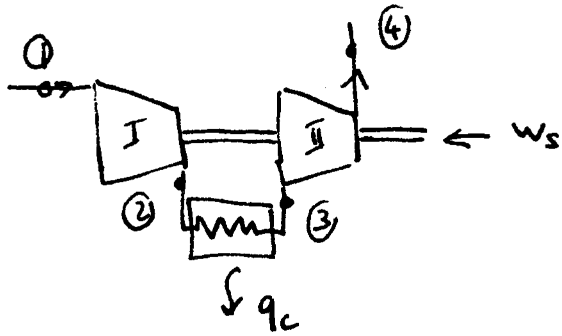
T6

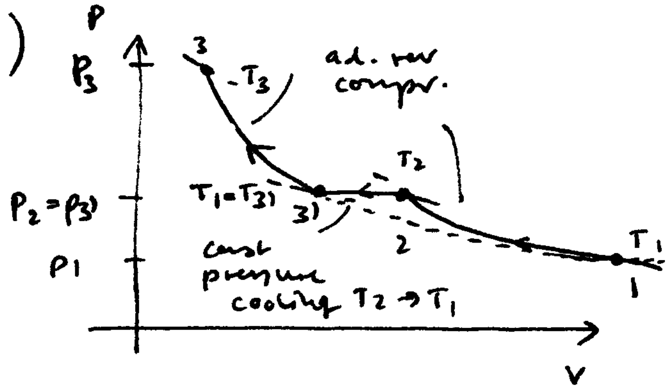
16. Unified Fall 08 Z5

a) 

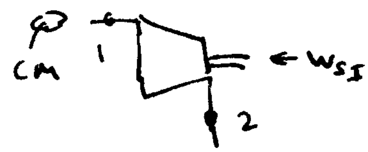
concepts:

- 1st law
- shaft and flow work
- adiabatic rev. process
- pv-diagrams

Assume: adiab rev. compressors, ideal gas, steady flow, no ΔKE, ΔPE

b) 

c) compressor I:



$$u_2 - u_1 = -(-p_1 v_1 - w_{sI} + p_2 v_2)$$

$$h_2 - h_1 = w_{sI} = c_p (T_2 - T_1)$$

adiab. rev: $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \to w_{sI} = c_p T_1 \left(\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$

compressor II: similarly $\to w_{sII} = c_p T_{3'} \left(\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right) = c_p T_1 \left(\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$

so $w_s = w_{sI} + w_{sII} \quad \to \quad \underline{w_s = c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 2 \right]}$

d) find minimum: $\frac{dw_s}{dp_2} = 0$

$$0 = c_p T_1 \left[\frac{\gamma-1}{\gamma} \left(\frac{p_2}{p_1}\right)^{\frac{-1}{\gamma}} \frac{1}{p_1} + \frac{\gamma-1}{\gamma} \left(\frac{p_3}{p_2}\right)^{\frac{-1}{\gamma}} \cdot \left(\frac{-p_3}{p_2^2}\right) \right]$$

$$0 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \cdot \frac{1}{p_2} - \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \cdot \frac{1}{p_2} \quad \to \quad \frac{p_2}{p_1} = \frac{p_3}{p_2}$$

find for minimum shaft work $\quad \underline{p_{2_{\min w_s}} = \sqrt{p_1 p_3}}$

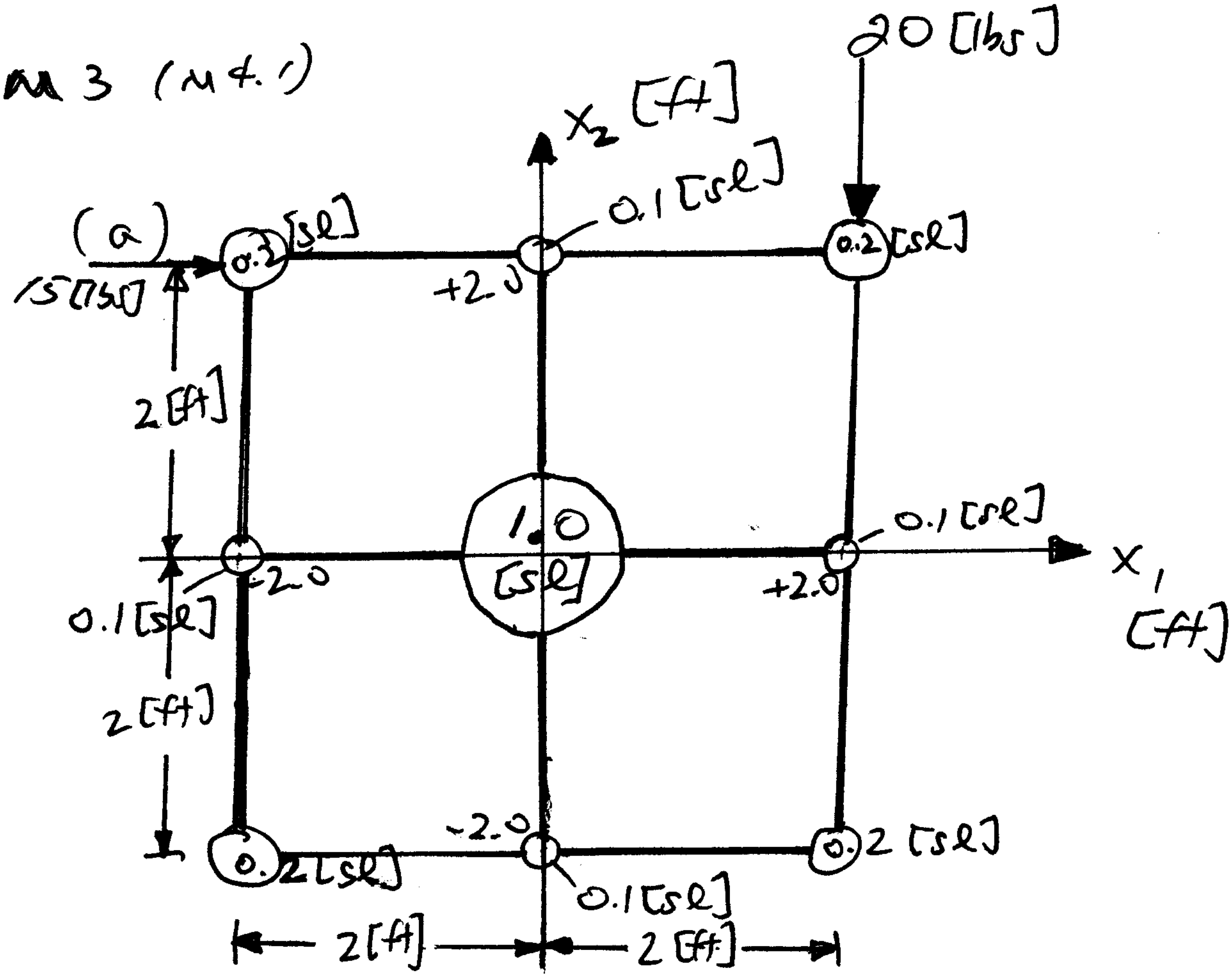
PAL
9/20/08

Unified Engineering

Homework SOLUTIONS

Week #4; Problem Set #3

M3 (M4.1)



[sl] = [slugs]

—— = rigid, massless rods

(b) This system is not in equilibrium. One can see that both its net force and net moment are non-zero.

Call the 20 lbs force $\underline{F}_1$ and the 15 lb force $\underline{F}_2$. Express each as vectors:

$$\underline{F}_1 = -20[lb]\ \hat{i}_2 \quad \text{at } (2ft, 2ft)$$

$$\underline{F}_2 = +15[lb]\ \hat{i}_1 \quad \text{at } (-2ft, 2ft)$$

Thus the net force, $\underline{F}_{net}$, is:

$$\underline{F}_{net} = \underline{F}_1 + \underline{F}_2 = +15[lb]\ \hat{i}_1 + (-20)[lb]\ \hat{i}_2$$

The acceleration of the rigidly connected system is:

$$\underline{a} = \frac{\underline{F}_{net}}{m}$$

where m is the total mass $= (4 \times 0.1[sl]) + (4 \times 0.2[sl]) + 1.0[sl] = 2.2\ [slugs]$

So:

$$\underline{a} = \frac{15\ lb}{2.2\ slugs}\ \hat{i}_1 - \frac{20\ lb}{2.2\ slugs}\ \hat{i}_2$$

Note the relationship of force in [lbs] to mass in [slugs]

Thus: $\underline{a} = 6.82\,[ft/s^2]\,\hat{i}_1 - 9.09\,[ft/s^2]\cdot\hat{i}_2$

So the system has a linear acceleration of 6.82 [ft/s²] in the $+x_1$-direction and 9.09 [ft/s²] in the $-x_2$-direction

There will also be a clockwise rotation about the origin

This is due to the net moment:

$$\underline{M}_{net} = \sum(\underline{r}_i \times \underline{F}_i)$$

where $\underline{r}_i$ = position vector to force vector from origin

One can find:

$$\underline{M}_{net} = (2ft)\,\hat{i}_1 \times (-20\,lbs)\,\hat{i}_2 + (2ft)\,\hat{i}_2 \times (+15\,lbs)\,\hat{i}_1$$

Also noting: $\hat{i}_1 \times \hat{i}_2 = +\hat{i}_3$
$\hat{i}_2 \times \hat{i}_1 = -\hat{i}_3$

$$\Rightarrow \underline{M}_{net} = (-40\ [ft \cdot lbs])(+\hat{i}_3) + (30\ [ft \cdot lbs])(-\hat{i}_3)$$
$$= -70\ [ft \cdot lbs]\ \hat{i}_3$$

$-$ $\Rightarrow$ clockwise

(c) By applying other forces and moments, one can achieve equilibrium if the net force and moment are zero.

Thus, by applying a force, one can get the net force to be zero. The applied force must be equal in magnitude and opposite in direction the the current net force (as it exists)

$$\Rightarrow \text{apply } \underline{F}_{applied} = (-15\ [lb])\ \hat{i}_1 + 20\ [lb]\ \hat{i}_2$$

and the linear acceleration will be zero.

BUT, by applying a force at the origin one cannot cause a moment about the origin as the position vector is: $\underline{r} = \underline{0}$. Thus the net moment cannot become zero.

$\Rightarrow \boxed{NO}$ equilibrium cannot be achieved

(d) The opposite occurs in this case. By applying a moment about the origin one can cause the net moment to be zero (by applying a moment that is equal in magnitude and opposite in direction)

$$\Rightarrow \underline{M}_{applied} = +70 \,[ft \cdot lb]\, \hat{\imath}_3$$

However, one cannot get the net force to be zero and thus there will be linear acceleration and motion

$\Rightarrow \boxed{NO}$ equilibrium cannot be achieved

(e) Go back to the reasonings of parts (c) and (d). Add these two (superposition applies), and equilibrium can be achieved.

$\Rightarrow \boxed{4 FS_1}$

Apply:

$$\boxed{\begin{aligned} \underline{F}_{applied} &= (-15\,[lbs])\,\hat{i}_1 + (20\,[lbs])\,\hat{i}_2 \\ \underline{M}_{applied} &= +70\,[ft \cdot lbs]\,\hat{i}_3 \end{aligned}}$$

(f) The same reasoning applies as in the answer to part (d). A couple results in a net moment but no net force. So a couple can cause the rotation not to occur, but the total net force will still be non-zero and thus there will be linear acceleration and motion.

$\Rightarrow \boxed{NO}$ equilibrium cannot be achieved

(g) A force can achieve a desired moment by proper placement. Thus using a similar reasoning as in part (e), a force of the proper magnitude and direction can be properly placed to give the needed opposite moment about the origin.

→ The force must be as in part (c) to achieve a net force of zero:

$$\underline{F}_{applied} = (-15\,[lbs])\,\hat{i}_1 + (20\,[lbs])\,\hat{i}_2$$

→ One must have a net moment, as per part (d), of:

$$\underline{M}_{applied} = +70\,[ft \cdot lbs]\,\hat{i}_3$$

This is achieved via:

$$\underline{M}_{applied} = \underline{r}_{applied} \times \underline{F}_{applied}$$

where $\underline{r}_{applied}$ is the position vector to the point (x_1, x_2) where $\underline{F}_{applied}$ is applied. Write this as:

$$\underline{r}_{applied} = x_1\,\hat{i}_1 + x_2\,\hat{i}_2$$

So this gives via the moment expression:

$$+70\,[ft \cdot lbs]\,\hat{i}_3 = (x_1\,\hat{i}_1 + x_2\,\hat{i}_2) \times \{(-15\,[lbs])\,\hat{i}_1 + (20\,[lbs])\,\hat{i}_2\}$$

$$\Rightarrow (-15x_2\,[\text{lbs}])(-\hat{i}_3) + (20x_1\,[\text{lbs}])(\hat{i}_3) = +70\,[\text{ft}\cdot\text{lbs}]\,\hat{i}_3$$

$$\Rightarrow \boxed{\frac{3}{14}x_2 + \frac{2}{7}x_1 = 1\,[\text{ft}]}$$

so this defines a line along which

$$\underline{F} = (-15\,[\text{lbs}])\,\hat{i}_1 + (20\,[\text{lbs}])\,\hat{i}_2$$

can be placed to get equilibrium

So: $\boxed{\text{YES}}$ equilibrium can be achieved

(NOTE: There are 4 points of interaction with the world of the system)

Solutions F4

1. From the continuity equation: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\bar{v}) = 0$

we have $\boxed{\nabla\cdot\bar{v} = 0}$ for a steady, incompressible flow.

The velocity vector is $\bar{v} = u\hat{i} + v\hat{j}$

So the condition $\nabla\cdot\bar{v} = 0$ reduces to $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$

If I know $u = u(x)$ $\Rightarrow$ $\boxed{v = -\int \frac{\partial u}{\partial x}\, dy}$

u comes from our measurements: $u = V_0\left(1+\left(\frac{x}{x_0}\right)^2\right)e^{-y/y_0}$

then: $\frac{\partial u}{\partial x} = \frac{2V_0}{x_0^2}\, x\, e^{-y/y_0}$

Integrate in y $\Rightarrow$ $v = -\int \frac{2V_0 x}{x_0^2} e^{-y/y_0}\, dy = +\frac{2V_0 x y_0}{x_0^2} e^{-y/y_0} + \underbrace{f(x)}_{\text{function of } x} + \underbrace{A}_{\text{constant}}$

Evaluate at $y=0$ $\Rightarrow$ $v_{y=0}(x) = \frac{2V_0 x y_0}{x_0^2} + f(x) + A$

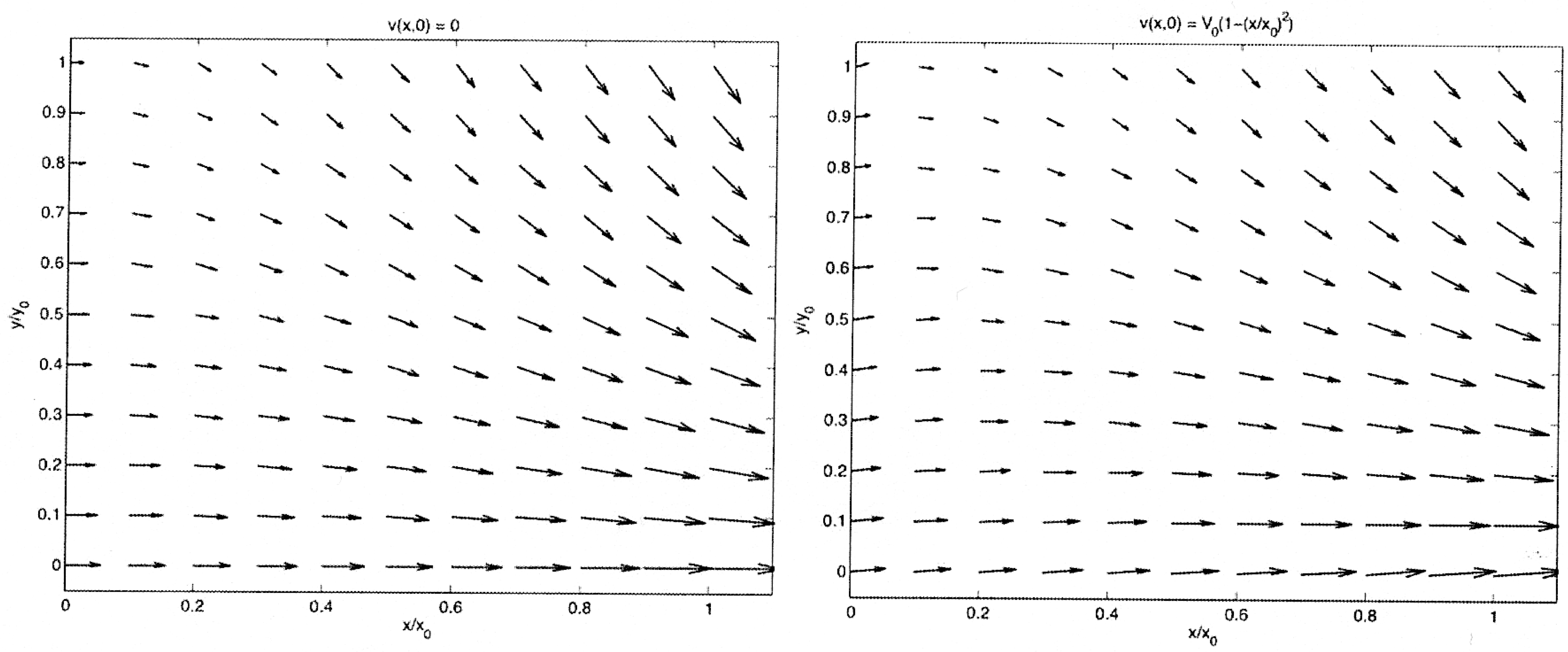
therefore $f(x) + A = v_{y=0}(x) - \frac{2V_0 x y_0}{x_0^2}$

the velocity vector is then:

$$\frac{\bar{v}}{V_0} = \underbrace{\left[1+\left(\frac{x}{x_0}\right)^2\right]e^{-y/y_0}}_{f_x}\,\hat{i} + \underbrace{\left\{\frac{2xy_0}{x_0^2}\left(e^{-y/y_0}-1\right) + \frac{v_{y=0}(x)}{V_0}\right\}}_{f_y}\hat{j}$$

We have a free parameter, the function $v_{y=0}(x)$, and we don't have information about it. To perform the plots, we need to make an assumption. We will do 2 limit cases: $v_{y=0}(x) = V_0\left(1+\left(\frac{x}{x_0}\right)^2\right)$ ← same as u

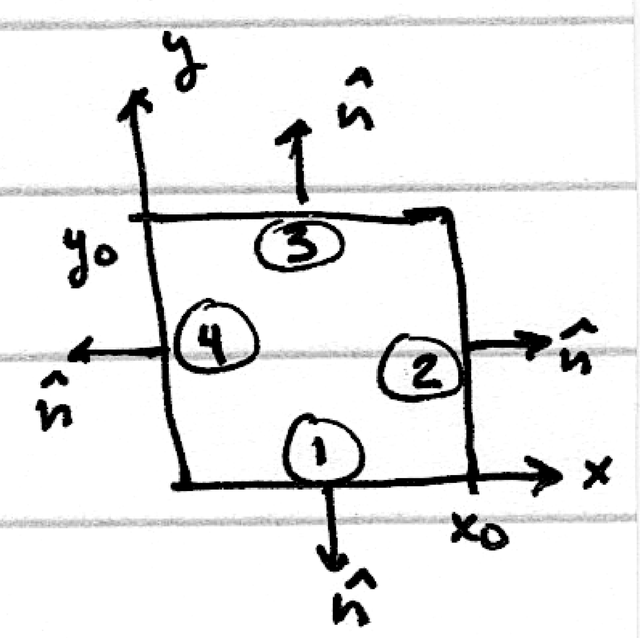
and $v_{y=0}(x) = 0$



The general shape of the field is very similar in both cases

3. Evaluate the flow integral: $\int \rho \bar{v}\cdot\hat{n}\,\frac{dA}{L}$ over the 4 faces

p. unit depth



const → $\rho V_0 \int \bigcirc \, dA$

Face 1: $\hat{n} = -\hat{j}$ then $-\int_0^{x_0} \frac{v_{y=0}(x)}{V_0}\,dx$
(y=0)

Face 2: $\hat{n} = \hat{i}$ then $\int_0^{y_0} 2e^{-y/y_0}\,dy = -2y_0(e^{-1}-1)$
(x=x_0)

Face 3: $\hat{n} = \hat{j}$ then $\int_0^{x_0}\left[\frac{2xy_0(e^{-1}-1)}{x_0^2} + \frac{v_{y=0}(x)}{V_0}\right]dx = y_0(e^{-1}-1) + \int_0^{x_0}\frac{v_{y=0}(x)}{V_0}\,dx$
(y=y_0)

Face 4: $\hat{n} = -\hat{i}$ then $-\int_0^{y_0} e^{-y/y_0}\,dy = y_0(e^{-1}-1)$
(x=0)

Adding all together: $-\int_0^{x_0}\frac{v_{y=0}(x)}{V_0}\,dx + \int_0^{x_0}\frac{v_{y=0}(x)}{V_0}\,dx - 2y_0(e^{-1}-1) + 2y_0(e^{-1}-1) = 0$

as it should be.

$\boxed{\nabla\cdot\bar{V} = 0 \Leftrightarrow \int \bar{V}\cdot\hat{n}\,dA}$

it is consistent.