

# Massachusetts Institute of Technology Department of Aeronautics and Astronautics <br> Cambridge, MA 02139 

16.001/16.002 Unified Engineering I, II Fall 2008

## Problem Set 6

Name: $\qquad$

Due Date: 10/17/2008

|  | Time Spent <br> (min) |
| :--- | :---: |
| F7 |  |
| M8 |  |
| M9 |  |
| T11 |  |
| T12 |  |
| T13 |  |
| Study <br> Time |  |

Announcements:

| Unified Engineering | Fall 2008 |
| :--- | :--- |
| Fluids | P. Lozano |

Problem F7
Consider the following density field in a Cartesian coordinate frame:

$$
\rho=\operatorname{At} \exp \left[-\frac{(x+y)}{L}\right]
$$

where $A$ and $L$ are constants. The $x$-component of the velocity vector is $u=2 \frac{L}{t}$.
(a) Find the $y$-component of the velocity vector. Write the complete vectorial expression.
(b) Calculate the substantial derivative of the density field.
(c) How is the density of the fluid at a given location changing with time? Explain.
(d) How is the density of a given fluid element changing with time? Explain.

## Unified Engineering Thermodynamics \& Propulsion

Fall 2008
Z. S. Spakovszky
(Add a short summary of the concepts you are using to solve the problem)

## Problem T11

Recently ecologists have expressed considerable concern over the thermal pollution produced by electrical power plants. However, there is another source of thermal pollution. In the summer, the increase in electrical power consumption is considerable. The bulk of this increase is used to operate air conditioning systems. For example, in New-York City, statistics compiled by the Consolidated Edison Co. show that the daily increase in electrical power consumption during the summer months is $2 \times 10^{9}$ watts. Let us assume that all of this increase is used to operate air conditioning systems. Let us also assume that we can model these air conditioning systems as Carnot refrigerators that operate in cycles while maintaining the temperature of the air conditioned spaces at a steady temperature of $21^{\circ} \mathrm{C}$. Model the environment (atmosphere) as a heat reservoir at $33^{\circ} \mathrm{C}$.
a) Estimate the rate of thermal pollution (watts) of the environment (atmosphere) due to these air conditioning systems.
b) Does this estimate represent a lower bound or upper bound on the rate of thermal pollution? Explain your answer.

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## Problem T12

An ideal Diesel engine operates on $0.01 \mathrm{~m}^{3}$ (measured at state 1) of air. Let $p_{1}=100 \mathrm{kPa}$, $T_{1}=60^{\circ} \mathrm{C}, r=14$, and let the cutoff be at $6 \%$ of the stroke (where cutoff is defined as $V_{3^{-}}$ $V_{2}$ ). You can assume $\gamma=1.4$ and $R=287 \mathrm{~J} / \mathrm{kg}-\mathrm{K}$ for air.
a) Draw the $p-V$ diagram and label all states.
b) Find $T_{2}, P_{2}, V_{2}, T_{3}, V_{3}, P_{4}$ and $T_{4}$.
c) Determine $Q_{i n}$ and $Q_{\text {out }}$.
d) Find $W$ and $\eta_{t h}$.

intake

compression
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power


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## Problem T13

A proposed Brayton cycle will use energy input from a thermonuclear (fusion) reactor employing lithium as an intermediate heat transfer fluid. The cycle is described in the figure below. The molecular mass of helium (monatomic gas) is $4 \mathrm{~g} / \mathrm{mol}$. Pressure losses in the heat exchangers can be neglected and the turbomachinery (compressors and turbine) can be assumed ideal. Changes in kinetic energy can also be neglected. Note: the regenerator effectiveness, $\varepsilon$, is defined as the ratio of actual temperature rise to the maximum possible temperature rise.
a) Sketch the p-V diagram for this cycle and label all states (include local isotherms). Also mark heat and work interactions in your diagram.
b) What is the heat transfer per unit helium mass flow, $q=\dot{Q} / \dot{m}$ in the heat exchangers?
c) Calculate the thermal efficiency of the cycle.
d) What is the required helium flow per MW of net cycle power?


M8 (M7.1) (10 M-points) A rigid bar of length 6L is supported by three springs as shown. The springs at the left and right hand supports are of the linear type with the spring located at the left support having a constant of 3 k , and the spring at the right support having a constant of k . The other spring at the center of the bar is of the torsional type and requires a moment of the same sense to the angle of rotation for displacement giving a constitutive relation of: $M=k_{T} \theta$, where $k_{T}$ is the torsional spring constant. This is in addition to any reaction(s) due to the pin support at that point. The bar is loaded due to the weight from gravity considerations of a linearly distributed mass across the full length of the bar with a total mass magnitude of M.

(a) Draw the free body diagram(s) for this situation (Consider the overall system and any appropriate subsystems).
(b) Determine whether this structural configuration is statically determinate or statically indeterminate and clearly explain your reasoning.
(c) Determine the manifestation of the Compatibility of Displacement for this configuration.
(d) Determine the reaction forces, the deflection of each spring, and the overall deflection of the bar.

M9 (M7.2) (10 M-points) A five-member truss arrangement between two walls in a building structure is used to resist a vertical load applied in the negative direction as in the illustration. Three bars are attached to a common pin support on the left wall, and each is attached to individual roller supports on the right wall. Bars, of length L, are also attached between these roller supports. The wall separation is also L. Each bar has the same cross-sectional area, A, and material modulus of elasticity, E, varying only in length and angle of orientation.

(a) Determine the manifestation of the Compatibility of Displacement for this configuration in terms of the deflections of each of the roller supports, and the associated deflections of each of the bars.
(b) If it is assumed that the deflections are small, can anything be done to linearize these equations? Clearly explain and show resulting equations.
(c) Comment on how the approach to solve an overall set of equations involving those from the Compatibility of Displacement is effected in the case where the deflections can be assumed "small" versus when they become relatively "large".

