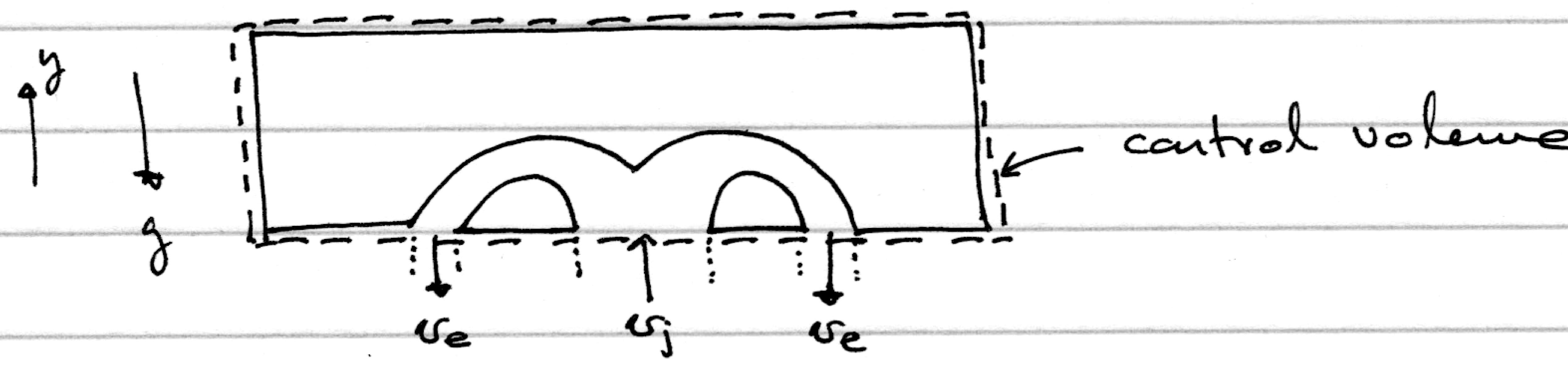
Solutions

Problem F5

(20%) (a)



We select this volume since we know the pressure distribution on it $\Rightarrow$ $\boxed{P = P_{atm}}$. Note: there is a small hydrostatic difference, but we can neglect that.

(40%) (b) Momentum eqn. (along y) $\Rightarrow$ $\int \rho v\, \bar{V}\cdot\hat{n}\, dA = -\frac{\partial}{\partial t}\int \rho v\, dV - \int p\, dA\, \hat{n}\cdot\hat{j} + \int \rho g_y\, dV + F_{4,y}$

In steady state $\frac{\partial}{\partial t} \to 0$ and on the control volume $F_{4,y} = 0$

The pressure integral is also $= 0$ (P_{atm} everywhere)

Then: $\underset{(+)}{\rho_w}\, v_j \underset{(-\hat{j}\cdot\hat{n})}{(-v_j)} \frac{\pi D_j^2}{4} + 2\rho_w \underset{(-)}{(-v_e)} \underset{(-\hat{j})\cdot(-\hat{n})}{(v_e)} A = -g(\rho_w V_w + \rho_{cu} V_{cu})$ ⟸ Remember your sign conventions

the volume of Cu is $V_{cu} = \frac{M}{\rho_{cu}}$ so $V_w = hs^2 - \frac{M}{\rho_{cu}}$

Also, from continuity: $\int \rho \bar{V}\cdot\hat{n}\, dA = -\frac{\partial}{\partial t}\int \rho\, dV = 0$ ← in steady state.

then $\dot{m}_j = 2\dot{m}_e$ or $v_j \frac{\pi D_j^2}{4} = 2 v_e A \Rightarrow v_e = \frac{\pi D_j^2}{8A} v_j$

we solve for $$\boxed{v_j = \sqrt{\frac{g\left[hs^2 + M\left(\frac{1}{\rho_w} - \frac{1}{\rho_{cu}}\right)\right]}{\frac{\pi D_j^2}{4}\left(1 + \frac{\pi D_j^2}{8A}\right)}}}$$

with our values: $\boxed{v_j = 19.5\ \frac{m}{s}}$

(30%) (c) $\boxed{v_e = \frac{\pi D_j^2}{8A} v_j = 68.9\ m/s}$ (d) $\boxed{\dot{m} = \rho_w v_j \frac{\pi D_j^2}{4} = 13.8\ \frac{kg}{s}}$

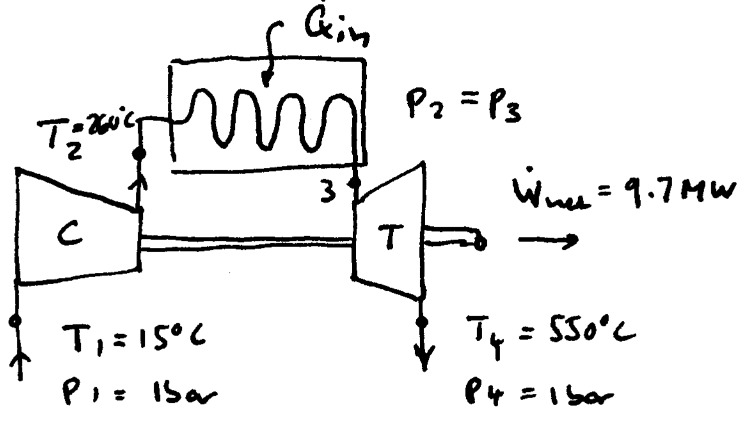
(10%) (e) What limits the A? (Qualitative only)

→ If A is too small, viscous forces dominate at the exit

→ If A is too big, flow separation will occur. (unsteady)

T7

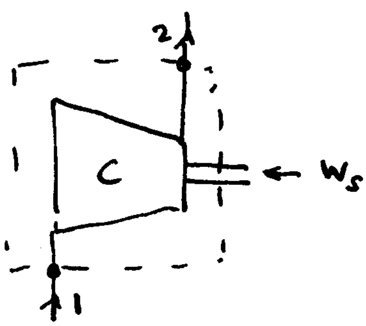
16. Unified Fall 08 Z5



Concepts: - 1st law CV
- ad. rev. processes

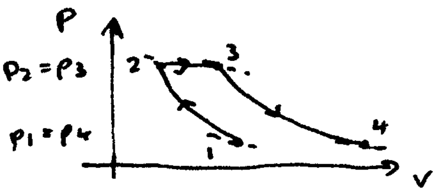
Assume: - ideal gas
- steady operation
- adiab. turbine, compr
- neglect ΔKE, ΔPE

a)



$$\underbrace{\frac{dE_{cv}}{dt}}_{0} = \underbrace{\sum \dot{Q}}_{0} + \sum \dot{W}_s + \underbrace{\sum \dot{W}_p}_{0} + \sum \dot{m}\Big(h + \underbrace{\tfrac{c^2}{2}}_{0} + \underbrace{gz}_{0}\Big)$$

have $0 = \dot{m} w_s + \dot{m}(h_1 - h_2)$



$w_s = h_2 - h_1$; $dh = c_p\,dT$ so $w_s = c_p(T_2 - T_1)$

$c_p = \frac{\gamma}{\gamma-1} R = 1004.5\ J/kg\,K$ $\qquad \underline{w_s = 246.1\ kJ/kg}$

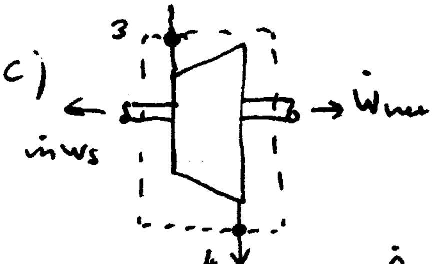
b) compressor pressure ratio = turbine pressure ratio

$(p_1 = p_4$ and $p_2 = p_3)$ so $\frac{p_2}{p_1} = \frac{p_3}{p_4} \xrightarrow[\text{rev. proc.}]{\text{ad.}}$ C and T have same temp. ratio

$$T_3 = T_4 \cdot \left(\frac{T_2}{T_1}\right) \leftarrow T_3 = T_4 \cdot \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = T_4 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_4 \cdot \left(\frac{T_2}{T_1}\right)$$

$\underline{T_3 = 1523\,K}$

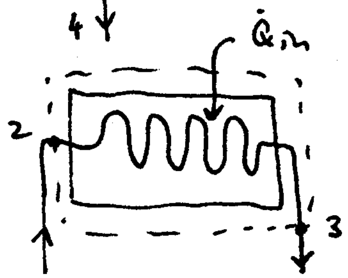
c)



1st law: $0 = -\dot{m} w_s - \dot{W}_{net} + \dot{m}(h_3 - h_4)$

$\dot{m} = \dot{W}_{net} / (c_p(T_3 - T_4) - w_s)$; $\underline{\dot{m} = 21.2\ kg/s}$

d)



1st law: $0 = \dot{Q}_{in} + \dot{m}(h_2 - h_3)$

$\dot{Q}_{in} = \dot{m} c_p (T_3 - T_2)$ $\qquad \underline{\dot{Q}_{in} = 21.1\ MW}$

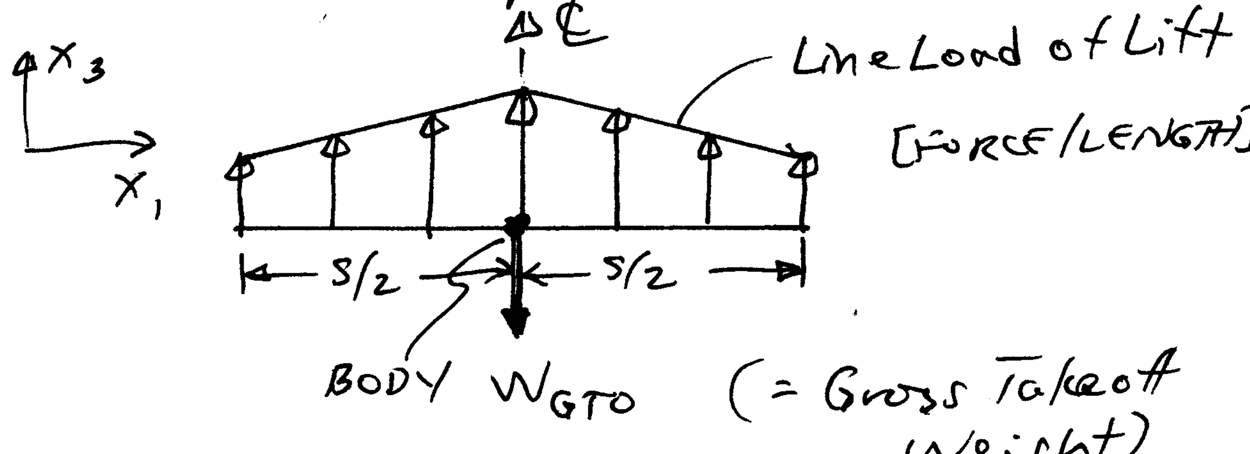
PAL
9/25/08

Unified Engineering
Problem Set #4 SOLUTIONS
Week #5 Fall, 2008

M4 (M5.1)

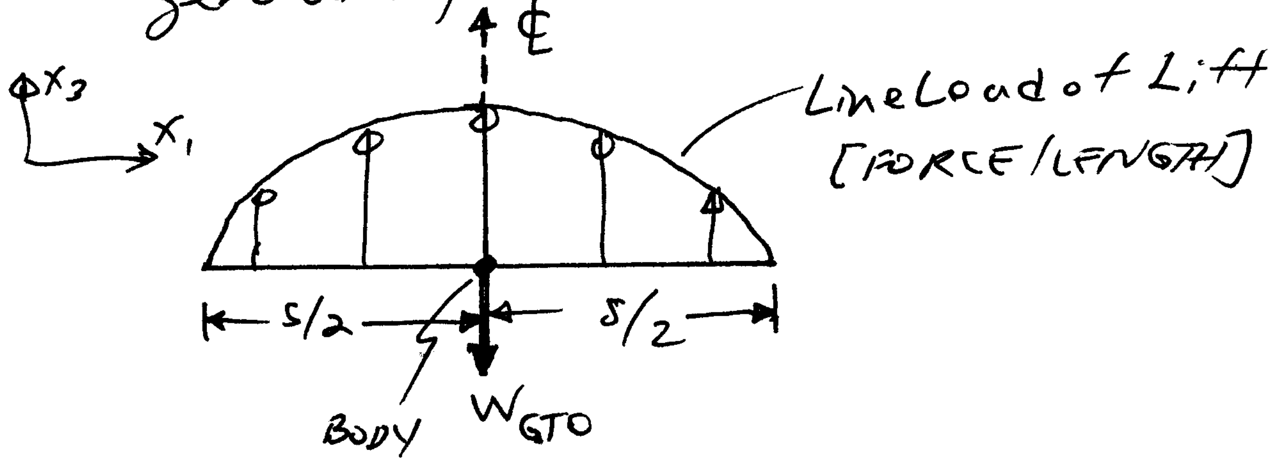
(a) The configuration is the same basic one for the two lift models except that the loading changes. So:

<u>Model ①</u>: Linear along span to half value at tip



s = span

Model ② : Quadratic along span to zero at tip



(b) For this it is necessary to consider each lift model and determine values to get an expression for the variation including the variation with respect to the span lengths.

→ Place the origin of the x_1-x_3 coordinate system at the root ($x_1 = 0$). Here are the steps:

- Step 1 : Each wing must carry half the weight of the airplane through the counteracting action of the force of lift for the plane to be in level flight (i.e. steady flight).

Generally can express lift varying along the wing as:

$$Lift = f(x_1)$$

for one wing in steady flight:

$$\frac{W_{GTO}}{2} = \int_{root}^{tip} f(x_1)\,dx_1$$

at root, $x_1 = 0$
at tip, $x_1 = S/2$

• <u>Step 2</u>: Get an expression for the lift as a function of x_1. This changes for each case:

① The lift is maximum at the root (call this L_R) and goes to half the value at the tip (Lift $= L_R/2$ @ $x_1 = S/2$) with a linear variation. The general expression for a linear variation is:

$$f_{1}(x_1) = m x_1 + b$$

First find the slope, m. This is the ratio of the change in lift to the change in direction:

$$m = \frac{(L_R - L_R/2)}{(0 - S/2)} \Rightarrow m = -\frac{L_R}{S}$$

To find the constant b, use the value of lift at the root: $f(x_1 = 0) = L_R$

$$\Rightarrow L_R = \left(-\frac{L_R}{s}\right)(0) + b$$

$$\Rightarrow b = L_R$$

Overall this gives:

$$\boxed{f_{1}(x_1) = -\frac{L_R}{s} x_1 + L_R}$$

(NOTE: This is only valid for $x_1 > 0$. For $x_1 < 0$, the sign of the slope must be changed)

(2) For the quadratic case, begin with the general quadratic equation:

$$\text{Lift: } f_{2}(x_1) = b - ax^2$$

start by finding the constant, b, by noting the value at the root $(x_1 = 0)$, again calling this L_R:

$$\Rightarrow \quad b = L_R$$

Also note that this is the maximum. Then use the other point of known value. This is at the tip $(x_1 = 0)$ where the lift goes to zero. Use this to determine the value for a:

$$f_{②}(x_1 = s/2) = 0 = L_R - a(s/2)^2$$

$$\Rightarrow \quad a = \frac{4L_R}{s^2}$$

Putting this together:

$$\boxed{\begin{aligned} f_{②}(x_1) &= L_R\left(1 - \frac{4}{s^2}x_1^2\right) \\ &= L_R\left\{1 - 4\left(\frac{x_1}{s}\right)^2\right\} \end{aligned}}$$

→ Now proceed to .....

• Step 3: Solve using the known values and the equations.

Model ①: $\dfrac{W_{GTO}}{2} = \displaystyle\int_0^{s/2} L_R\left(1 - \frac{x_1}{s}\right)dx_1$

$$\Rightarrow \frac{W_{GTO}}{2} = L_R\left(x_1 - \frac{x_1^2}{2s}\right)\Big]_0^{s/2}$$

$$= L_R\left(s/2 - \frac{s}{8}\right)$$

$$\Rightarrow \quad W_{GTO} = 2L_R\left(\frac{3s}{8}\right)$$

Finally: $\boxed{L_R = \dfrac{4\,W_{GTO}}{3\,s}}$ at the wing root $(x_1 = 0)$

This is the maximum value.

NOTE: This has units of [force/length]. This is as indicated in the description and thus checks

Model ②: $$\frac{W_{GTO}}{2} = \int_0^{S/2} L_R\left\{1 - 4\left(\frac{x_1}{S}\right)^2\right\} dx_1$$

$$\Rightarrow \frac{W_{GTO}}{2} = L_R\left(x_1 - \frac{4x_1^3}{3S^2}\right)\Bigg]_0^{S/2}$$

$$= L_R\left(S/2 - S/6\right)$$

$$= L_R\left(3S/6 - S/6\right)$$

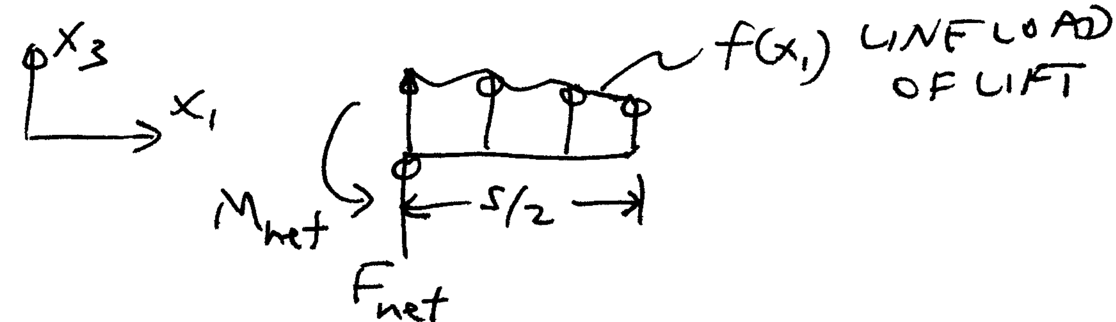
$$\Rightarrow W_{GTO} = 2L_R\left(S/3\right)$$

Finally: $\boxed{L_R = \frac{3W_{GTO}}{2S}}$ at the wing root $(x_1 = 0)$

Again, this is the maximum value and has the proper units of intensity as before: [force/length] $\Rightarrow$ it checks.

(c) An equipollent force system is the net sum of forces and moments on the system. Considering this at the root gives an idea of the loading the wing must take at the attachment to the body based on the different models of lift.

This can be drawn as "cutting off" the wing and looking at the net force and net moment at the root:



To determine the net force and net moment, $f(x_1)$ needs to be integrated:

$$\text{Net force: } F_{net} = \int_0^{s/2} f(x_1)\,dx_1$$

$$\text{Net moment: } M_{net} = \int_0^{s/2} f(x_1)\,\underbrace{x_1}_{\text{moment arm}}\,dx_1$$

with a positive moment acting counterclockwise (+ccw)

Do this for each of the two models:

Model (1): $F_{net_{(1)}} = \int_0^{s/2} \frac{4W_{GTO}}{3s}\left(1-\frac{x_1}{s}\right)dx_1$

$$= \frac{4W_{GTO}}{3s}\left(x_1 - \frac{x_1^2}{2s}\right)\Big]_0^{s/2}$$

$$= \frac{4W_{GTO}}{3s}\left(s/2 - s/8\right)$$

$$= \frac{4W_{GTO}}{3s}\left(\frac{3s}{8}\right)$$

Finally:

$+\uparrow \Rightarrow \boxed{F_{net_{(1)}} = \frac{W_{GTO}}{2}}$

NOTE: This is consistent as the statement is that each wing must carry half the overall weight in steady flight.

Now: $M_{net_{(1)}} = \int_0^{s/2} \frac{4W_{GTO}}{3s}\left(1-\frac{x_1}{s}\right)x_1\, dx_1$

$$= \frac{4W_{GTO}}{3s}\left(\frac{x_1^2}{2} - \frac{x_1^3}{3s}\right)\Big]_0^{s/2}$$

continuing....

$$M_{net_{①}} = \frac{4W_{GTO}}{3s}\left(\frac{s^2}{8} - \frac{s^2}{24}\right)$$

$$= \frac{4W_{GTO}}{3s}\left(\frac{3s^2}{24} - \frac{s^2}{24}\right)$$

$$= \frac{4W_{GTO}}{3s}\left(\frac{s^2}{12}\right)$$

Finally:

$$\circlearrowleft + \Rightarrow \boxed{M_{net_{①}} = \frac{W_{GTO}\, s}{9}}$$

NOTE: This has units of [force · length] which is correct for a moment.

Model ②:

$$F_{net_{②}} = \int_0^{s/2} \frac{3W_{GTO}}{2s}\left\{1 - 4\left(\frac{x_1}{s}\right)^2\right\} dx_1$$

$$= \frac{3W_{GTO}}{2s}\left(x_1 - \frac{4x_1^3}{3s^2}\right)\Big]_0^{s/2}$$

$$= \frac{3W_{GTO}}{2s}\left(s/2 - s/6\right)$$

$$= \frac{3W_{GTO}}{2s}\left(s/3\right)$$

giving:

$$+\uparrow \Rightarrow \boxed{F_{net_{②}} = \frac{W_{GTO}}{2}}$$

Again, consistent with the condition that each wing carry half the overall weight

continuing:

$$M_{net_{②}} = \int_0^{s/2} \frac{3W_{GTO}}{2s}\left\{1 - 4\left(\frac{x_1}{s}\right)^2\right\} x_1\, dx_1$$

$$= \frac{3W_{GTO}}{2s}\left(\frac{x_1^2}{2} - \frac{x_1^4}{s^2}\right)\Bigg]_0^{s/2}$$

$$= \frac{3W_{GTO}}{2s}\left(\frac{s^2}{8} - \frac{s^2}{16}\right)$$

Finally:

$$\circlearrowleft + \Rightarrow \boxed{M_{net_{②}} = \frac{3W_{GTO}\, s}{32}}$$

Again, this has the proper units for moment of [force · length]

→ As to plotting how these results vary with wingspan:

- There is no need to plot F_{net} as this is the same $\left(\frac{W_{GTO}}{2}\right)$ irrespective of spanlength and lift model since in steady flight each wing must carry half the overall weight.

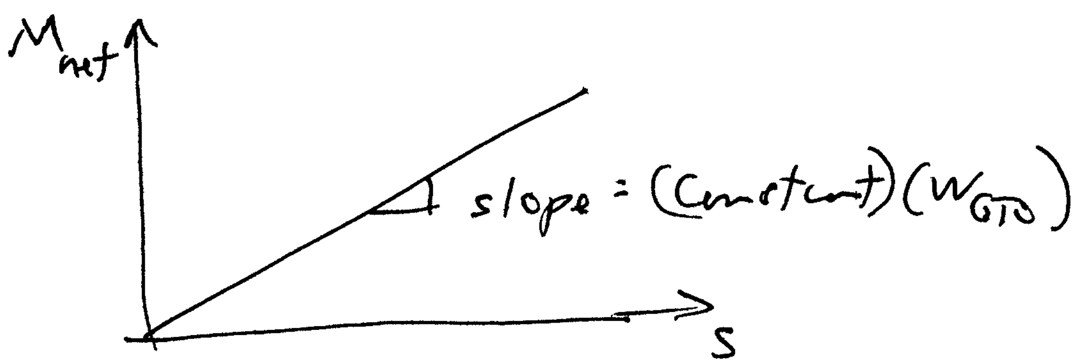
• The moment does change with lift model. Both are of the form:

$$M_{net} = (\text{Constant})(W_{GTO})(s)$$

where the constants are:

$$\text{Model } 1 = 1/9 = 0.111$$
$$\text{Model } 2 = 3/32 = 0.094$$

• As to variation with span length, the moment is linearly related. So:



• The Boeing 787 values are given just to allow checks/references:

$$F_{net} = 270{,}000 \text{ lbs.}$$

$$M_{net_1} = 12.0 \times 10^6 \text{ lbs} \cdot \text{ft}$$

$$M_{net_2} = 10.13 \times 10^6 \text{ lbs} \cdot \text{ft}$$

M5 (M5.2)

(a) In order to draw the free body diagram, it is necessary to convert the masses into loads (i.e. weight due to gravity):

$$\text{Force} = \text{mass} \times g$$

in negative x_3-direction

→ for vehicle: $\text{weight}_v = m_v g$

→ for mass hanging on pulley: $\text{weight}_p = m_p g$

→ An equation is needed for the distribution of the beam mass. Using the left end of the beam at the pinned support as the origin of the axis system (i.e. $x_1 = 0$), it is noted that:

$$\text{mass intensity } (x_1 = 0) = 2 \text{ mass intensity } @ (x_1 = 50 \text{ m})$$

The mass produces a lineload that varies linearly. Thus:

$$\text{mass/unit length} = f(x_1) = m x_1 + b$$

Two conditions are needed to determine the constants, m and b, of this equation. These are:

$$\text{Total mass} = m_b = \int_0^{50m} f(x_1)\,dx_1$$

$$f(x_1 = 0) = 2f(x_1 = 50m)$$

- Using the second gives:

$$b = 2\left(m(50m) + b\right)$$

$$\Rightarrow -b = m(100m)$$

- Using the first gives:

$$m_b = \int_0^{50m} (mx_1 + b)\,dx_1$$

$$= \frac{m}{2}x_1^2 + bx_1 \Big]_0^{50m}$$

$$\Rightarrow m_b = \frac{m}{2}(2500m^2) + b(50m)$$

Now using the earlier result for b:

$$m_b = \frac{m}{2}(2500m^2) - m(5000m^2)$$

$$\Rightarrow m_b = -m(3750m^2)$$

giving $m = \dfrac{-m_b}{3750m^2}$

Use this to get:

$$b = \frac{m_b}{3750\,m^2}(100\,m)$$

$$\Rightarrow b = \frac{m_b}{37.5\,m}$$

Putting this together:

$$m_b(x_1) = \frac{m_b}{3750\,m^2}(-x_1 + 100\,m)$$

NOTE: The units are $\left[\frac{\text{mass} \cdot \text{length}}{\text{length}^2}\right]$ giving $\left[\frac{\text{mass}}{\text{length}}\right]$ giving mass intensity which is consistent.

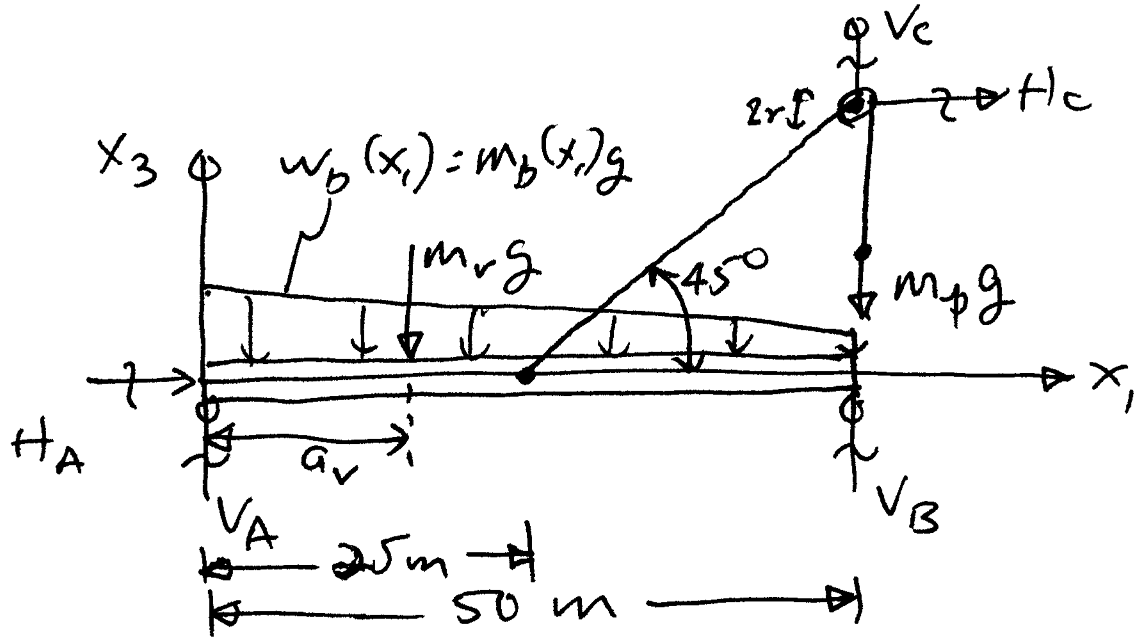
also, the value @ $x_1 = 0$ is twice that @ $x_1 = 50\,m$ as specified.

→ finally to get weight (intensity), multiply by gravity:

$$\text{beam weight (intensity)} = w_b(x_1) = m_b(x_1)\,g$$

$$= \frac{m_b g}{3750\,m^2}(-x_1 + 100\,m)$$

Now the Free Body Diagram can be drawn:



where:

A is the pin support at $x_1 = 0$ with horizontal and vertical reactions

B is the roller support at $x_1 = 50$ m with vertical reaction

C is the pin support for the pulley with vertical and horizontal reactions

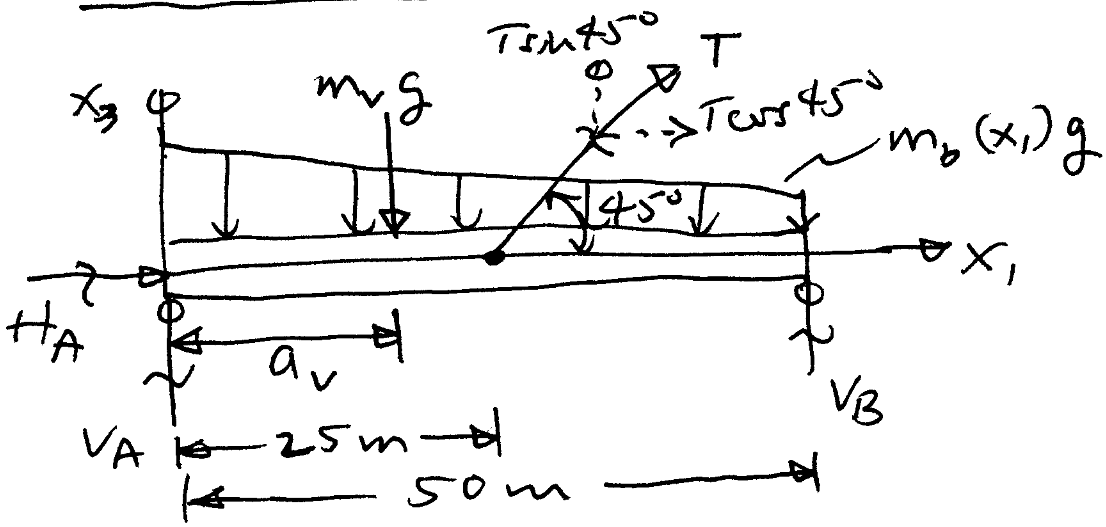
a_v is the x_1-location of the vehicle

It is useful in a problem such as this to split the FBD (Free Body Diagram) into two "subsystems", each with their own Free Body Diagram. (The individual FBD's must "sum" to the overall FBD).

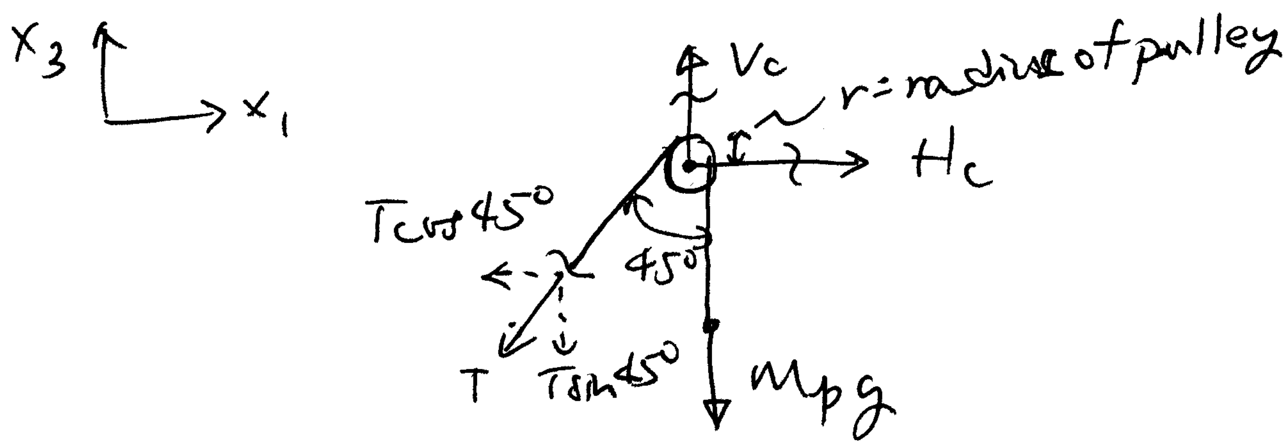
Why do this? There are actually two structures -- the beam and the cable. Split this by replacing the beam/cable connection by a force representing that connection. In this case, it is the tension in the cable, T.

Thus:

Beam sub-system



Pulley sub-system



Note: T is in opposite directions at the "cut" in the cable between the two sub-systems. Thus, in adding these two FBD's, the T "goes internal" (i.e. is zero externally) and the FBD for the overall system results

(b) For it to be possible to determine the reactions, the system must be statically determinate (or constitutive relations are needed). In this case, this means that each of these subsystems must be statically determinate.

→ Beam sub-system:

# of dof = 3
- lateral in x_1
- lateral in x_3
- rotation in x_1-x_3 plane

# of reactions = 3 (V_A, V_B, H_B)

(# of dof) = (# of reactions) ⇒ Statically Determinate

→ <u>Pulley sub-system</u>

# of dof = 2
- lateral in x_1
- lateral in x_3

(<u>NOTE</u>: A cable is not rigid, so it cannot rotate as a rigid body)

# of reactions = 2 (H_c, V_c)

(# of dof) = (# of reactions) ⇒ <u>Statically Determinate</u>

→ what about T?

This is a transmission across the two sub-systems so there will be "an equation" as well as "across the cable..." (that is: $T = T$)

So apply equilibrium within each of the subsystems:

→ <u>Beam sub-system</u>:

$\sum F_1 = 0 \quad \xrightarrow{+} \Rightarrow H_A + 0.707\, T = 0 \quad (1)$

$$\Sigma F_3 = 0 \quad \uparrow + \Rightarrow V_A + V_B + 0.707T - m_v g - \int_0^{50m} m_b(x_1) g \, dx_1 = 0 \quad (2)$$

$$\Sigma M = 0 \quad \circlearrowleft + \Rightarrow (0.707T)(25m) - (m_v g)(a_v) + (V_B)(50\,m) - \int_0^{50m} m_b(x_1) x_1 g \, dx_1 = 0 \quad (3)$$

(take about $x_1 = 0$)

→ Pulley sub-system:

$$\Sigma F_1 = 0 \quad \xrightarrow{+} \Rightarrow H_c - 0.707\,T = 0 \quad (4)$$

$$\Sigma F_3 = 0 \quad \uparrow + \Rightarrow V_c - 0.707T - m_p g = 0 \quad (5)$$

$$\Sigma M = 0 \quad \circlearrowleft + \Rightarrow Tr - (m_p g) r = 0 \quad (6)$$

(about pulley pin) where: r = pulley radius

Rearrange and work these equations. First go at the two integrals:

$$\int_0^{50m} \frac{m_b g}{3750 m^2} (-x_1 + 100m) \, dx_1$$

$$= \frac{m_b g}{3750 m^2} \left\{ -\frac{x_1^2}{2} + (100m) x_1 \right\} \Big]_0^{50m}$$

$$= \frac{m_b g}{3750 m^2} \left(-1250 m^2 + 5000 m^2 \right) = m_b g$$

(as it should be!)

and the other integral with the moment arm:

$$\int_0^{50\,m} \frac{m_b g}{3750\,m^2}\left\{-x_1^2 + (100\,m)x_1\right\} dx_1$$

$$= \frac{m_b g}{3750\,m^2}\left\{\frac{-x_1^3}{3} + \frac{(100\,m)x_1^2}{2}\right\}\Bigg]_0^{50\,m}$$

$$= \frac{m_b g}{3750\,m^2}\left(-\frac{125{,}000\,m^3}{3} + \frac{250{,}000\,m^3}{2}\right)$$

$$= m_b g\,(22.2\,m)$$

NOTE: This has units of moment as it should.

Now summarizing the equations:

$$H_A = -0.707\,T \qquad (1)$$

$$V_A + V_B = m_v g + m_b g - 0.707\,T \qquad (2)$$

$$2V_B = \frac{m_v g\,a_v}{25} - 0.707\,T + 0.889\,m_b g \qquad (3)^*$$

$$H_C = 0.707\,T \qquad (4)$$

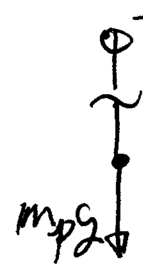
$$V_C = m_p g + 0.707\,T \qquad (5)$$

$$T = m_p g \qquad (6)^{**}$$

*, ** See notes on next page

* <u>NOTE 1</u>: The unit [m] has been cancelled on both sides of the equation so the distance a_v to the vehicle must be in [m].

** <u>NOTE 2</u>: Can also get this equation by noting that a cable transmits axial force only, so the tension in the cable must be equal to the weight hanging at its end. So by cutting the cable near the end:



$\Rightarrow$ $T = m_p g$ as per (6)

Now, use (6) to express T in the other equations and get the following for the reactions expressed in terms of given masses and the variable distance to the vehicle, a_v:

$$\boxed{\begin{aligned} H_C &= 0.707\, m_p g \\ V_C &= 1.707\, m_p g \\ H_A &= -0.707\, m_p g \\ V_B &= \frac{m_v g a_v}{50} - 0.354\, m_p g + 0.444\, m_b g \\ V_A &= m_v g(1 - 0.02 a_v) - 0.353\, m_p g + 0.555\, m_b g \end{aligned}}$$

with a_v in [m]

→ This can be checked by performing an equilibrium assessment on the overall free body diagram first drawn:

$$\sum F_1 = 0 \ \overset{+}{\rightarrow} \Rightarrow H_A + H_C \overset{?}{=} 0 \qquad \checkmark \ \underline{\text{yes}}$$

$$\sum F_3 = 0 \ \uparrow + \Rightarrow V_A + V_B + V_C - m_v g - m_b g - m_p g \overset{?}{=} 0$$

write this out:

$$\cancelto{1}{(1-0.02a_v)m_v g} - \cancelto{2}{0.353 m_p g} + \cancelto{3}{0.555 m_b g} + \cancelto{1}{0.02 m_v g a_v} - \cancelto{2}{0.354 m_p g} + \cancelto{3}{0.444 m_b g} + \cancelto{2}{1.707 m_p g} - \cancelto{1}{m_v g} - \cancelto{3}{m_b g} - \cancelto{2}{m_p g} \overset{?}{=} 0$$

$\checkmark$ <u>yes</u>

A similar thing <u>can</u> be done with moments.

(c) If there is no cable, then $H_C = 0$, $V_C = 0$ and T does not exist. Using this in equations (1)-(3) results in:

$$H_A = 0$$

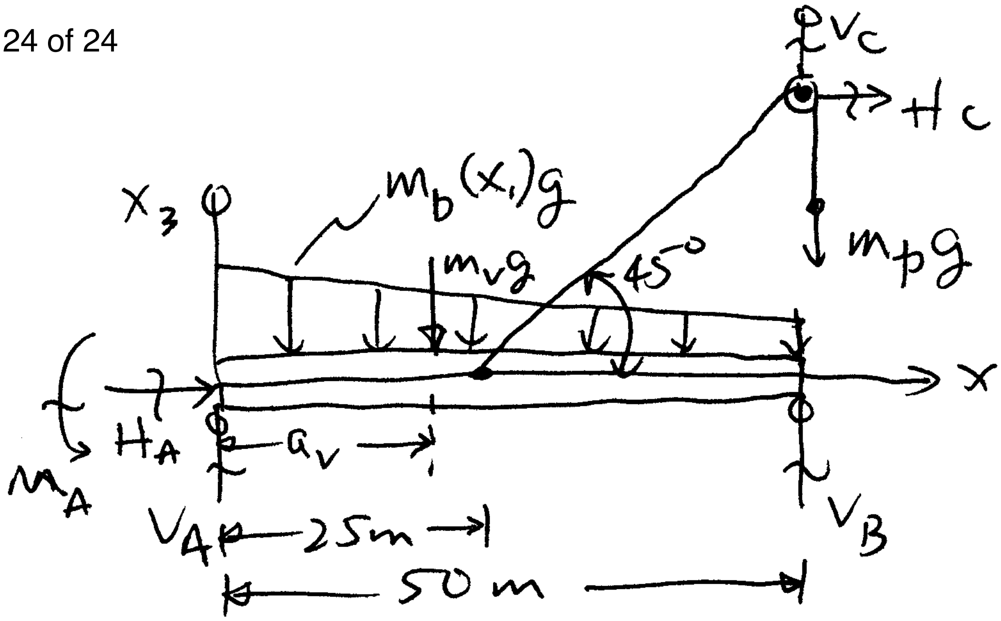
$$V_B = \frac{m_v g a_v}{50} + 0.444\, m_b g$$

$$V_A = m_v g (1 - 0.02 a_v) + 0.555\, m_b g$$

→ Since $m_b g$ is positive, V_A and V_B <u>decrease</u> with the cable support. However, H_A then becomes nonzero.

One can argue that the cable suspends the beam to some extent and relieves the load and thus the vertical reactions, V_A and V_B. However, there is a horizontal "pull" of this cable and this results in a nonzero horizontal reaction, H_A.

(d) If a clamped support replaces a pin, a moment reaction is added. Thus the Free Body Diagram (overall) becomes:



The extra moment reaction, M_A, is thus added to the Beam sub-system. This adds this term to equilibrium equation (3) ($\Sigma M = 0$ for Beam sub-system) giving:

$$(0.707T)(25m) - (m_v g)(a_v) - \int_0^{50m} m_b(x_1)x_1 g\,dx_1 + (V_B)(50m) + M_A = 0 \quad (3)'$$

There is now an additional reaction giving $\underline{\underline{4}}$ reactions with $\underline{\underline{3}}$ degrees of freedom

$\Rightarrow$ **Statically Indeterminate**

Can express all the reactions in terms of one unknown reaction.