

# Massachusetts Institute of Technology Department of Aeronautics and Astronautics <br> Cambridge, MA 02139 

16.001/16.002 Unified Engineering I, II Fall 2008

## Problem Set 4

Name: $\qquad$

Due Date: 10/3/2008

|  | Time Spent <br> (min) |
| :--- | :---: |
| F5 |  |
| M4 |  |
| M5 |  |
| T7 |  |
| Study <br> Time |  |

Announcements:

Unified Engineering
Fluids

## Problem F5



Suppose you have a cylindrical water jet ( $\rho_{w}=1 \mathrm{~g} / \mathrm{cm}^{3}$ ) of diameter $D_{j}=3 \mathrm{~cm}$ that "levitates" a copper block ( $\rho_{C u}=8.96 \mathrm{~g} / \mathrm{cm}^{3}$ ) of mass $M=120 \mathrm{~kg}$ and dimensions $h=20$ cm and $s=30 \mathrm{~cm}$ (the block is square). The jet is divided in two identical paths and deflected back as shown in the figure. Each turning pipe changes its cross sectional shape until becoming a square of area $A=1 \mathrm{~cm}^{2}$ at the exit. Neglect viscosity and friction with the stabilizing walls. Use your knowledge on mass and momentum conservation to answer the following:
(a) Make a sketch of the system and select an appropriate control volume.
(b) Find an expression for the cylindrical jet velocity $v_{j}$ and calculate its value.
(c) What is the exit velocity $v_{e}$ from each turning pipe?
(d) What is the mass flow rate $\dot{m}$ ?
(e) What is the valid range of exit areas $(A)$ in your model?

M4 (M5.1) (15 M-points) In this problem, we consider overall configuration of the wing-body of an airplane and learn about the modeling of the lift on a wing in order to consider the loads acting on the structure. The overall weight can be considered to act at the center of the body (sometimes known as the fuselage). The half-span of the wing is the distance from the root (where the wing connects to the body) to the tip. (See the simple diagram of the geometry below.)


For the purposes of structural analysis, we can model an airplane wing in two dimensions as a linear structural member of length s/2 (the half-span) emanating from the root as shown above. Consider the body to be a point for this initial structural consideration (not too comfortable for any passengers or useful for freight transport!).

The lift (pressure differential between the top and bottom surfaces) on the wing can be represented as a lineload and thus has dimensions of [force/length]. We can consider different possible models for the lift distribution that may be connected to aerodynamic analysis, different spans for the wings, and the resulting overall load produced by these various configurations. We will consider two specific lift distributions: (1) linear variation along the span of the wing with its maximum value at the root and with a value at the tip of half that at the root; and (2) quadratic variation along the span of the wing such that Lift $=\mathrm{b}-\mathrm{ax}^{2}$ with its maximum value at the root and with a value of zero at the tip. For each case, assume that in steady flight, each wing must provide sufficient lift to support half the total weight.

We work to see how the model for lift, as one might get from
fluid / aerodynamic analysis, and the span used for the wing length changes the following results. For each of the lift models:
(a) Draw this configuration showing all loads.
(b) Determine the maximum magnitude of the lineload of lift, where it occurs, and its dependence on the span length.
(c) For one wing, determine the equipollent force system at the root for the lift. In this latter case, plot how these will vary with wingspan. For reference, the new Boeing 787 has approximate values for the gross takeoff weight of up to 540,000 pounds and a wing half-span of about 100 feet.

M5 (M5.2) (15 M-points) (SOME LOOK-AHEAD: Use M1.3 notes, CDL 1.7, 1.8)
A 50-meter long bridge is modeled as a 50-meter long beam pinned at one end and attached via a roller at the other end. This support configuration is known as simply-supported. In addition to this basic configuration, the beam has a cable attached to it at the midpoint which transfers load through an overhead system. This cable configuration can be modeled as a pulley, with a radius of $r$, attached by a pin to an overhead support directly above the roller support. The cable makes a $45^{\circ}$ angle with the beam where it is connected. At the other end of the cable is a suspended mass, $\mathrm{m}_{\mathrm{p}}$. The overall structural configuration must be able to carry two basic loads -- the weight of the beam and the weight of a vehicle of mass, $\mathrm{m}_{\mathrm{v}}$, that can move anywhere along this beam, although it is stationary at any point in time. The total mass of the structural beam, $\mathrm{m}_{\mathrm{b}}$, can be represented as producing a lineload that varies linearly from the roller support to the pin, being twice in value at the pin end to the roller end.
(a) Draw the free body diagram for this situation (choose any location for the vehicle).
(b) If possible, determine the reaction forces as a function of the point on the beam at which the vehicle is located.
(c) With the vehicle at any location, determine if having the cable support at the mid-point reduces or increases the overall load carried by the configuration as opposed to the cable not being present. Clearly explain your reasoning.
(d) The end with the pin is now clamped. Draw the free body diagram for this case and then determine the reaction forces, or if the reaction forces cannot be exactly determined, clearly explain the reasons for that.


## Unified Engineering <br> Thermodynamics \& Propulsion

Fall 2008
(Add a short summary of the concepts you are using to solve the problem)

## Problem T7

A simple gas turbine power plant operating at steady state is illustrated schematically in the figure below. The power plant consists of an ideal air compressor mounted on the same shaft as the ideal turbine. Relevant data are given in the figure. Kinetic and potential energy effects are negligible. Using the ideal gas model with $\gamma=1.4$ and $\mathrm{R}=$ $287 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$, determine:
a) the specific shaft work input to the compressor, in J per kg of air flowing,
b) the temperature at the turbine inlet, in K ,
c) the mass flow, in kg ,
d) the heat transfer rate, $\dot{Q}_{\text {in }}$, in $W$.


