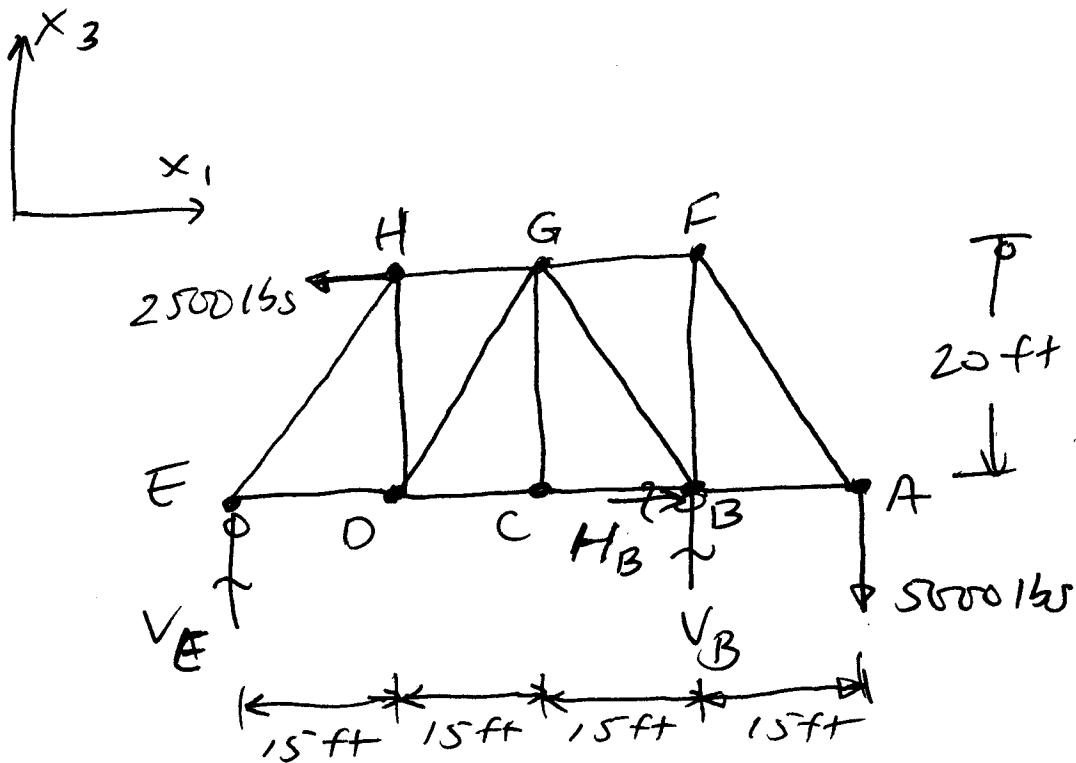


Unified Engineering - Fall 2008
Problem set #5 week #6

SOLUTIONS

M6 (M6.1)

(a) Draw the Free Body Diagram



(b) Determine the reaction forces by applying the equilibrium equations:

$$\sum F_x = 0 \quad \rightarrow \Rightarrow H_B - 2500 \text{ lbs} = 0$$

$$\Rightarrow H_B = +2500 \text{ lbs}$$

$$\sum F_y = 0 \quad \uparrow \Rightarrow V_E + V_B - 5000 \text{ lbs} = 0$$

and

$$\sum M = 0 \quad (\uparrow \Rightarrow -V_E (45 \text{ ft}) + 2500 \text{ lbs} (20 \text{ ft}) - 5000 \text{ lbs} (10 \text{ ft}) = 0$$

(about point B)

$$\text{giving: } -V_E (45 \text{ ft}) - 2500 \text{ ft} \cdot 10 = 0$$

$$\Rightarrow V_E = -5555 \text{ lbs}$$

use this in the $\sum F_y = 0$ equation:

$$-5555 \text{ lbs} + V_B - 5000 \text{ lbs} = 0$$

$$\Rightarrow V_B = +5555 \text{ lbs}$$

Summarizing and bringing to significant digits:

$$\begin{aligned} H_B &= 2500 \text{ lbs} \\ V_B &= 5555 \text{ lbs} \\ V_E &= -5555 \text{ lbs} \end{aligned}$$

NOTE: One thing that can be done is to check results by taking moment equilibrium about another point. Summing to zero must occur about any point.

choose H:

$$\sum M_H = 0 \quad \left(\begin{array}{l} + \\ \curvearrowright \end{array} \Rightarrow \begin{array}{l} V_F \\ H_B \end{array} \right)$$

$$+ (-5551 \text{ lbs})(15 \text{ ft}) + (2500 \text{ lbs})(20 \text{ ft}) + (5555 \text{ lbs})(30 \text{ ft}) - (5000 \text{ lbs})(45 \text{ ft}) \stackrel{?}{=} 0$$

applied load at ~~A~~

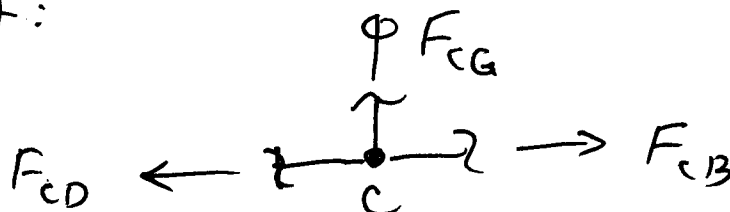
solved:

$$8325 \text{ ft} \cdot \text{lbs} + 50,000 \text{ ft} \cdot \text{lbs} + 166,650 \text{ ft} \cdot \text{lbs} - 225,000 \text{ ft} \cdot \text{lbs} \stackrel{?}{=} 0$$

YES ✓

moving on.....

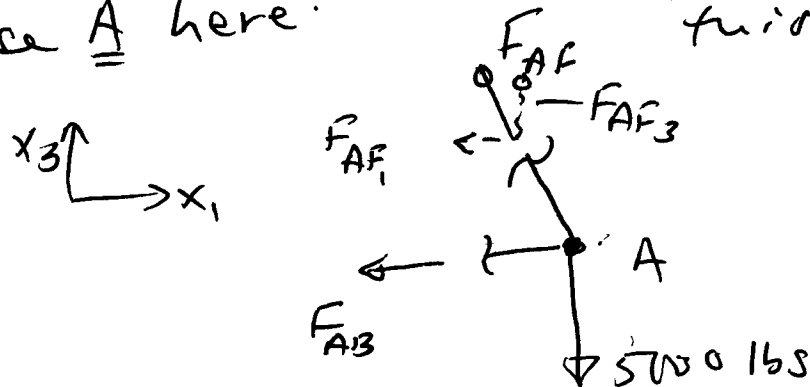
(c) YES! Consider point C and vertical (x_3) equilibrium at this point:



Since only the bar CG has a vertical (i.e. x_3) direction, only it can carry a vertical load (x_3 -direction). There is no load applied at point C, so this gives $F_{CG} = 0$ by equilibrium in the vertical (x_3) direction.

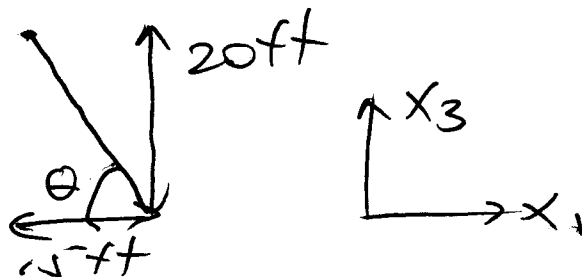
(d) using the Method of Joints, choose an end from which to start.

Choose A here: (NOTE: Multiple starting points and ways through this can be chosen)



Note that the diagonal bars make an angle to the axes that must be determined.

Use:



$$\tan \theta = \frac{20\text{ft}}{15\text{ft}} = 1.33 \Rightarrow \theta = 53^\circ$$

$$\begin{aligned} \text{giving } \sin \theta &= 0.8 \\ \cos \theta &= 0.6 \end{aligned}$$

and note that:

$$F_{AF_1} = F_{AF} \cos \theta = 0.6 F_{AF}$$

$$F_{AF_3} = F_{AF} \sin \theta = 0.8 F_{AF}$$

Now use equilibrium:

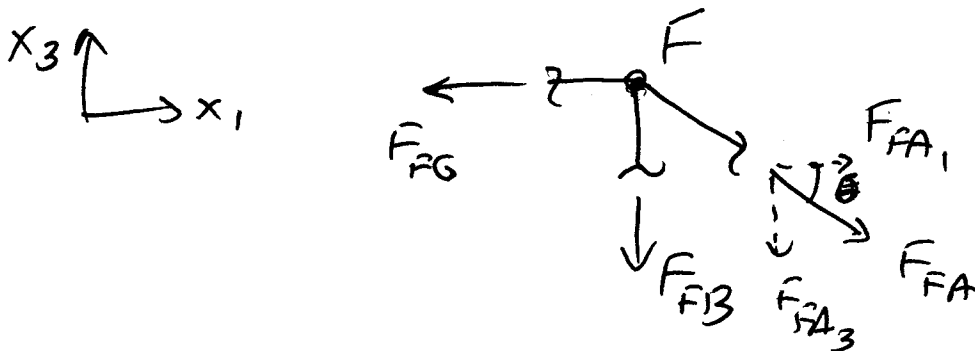
$$\sum F_x = 0 \quad \rightarrow \Rightarrow -F_{AB} - 0.6 F_{AF} = 0$$

$$\sum F_z = 0 \quad \uparrow \Rightarrow 0.8 F_{AF} - 5000 \text{ lbs} = 0$$

second equation gives: $F_{AF} = +6250 \text{ lbs}$

using the first gives: $F_{AB} = -3750 \text{ lbs}$

Proceed to Joint F



and use $F_{FA} = F_{AF} \dots$ and $F_{FA_1} = F_{FA} \cos \theta$
 $F_{FA_3} = F_{FA} \sin \theta$

with equilibrium:

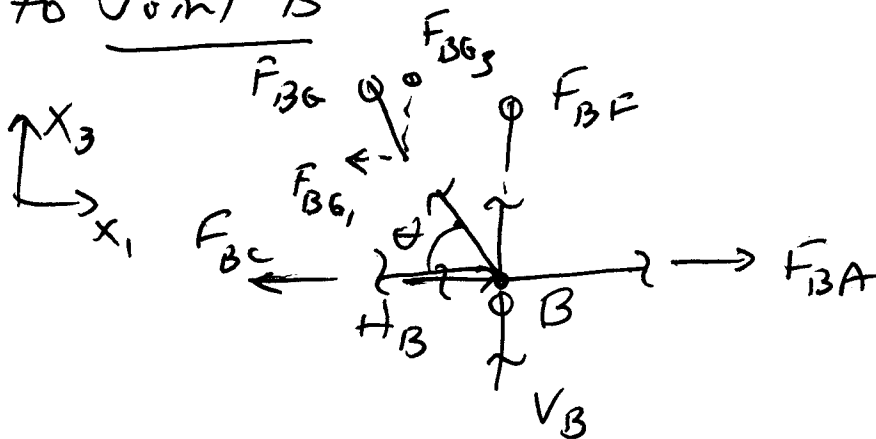
$$\sum F_1 = 0 \quad \rightarrow \Rightarrow -F_{FG} + 0.6 F_{FA} = 0$$

$$\Rightarrow \boxed{F_{FG} = +3750 \text{ lbs}}$$

$$\sum F_3 = 0 \quad \uparrow \Rightarrow -F_{FB} - 0.8 F_{FA} = 0$$

$$\Rightarrow \boxed{F_{FB} = -5000 \text{ lbs}}$$

to Joint B



Values for V_B , H_B , F_{BA} , and F_{BF} are known, so there are 2 equations and 2 unknowns ... GOOD! (and the same relation for θ)

$$\sum F_1 = 0 \quad \rightarrow \Rightarrow -F_{BC} - 0.6 F_{BG} + H_B + F_{BA} = 0$$

$$\text{giving: } -F_{BC} - 0.6 F_{BG} + 2500 \text{ lbs} - 3750 \text{ lbs} = 0$$

$$\Rightarrow -F_{BC} - 0.6 F_{BG} = 1250 \text{ lbs}$$

$$\sum F_3 = 0 \quad \uparrow \Rightarrow V_B + F_{BF} + 0.8 F_{BG} = 0$$

giving: $5555 \text{ lbs} - 5000 \text{ lbs} + 0.8 F_{BG} = 0$

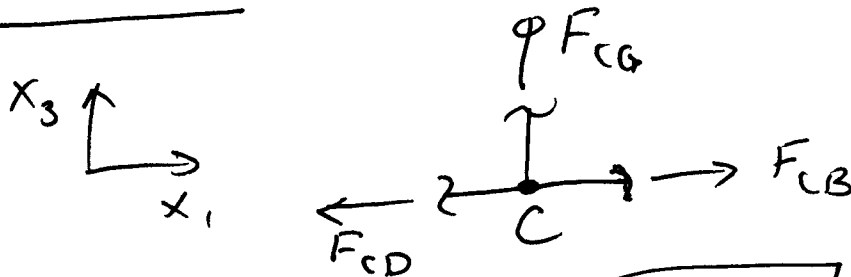
$$\Rightarrow F_{BG} = -694 \text{ lbs}$$

using the previous equation:

$$-F_{BC} + 416 \text{ lbs} = 1250 \text{ lbs}$$

$$\Rightarrow F_{BC} = -834 \text{ lbs}$$

to Joint C

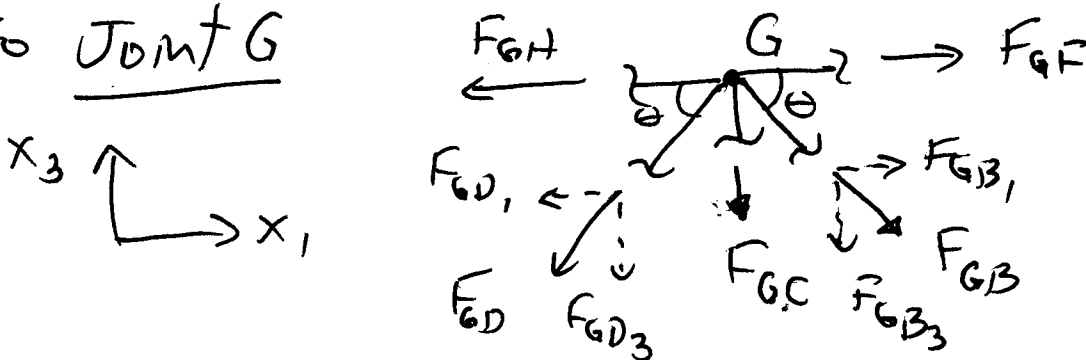


$\sum F_3 = 0$ \uparrow gives $F_{CG} = 0$ as determined in part (c)

$$\sum F_1 = 0 \quad \rightarrow \Rightarrow F_{CB} - F_{CD} = 0$$

$$\Rightarrow F_{CD} = -834 \text{ lbs}$$

to Joint G



The values for F_{GF} , F_{GB} , and F_{GC} are known leaving two unknowns (F_{GH} and F_{GD}). So using equilibrium and the relations for θ -----

$$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow F_{GF} + \overbrace{0.6 F_{GB}}^{F_{GB}} - F_{GH} - \overbrace{0.6 F_{GD}}^{F_{GD}} = 0$$

giving: $3750 \text{ lbs} - 416 \text{ lbs} - F_{GH} - 0.6 F_{GD} = 0$

$$\Sigma F_3 = 0 \uparrow \Rightarrow -\overbrace{0.8 F_{GB}}^{F_{GB}} - F_{GC} - \overbrace{0.8 F_{GD}}^{F_{GD}} = 0$$

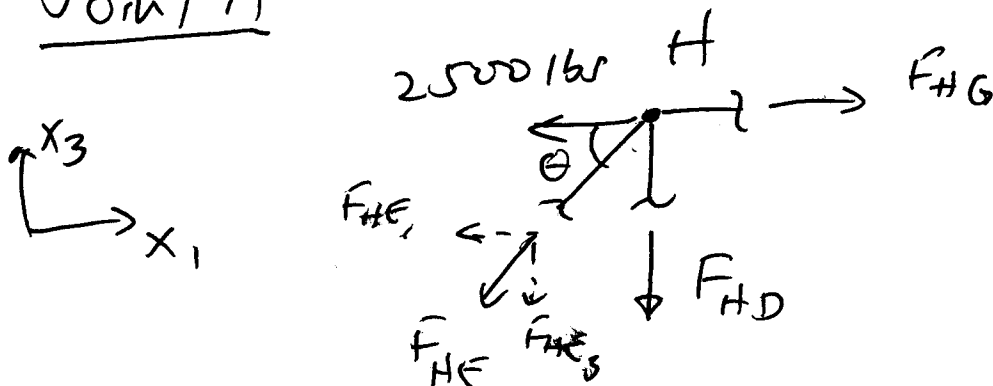
giving: $-F_{GB} = F_{GD} \Rightarrow \boxed{F_{GD} = +694 \text{ lbs}}$

using this in the previous equation:

$$3334 \text{ lbs} - F_{GH} - 416 \text{ lbs} = 0$$

$$\Rightarrow \boxed{F_{GH} = +2918 \text{ lbs}}$$

to Joint H



Again, there are only 2 unknowns
(F_{HD} and F_{HE}), so with the relations
for θ , move forward:

$$\sum F_1 = 0 \quad \rightarrow \Rightarrow -2500 \text{ lbs} + F_{HG} - \overbrace{0.6 F_{HE}}^{F_{HE1}} = 0$$

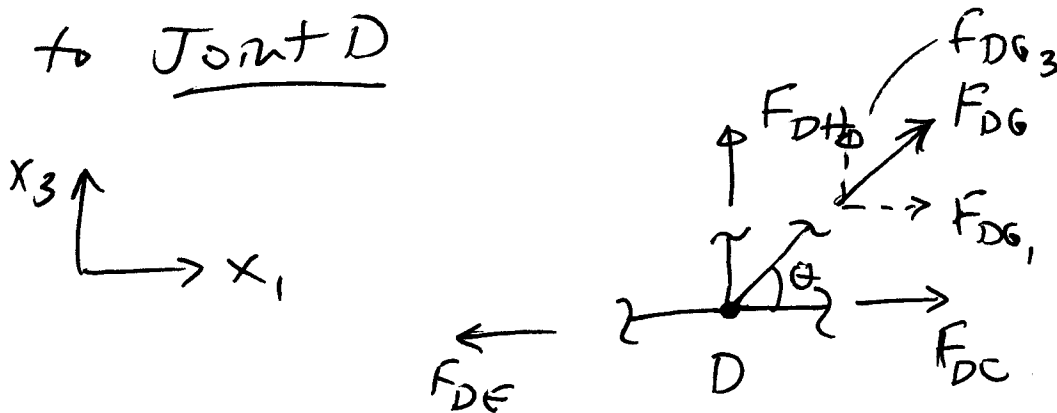
$$\text{giving: } -2500 \text{ lbs} + 2918 \text{ lbs} = 0.6 F_{HE}$$

$$\Rightarrow \boxed{F_{HE} = +697 \text{ lbs}}$$

$$\sum F_3 = 0 \quad \uparrow \Rightarrow -\overbrace{0.8 F_{HE}}^{F_{HE3}} - F_{HD} = 0$$

$$\Rightarrow \boxed{F_{HD} = -558 \text{ lbs}}$$

to Joint D



Here there is only one unknown (F_{DF}).
In such a situation, the other equilibrium
equation can be used as a check as it
still must be satisfied.

for the unknown:

$$\sum F_x = 0 \rightarrow \Rightarrow -F_{DE} + F_{DC} + \overbrace{0.6 F_{DG}}^{F_{DG1}} = 0$$

using values:

$$F_{DE} = -834 \text{ lbs} + 416 \text{ lbs}$$

$$\Rightarrow \boxed{F_{DE} = -418 \text{ lbs}}$$

then check via:

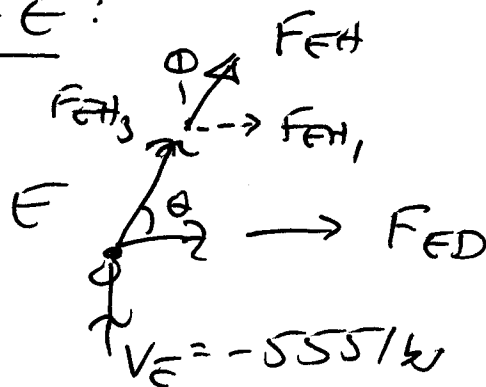
$$\sum F_z = 0 \uparrow \Rightarrow F_{DH} + \overbrace{0.8 F_{DG}}^{F_{DG3}} = 0$$

$$\text{continuing: } -558 \text{ lbs} + 555 \text{ lbs} \stackrel{?}{=} 0$$

YES
CHUCKS ✓

(NOTE: some round off "errors" -- less than 1%)

Finally to Joint E:



All values are determined, so use this to check results ---.

$$\sum F_x = 0 \xrightarrow{+} \Rightarrow F_{ED} + \overbrace{0.6 F_{EH_1}} = 0$$

with values:

$$-418168 + 418168 \stackrel{?}{=} 0$$

✓ F₁ checks ✓

$$\sum F_z = 0 \uparrow \Rightarrow V_E + \overbrace{0.8 F_{EH_3}} = 0$$

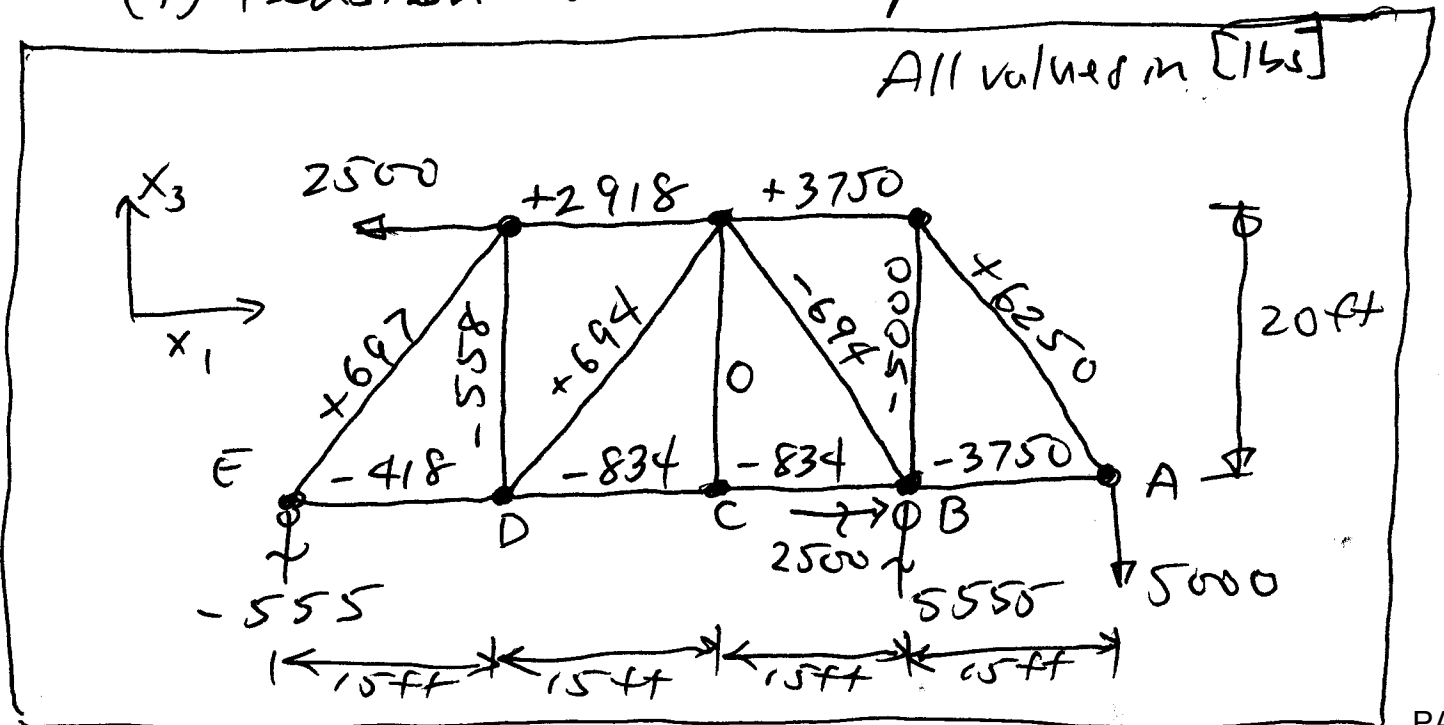
with values:

$$-555168 + 558168 \stackrel{?}{=} 0$$

✓ F₃ checks ✓

(again within errors of calculation)

→ Summarize by drawing the truss and placing the bar load above each bar with (+) tension and (-) compression noted.

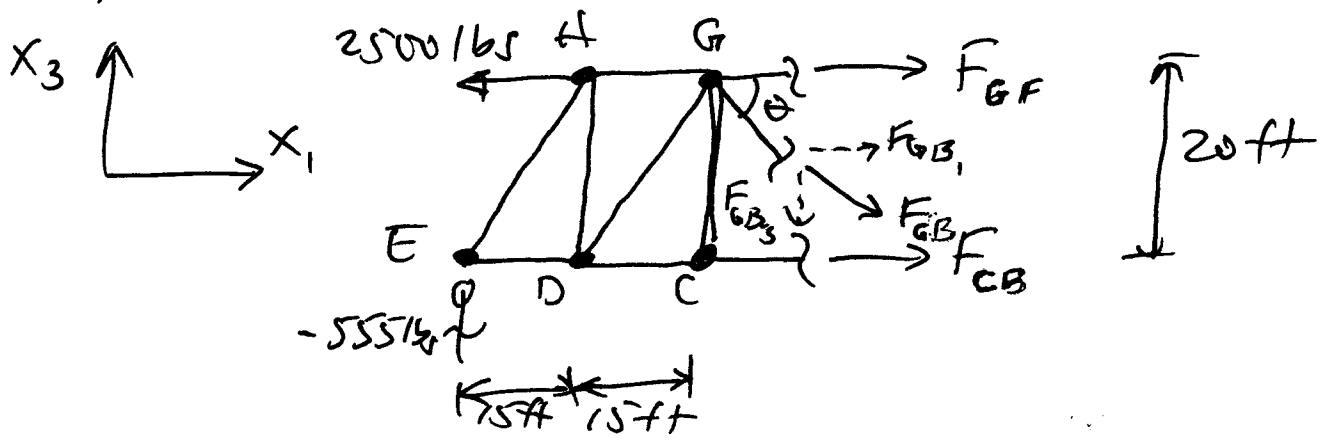


A few notes

- There are round off errors that propagate through the work. This should be noted and checked via the points allowing such.
- Engineers use whatever opportunities that present themselves to check work.
- Once calculated, reactions can be drawn as originally represented with the sign that comes from the calculation OR the reaction can be drawn in the direction such that only its magnitude needs to be shown. Be sure one is CONSISTENT in such.

(e) To determine the loads in the bars of the third bay noted (CB, GB, GF) by the Method of Sections, a "sectional cut" must be made that goes through these bars. There must be only three bars of unknown load so that the three equilibrium equations yield a determinate solution.

→ Choose the following cut from the end of point F (NOTE: the same could be done for point B)



The proper and detailed figure is the key first point.

→ Now apply the equations of equilibrium (using the relations for θ determined earlier)

$$\Sigma F_x = 0 \xrightarrow{+} \Rightarrow -2500 \text{ lbs} + F_{CB} + F_{GF}$$

$$+ \underbrace{0.6 F_{GB}}_{F_{GB_1}} = 0$$

$$\Sigma F_z = 0 \quad \uparrow + \Rightarrow -555 \text{ lbs} - \underbrace{0.8 F_{GB}}_{F_{GB_2}} = 0$$

$$\Rightarrow \boxed{F_{GB} = -694 \text{ lbs}}$$

✓
check with
result from (d)

$$\Sigma M = 0 \quad (\text{about point G}) \quad \left(\begin{array}{l} + \\ \rightarrow \end{array} \right) \Rightarrow -(-555 \text{ lbs})(30 \text{ ft}) + F_{CB}(20 \text{ ft}) = 0$$

↓
only get 1 unknown...

$$\Rightarrow \boxed{F_{CB} = -833 \text{ lbs}}$$

check ✓

Putting these two results in the $\Sigma F_x = 0$ equation:

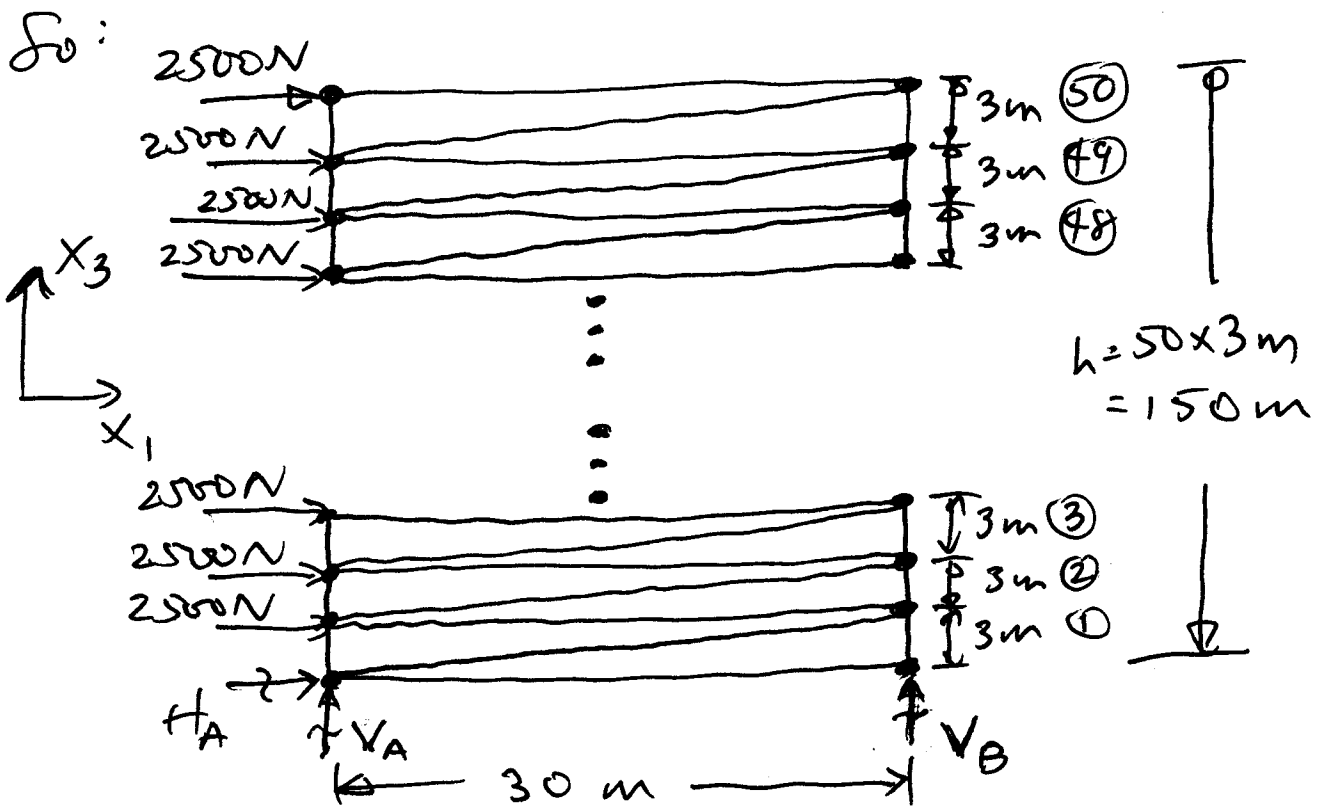
$$-2500 \text{ lbs} - 833 \text{ lbs} + F_{GF} - 416 \text{ lbs} = 0$$

$$\Rightarrow \boxed{F_{GF} = +3749}$$

check ✓
(within calculation
error of roundoff...)

M7 (MG.2)

(a) The first step is to draw the Free Body Diagram. One could draw the entire structure or do as done in the problem statement showing the applied load with a magnitude of 2500 N at each floor juncture/joint.



(b) To determine the reaction, use equilibrium. There are 3 reactions and 3 rigid body degrees of freedom for the Presidential Center with this model.

⇒ Statically Determinate

$$\Sigma F_1 = 0 \quad \rightarrow \Rightarrow 2500N(50) + H_A = 0$$

there are
50 points of
applied load -- at
each floor but the
ground.

$$\Rightarrow H_A = -125,000 N$$

$$\Sigma F_3 = 0 \quad \uparrow \Rightarrow V_A + V_B = 0$$

$$\Sigma M = 0 \quad \left(\begin{array}{l} \uparrow \\ \text{choose point} \\ A \end{array} \right) \Rightarrow V_B(30m) - \sum_{i=1}^{50} (2500N)(3m)i = 0$$

At each floor there is a force of 2500 N with a moment arm of (3m)(i). Thus the sum of i from 1 to 50 is needed:

$$\sum_{i=1}^{50} i = 1275$$

giving:

$$V_B(30m) = 9,562,500 N \cdot m$$

$$\Rightarrow V_B = +318,750 \text{ N}$$

using this in the $\Sigma F_z = 0$ equation gives

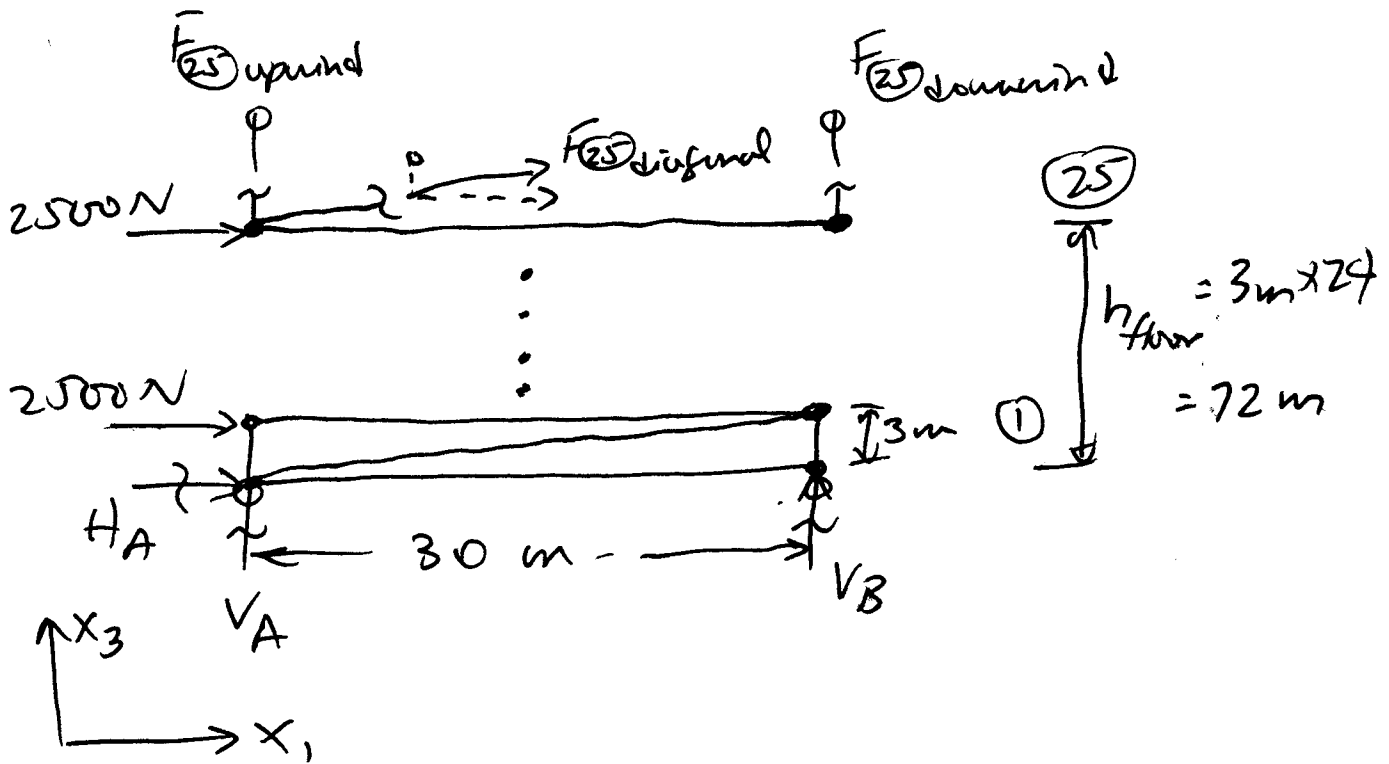
$$V_A = -318,750 \text{ N}$$

Summarizing:

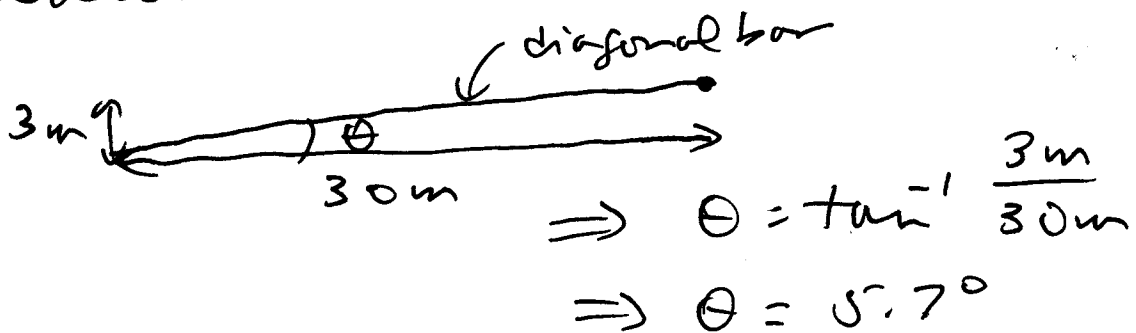
$H_A = -125,000 \text{ N}$ $V_A = -318,750 \text{ N}$ $V_B = +318,750 \text{ N}$
--

(c) To get the bar loads in the bars modeling the bay representing the 25th floor, one could use the Method of Joints or the Method of Sections. Using the Method of Joints requires working from a point with sufficient known loads and progressing to the desired point. This can involve significant points and equations. The Method of Sections is generally more efficient in such situations. Make a cut through the bars of the 25th floor (vertical and diagonal as desired) and draw the

resulting free body diagram:



It is necessary to resolve $F_{25}^{diagonal}$ into x_1 and x_3 components. Based on the geometry of each floor:



and

$$F_{25}^{diagonal}(x_1) = F_{25}^{diagonal} \cos \theta = 0.99 F_{25}^{diagonal}$$

$$F_{25}^{diagonal}(x_3) = F_{25}^{diagonal} \sin \theta = 0.1 F_{25}^{diagonal}$$

→ Now apply equilibrium ...
 24 occurrences of top of
 1st floor
 below 25th

$$\sum F_x = 0 \rightarrow \Rightarrow H_A + (2500\text{N})(24) + 0.99 \overset{\text{diagonal}}{F_{25}} = 0$$

using value for H_A :

$$0.99 \overset{\text{diagonal}}{F_{25}} = + 65,000\text{N}$$

$$\Rightarrow \overset{\text{diagonal}}{F_{25}} = + 65,600\text{N}$$

$$\sum F_z = 0 \uparrow \Rightarrow V_A + V_B + \overset{\text{upward}}{F_{25}} + \overset{\text{downward}}{F_{25}} + 0.1 \overset{\text{diagonal}}{F_{25}} = 0$$

using known values yields:

$$- 318,750\text{N} + 318,750\text{N} + \overset{\text{upward}}{F_{25}} + \overset{\text{downward}}{F_{25}} + 6,560\text{N} = 0$$

$$\Rightarrow \overset{\text{upward}}{F_{25}} + \overset{\text{downward}}{F_{25}} = -6,560\text{N}$$

and ...

$$\sum M = 0 \quad (\rightarrow) \Rightarrow + (H_A)(72\text{m}) + (V_B)(30\text{m}) + \overset{\text{downward}}{F_{25}}(30\text{m}) + \sum_{i=1}^{24} (2500\text{N})(3\text{m})(24-i) = 0$$

(about upward side of floor)

↳ gives only 1 unknown

direction is +; each floor to 24th has magnitude of 2500N; distance is (3m)(24-i) - - - - -

Consider further the sum:

$$\sum_{i=1}^{24} (24-i)$$

This has a value of 0 for $i=24$ and then 1 through 23 as i decreases from 23 to 1. So this is the same as

$$\sum_{i=1}^{23} i$$

This is equal to 276

using values and this result:

$$\begin{aligned} & (-125,000 \text{ N})(72 \text{ m}) + (318,750 \text{ N})(30 \text{ m}) \\ & + (2500 \text{ N})(3 \text{ m})(276) + F_{25}^{\text{downward}}(30 \text{ m}) = 0 \end{aligned}$$

$$\begin{aligned} \text{gives: } & -300,000 \text{ N} + 318,750 \text{ N} \\ & + 69,000 \text{ N} + F_{25}^{\text{downward}} = 0 \end{aligned}$$

$$\Rightarrow F_{25}^{\text{downward}} = -87,750 \text{ N}$$

using this in the $\sum F_3 = 0$ equation gives:

$$F_{25}^{\text{upward}} = +81,190 \text{ N}$$

Summarizing:

$$F_{(25) \text{ upward}} = +81,190 \text{ N}$$

$$F_{(25) \text{ downward}} = -87,750 \text{ N}$$

$$F_{(25) \text{ diagonal}} = +65,600 \text{ N}$$

Solutions F6

This 2D, unsteady, incompressible flow has pathlines:

$$\begin{aligned}x &= 1 + t \\y &= 1 - t^2\end{aligned} \quad \boxed{t \text{ is non-dim time}}$$

(At $t=0$), these equations describe the trajectory of a pathline that goes through $(x=1, y=1)$

(20%) (a) Pathlines result from integrating the velocity components in time:

$$\begin{aligned}x(t) &= x_0 + \int_{t_0}^t u(\tau) d\tau \\ \text{and} \\ y(t) &= y_0 + \int_{t_0}^t v(\tau) d\tau\end{aligned} \quad \begin{array}{l} (x_0, y_0) \text{ are the points on the} \\ \text{pathline at time } = t_0 \end{array}$$

Taking the derivative (in time) of these expressions, we find u and v :

$$\frac{dx}{dt} = \frac{dx_0}{dt} + \frac{d}{dt} \int_{t_0}^t u(\tau) d\tau = \frac{dx_0}{dt} + \int_{t_0}^t \frac{du(\tau)}{d\tau} d\tau = \frac{dx_0}{dt} + u(t) - u(t_0)$$

$$\text{and} \quad \frac{dy}{dt} = \frac{dy_0}{dt} + v(t) - v(t_0)$$

$$\text{Therefore:} \quad u(t) = \frac{dx}{dt} + u(t_0) \quad v(t) = \frac{dy}{dt} + v(t_0)$$

and using $x=1+t$ and $y=1-t^2$:

$$\boxed{u(t) = 1 + u(t_0) \quad v(t) = -2t + v(t_0)}$$

The velocity vector is: $\vec{v} = [1 + u(t_0)] \hat{i} + [-2t + v(t_0)] \hat{j}$

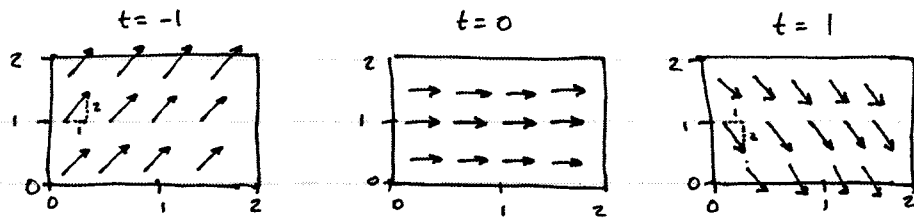
$$(20\%) \quad (b) \text{ Integrating } u \text{ \& } v \Rightarrow \begin{aligned}x(t) &= x_0 + (1 + u(t_0))(t - t_0) \\ y(t) &= y_0 + v(t_0)(t - t_0) - (t^2 - t_0^2)\end{aligned}$$

$$\text{For the pathline going through } (1, 1) \Rightarrow x = 1 + t \quad y = 1 - t^2$$

$$\text{therefore: } x_0 = 1, u(t_0) = 0, t_0 = 0 \text{ and } v(t_0) = 0$$

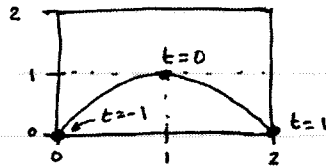
$$\text{then: } \boxed{\vec{v} = \hat{i} - 2t \hat{j}}$$

(b) contd'.



(c) the pathline going through $(1,1)$ is: $x = 1+t$ $y = 1-t^2$
 or $y = 1 - (x-1)^2$

then

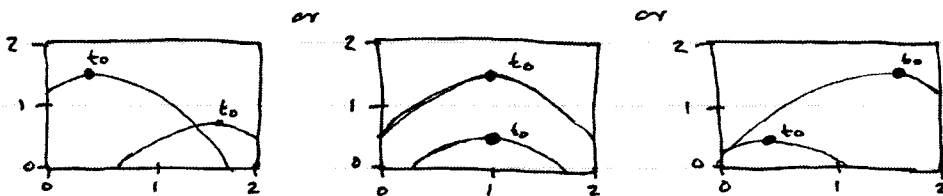


(20%)

2 different pathlines can be found from: $x(t) = x_0 + (1 + x'(t_0))(t - t_0)$
 $y(t) = y_0 + y'(t_0)(t - t_0) - (t^2 - t_0^2)$

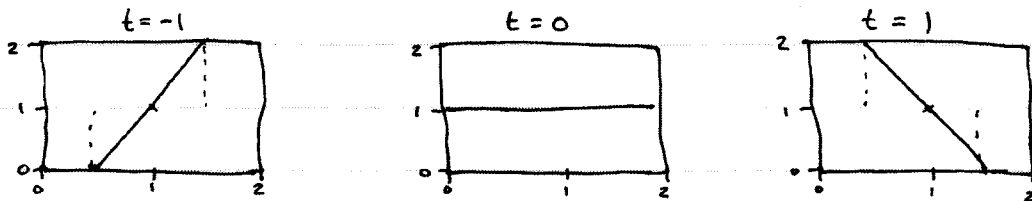
at $t = t_0 = 0$, we could pick any 2 points (x_{01}, y_{01}) (x_{02}, y_{02}) and the inverted parabolas will have their maximum at those points:

For example:



(20%)

(d) By definition, streamlines are tangent to velocity vectors, then, from (b):



(20%)

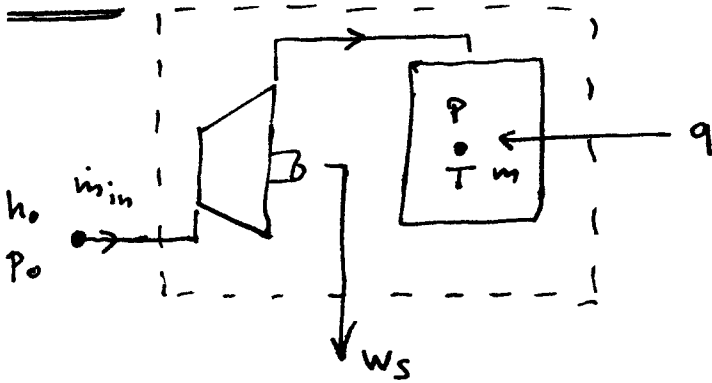
(e) Continuity is satisfied if $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$

Since the flow is incompressible $\boxed{\nabla \cdot \vec{V} = 0}$ $\rho \equiv \text{constant}$.

We verify that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0 \Rightarrow$ continuity is satisfied, even though the situation is unsteady.

T8

16. Unified Fall 08
ZS



Concepts: - 1st Law CV form
- cons. of mass

Assume: - adiabatic turbine
- ideal gas N_2
- neglect $\Delta KE, \Delta PE$

know: $T_f = 250^\circ C$, $T_0 = 300^\circ C$
 $W_s = 45 \text{ kJ/kg}$

find: q , direction of q

unsteady process stops
when $p = p_0$ in tank
initially tank evacuated
($E_i = 0$, $m_i = 0$)

1st Law CV: $\frac{dE}{dt} = \dot{Q} - \dot{W}_s + \dot{m}_i h_0$

Cons of mass: $\frac{dm}{dt} = \dot{m}_i$; so $dE = \dot{Q} dt - \dot{W}_s dt + \frac{dm}{dt} h_0 dt$

Integrate: $\int_{E_i}^{E_f} dE = \int_0^Q dQ - \int_0^{W_s} dW_s + h_0 \int_{m_i}^{m_f} dm$ but $E_i = 0$, $m_i = 0$

$$E_f = m_f c_v T_f = Q - W_s + h_0 m_f$$

$$\text{or } c_v T_f = q - w_s + h_0$$

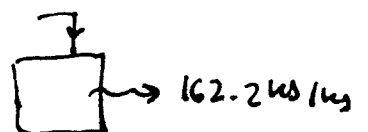
get $q = c_v T_f - c_p T_0 + w_s$

$M_{N_2} = 28 \text{ g/mol}$ $R = 8.31 \text{ kJ/kmol}\cdot\text{K}$ so $R_{N_2} = \frac{R}{M} = 296.8 \text{ J/kg}\cdot\text{K}$

$c_v = \frac{1}{\gamma - 1} R_{N_2}$, $c_p = \frac{\gamma}{\gamma - 1} R_{N_2}$ | $\gamma = 1.4$
 $c_v = 742 \text{ J/kg}\cdot\text{K}$
 $c_p = 1038.8 \text{ J/kg}\cdot\text{K}$

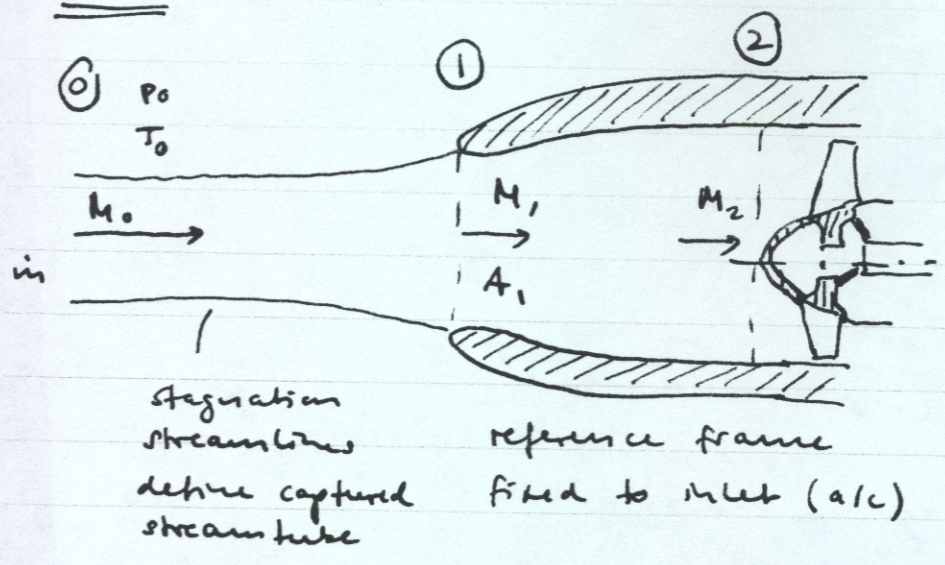
find $q = -162.2 \text{ kJ/kg}$

↑ heat is rejected from tank



T9

16. Unipid Fall 08
25



Concepts: - stagnation states
- adiabatic-reversible process

Assume: - steady, ad. flow
- ideal gas

know: M_0, M_2, A_1, P_0, T_0
 γ, R

a) 1st law: $h_{t0} = h_{t1}$ (no work done, adiabatic, steady flow)

so $T_{t1} = T_{t0} = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)$; $T_{t1} = 250 \text{ K}$

Bernoulli: $P_{t0} = P_{t1}$; $P_{t1} = P_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}}$; $P_{t1} = 1.45 \times 10^4 \text{ Pa}$

b) 1st law: $h_{t0} = h_{t1} = h_{t2}$ $\rightarrow T_2 = T_{t2} \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{-1}$; $T_2 = 231 \text{ K}$

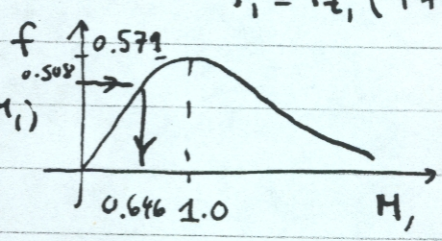
Bernoulli: $P_{t0} = P_{t1} = P_{t2}$ $\rightarrow P_2 = P_{t2} \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{-\frac{\gamma}{\gamma-1}}$; $P_2 = 1.299 \times 10^4 \text{ Pa}$

c) express $\dot{m} = \rho_1 A_1 C_1$ in terms of $P_{t1}, T_{t1}, A_1, \gamma$ and R

$\dot{m} = \frac{P_1}{RT_1} \cdot A_1 \cdot M_1 \cdot \sqrt{\gamma RT_1} = M_1 A_1 \frac{P_1}{\sqrt{T_1}} \cdot \sqrt{\frac{\gamma}{R}}$ and $P_1 = P_{t1} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{\gamma}{\gamma-1}}$
 $T_1 = T_{t1} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-1}$

get so called corrected flow:

$\frac{\dot{m} \sqrt{T_{t1}} \sqrt{R}}{P_{t1} A_1 \sqrt{\gamma}} = \frac{M_1}{\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = f(M_1)$



find M_1 iteratively or graphically:

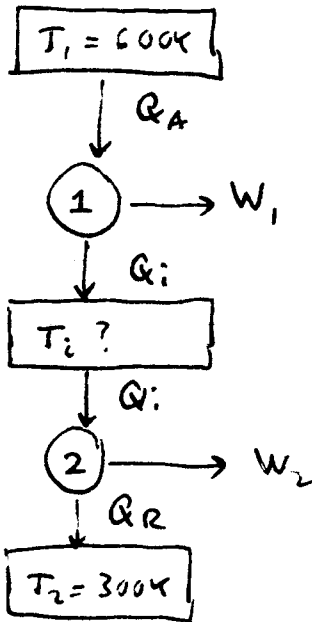
$\frac{\dot{m} \sqrt{T_{t1}} \sqrt{R}}{P_{t1} A_1 \sqrt{\gamma}} = 0.508 \rightarrow M_1 = 0.646$

d) $P_{t1} = P_{t2}$ so $P_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} = P_2 \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$

and $P_1/P_2 = \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \rightarrow \frac{P_1}{P_2} = 0.844$

T10

16. Unifed Fall 08
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- Concepts:
- Carnot cycles
 - 1st law of thermo
 - thermal efficiency

know: $\eta_{c1} = \eta_{c2} + 0.2$

Carnot 1: $\eta_{c1} = 1 - \frac{T_i}{T_1}$

Carnot 2: $\eta_{c2} = 1 - \frac{T_2}{T_i}$

thus $1 - \frac{T_i}{T_1} = 1 - \frac{T_2}{T_i} + 0.2$

$$T_i^2 + 0.2 T_1 T_i - T_1 T_2 = 0$$

$$T_i = -0.1 T_1 + \sqrt{(0.1 T_1)^2 + T_2 T_1}$$

find $T_i = 368.5 \text{ K}$