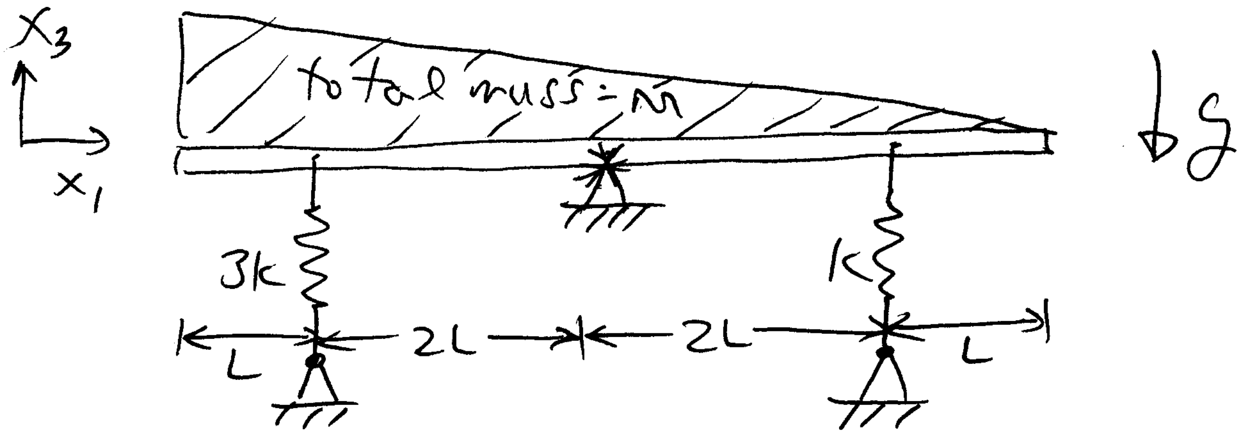
PAL
10/9/08

Unified Engineering Problem Set 6
Week 7 Fall, 2008
SOLUTIONS

M8 (M7.1)



(a) Overall there are three pin supports which provide reactions for this system. Label these as A, B, C. But before drawing the Free Body Diagram, need an expression for the mass distribution and the resulting load.

With $x_1 = 0$ at tip of bar:

$$\int_0^{6L} m(x_1)\,dx_1 = M$$

total mass = M

and $m(x_1) = 0$ at $x_1 = 6L$ and is linear.

So:

$$m(x_1) = a x_1 + b$$

using the integral:

$$\int_0^{6L} (a x_1 + b)\, dx_1 = M$$

$$\Rightarrow \quad \frac{a}{2} x_1^2 + b x_1 \Big]_0^{6L} = M$$

$$\Rightarrow \quad 18aL^2 + 6bL = M$$

using $m(6L) = 0 \Rightarrow 0 = 6La + b$

giving $b = -6La$

use that in the equation for M:

$$18aL^2 - 36aL^2 = M$$

$$\Rightarrow \quad a = -\frac{M}{18L^2}$$

giving $b = \dfrac{M}{3L}$

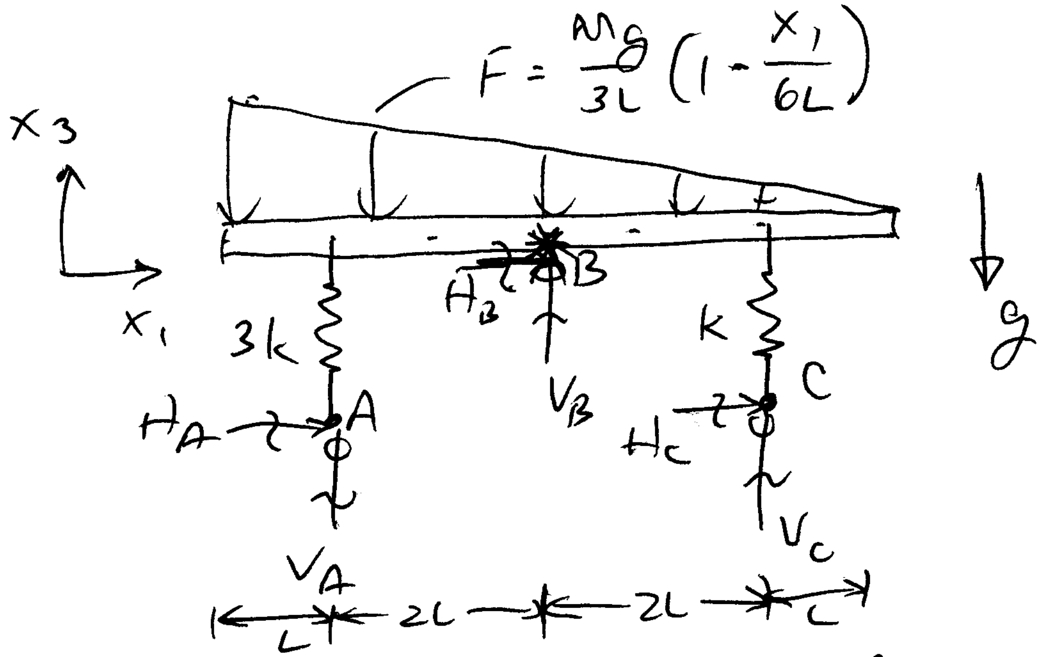
finally:

$$m(x_1) = \frac{M}{3L}\left(1 - \frac{x_1}{6L}\right)$$

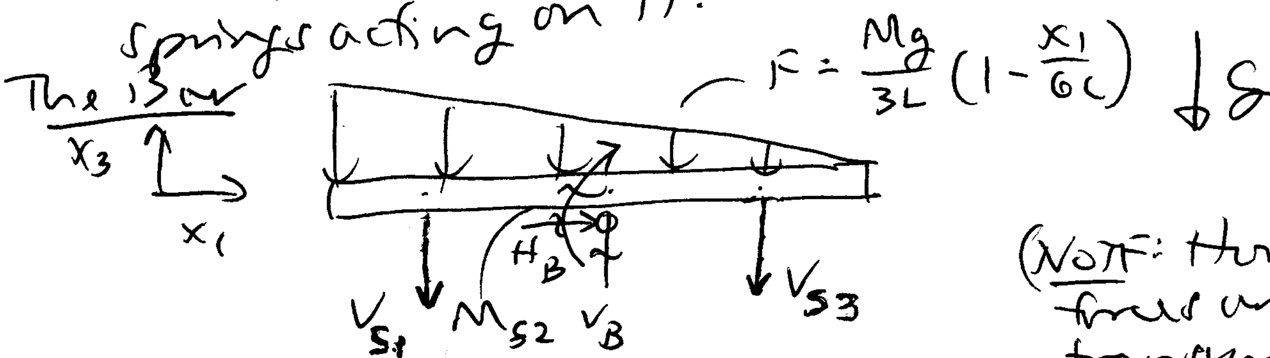
resulting in a loading due to the mass of:

$$\frac{Mg}{3L}\left(1 - \frac{x_1}{6L}\right)$$

Draw the Free Body Diagram:



This can be looked at as four different subsystems: the bar and each of the three springs acting on it:

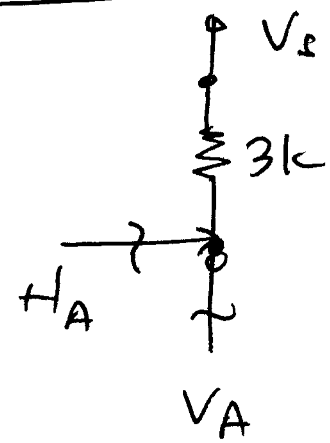


(NOTE: Horizontal forces are not transmitted through linear springs)

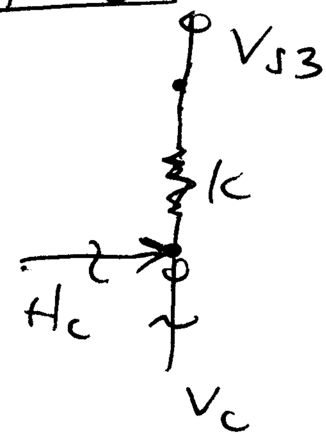
The left and right springs provide vertical forces. The center (torsional) spring provides a moment. The forces or moments on the bar from the springs must be equal and opposite to those on the springs. [Sum to zero for the overall system]

Looking at the springs:

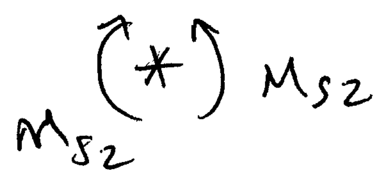
Spring 1



Spring 3



Spring 2



(Final Note: Put these 4 subsystems together and the internal spring loads sum to zero and the overall FBD results)

(b) This is a 2-D system and has a potential for 3 degrees of freedom of motion:

- lateral in x_1
- lateral in x_3
- rotational in $x_1 - x_3$ plane (about x_2)

There are 6 reactions.

reactions > # d.o.f.

$\Rightarrow$ $\boxed{\text{statically indeterminate}}$

(c) There are two parts that make up the Compatibility of Displacement for this configuration. The primary is that the bar is rigid. Thus, the displacement in x_1 ~~must be~~ a linear function of x_1:

$$\delta_3 = m x_1 + b$$

displacement in x_3 with $x_1 = 0$ at left end

Now look at further details. The bar is pinned with the torsional spring at $x_1 = 3L$ $\Rightarrow$ $\delta_3 = 0$ at $x_1 = 3L$

Place in expression for δ_3:

$$0 = 3Lm + b$$

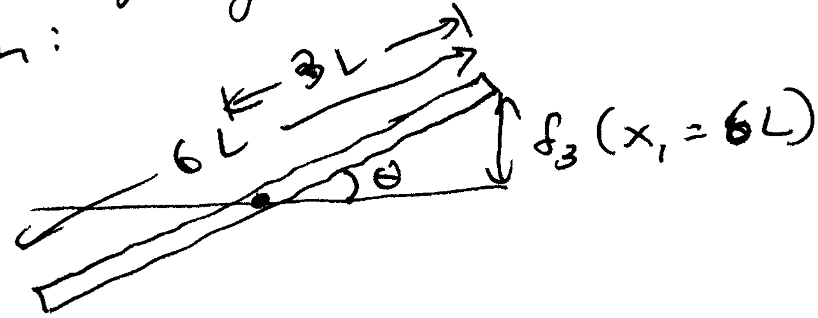
$$\Rightarrow b = -3Lm$$

So can write:

$$\boxed{\delta_3 = m(x_1 - 3L)} \qquad (1)$$

This does not yet give δ_3. Combination with the other Principles is needed.

One can also get an expression for the rotation of the bar to use with the rotational springs: Consider the bar deflection:



$$\Rightarrow (3L)\sin\theta = \delta_3(x_1 = 6L)$$

using the equation for δ_3:

$$(3L)\sin\theta = 3Lm$$

$$\Rightarrow \quad \theta = \sin^{-1}(m)$$

for small displacements:

$$\boxed{\theta = m} \qquad (2)$$

Finally, get the displacements at the location of the two linear springs. This will be needed subsequently:

Spring 1: $\delta_3(x_1 = L) = -2L\,m$ (1a)

Spring 3: $\delta_3(x_1 = 5L) = 2L\,m$ (1b)

(d) Now use all the equations.

→ First Equilibrium. Doing this for the bar system:

$$\sum F_{x_1} = 0 \ \xrightarrow{+} \ \Rightarrow \ H_B = 0$$

$$\sum F_{x_3} = 0 \ \uparrow + \ \Rightarrow \ -V_{S1} + V_B - V_{S3} - Mg = 0 \quad (3)$$

$$\sum M_B = 0 \ \circlearrowleft + \ \Rightarrow \ +V_{S1}(2L) - V_{S3}(2L) - M_{S2} + \int_0^{6L} \frac{Mg}{3L}\left(1 - \frac{x_1}{6L}\right)(3L - x_1)\,dx_1 = 0$$

working this:

$$\int = \int_0^{6L} \frac{Mg}{3L}\left(3L - \frac{x_1}{2} - x_1 + \frac{x_1^2}{6L}\right)dx_1$$

$$= \frac{Mg}{3L}\int_0^{6L}\left(3L - \frac{3x_1}{2} + \frac{x_1^2}{6L}\right)dx_1$$

$$= \frac{Mg}{3L}\left(3Lx_1 - \frac{3x_1^2}{4} + \frac{x_1^3}{18L}\right)\Bigg]_0^{6L}$$

$$= \frac{Mg}{3L}\left(18L^2 - 27L^2 + 12L^2\right) = MgL$$

$$\Rightarrow \quad V_{s_1}(2L) - V_{s3}(2L) + MgL - M_{s2} = 0 \quad (4)$$

for spring 1:

$$\Sigma F_{x_1} = 0 \quad \xrightarrow{+} \quad \Rightarrow \quad H_A = 0$$

$$\Sigma F_{x_3} = 0 \quad \uparrow + \quad \Rightarrow \quad V_A + V_{s_1} = 0$$

$$\Rightarrow V_A = -V_{s1} \qquad (5)$$

no moments

for spring 3:

$$\Sigma F_{x_1} = 0 \quad \xrightarrow{+} \quad \Rightarrow \quad H_c = 0$$

$$\Sigma F_{x_3} = 0 \quad \uparrow + \quad \Rightarrow \quad V_c + V_{s3} = 0$$

$$\Rightarrow V_c = -V_{s3} \qquad (6)$$

again, no moments

for spring 2:

$$\Sigma M_B = 0 \quad \circlearrowleft + \quad \Rightarrow \quad M_{s2} - M_{s2} = 0$$

$$\Rightarrow M_{s2} = M_{s2} \quad \text{(somewhat obvious...)}$$

→ finally, include the Constitutive Relations:

Spring 1: $V_{s_1} = 3k\,\delta(x_1 = L)$ (7)

Spring 2: $M_{s2} = k_T\,\theta$ (8)

Spring 3: $V_{s3} = k\,\delta(x_1 = 5L)$ (9)

→ with the Compatibility equations from part (c) -- equations (1a) (1b) (2)

There are 10 equations with the following unknowns:

$\delta_3(x_1 = L)$, $\delta_3(x_1 = 5L)$, θ, m, V_A, V_{S1}, V_B, M_{S2}, V_C, V_{S3} $\Rightarrow$ 10 unknowns

So progress to work through the equations already having shown: $H_A = H_B = H_C = 0$ (no applied horizontal loads)

use (15) in (9) $\Rightarrow V_{S3} = 2kmL$ (10)

and in (6) $\Rightarrow V_C = -2kmL$ (11)

use (1a) in (7) $\Rightarrow V_{S1} = -6kmL$ (12)

and in (5) $\Rightarrow V_A = 6kmL$ (13)

use (2) in (8) $\Rightarrow M_{S2} = k_T m$ (14)

Now use these in (4):

$$(-6kmL)(2L) - (2kmL)(2L) + MgL - k_T m = 0$$

$$\Rightarrow -12kmL^2 - 4kmL^2 + MgL - k_T m = 0$$

proceeding:

$$-m\left(k_T + 16kL^2\right) = -MgL$$

giving:

$$m = \frac{MgL}{\left(k_T + 16kL^2\right)} \qquad (15)$$

(Notes: • m is positive ⇒ a positive slope giving a negative displacement on left half of beam checking with greater amount of mass there

• m approaches zero as either spring constant approaches infinity.)

Use this result for m with others in (3):

$$-V_{S1} + V_B - V_{S3} - Mg = 0$$

$$\Rightarrow \quad +6kmL + V_B - 2kmL - Mg = 0$$

$$\Rightarrow V_B + kL\left(\frac{MgL}{k_T + 16kL^2}\right) - Mg = 0$$

giving:

$$V_B = Mg - 4kL\left(\frac{MgL}{k_T + 16kL^2}\right)$$

or expressed as:

$$V_B = Mg - \frac{4MgkL^2}{k_T + 16kL^2} \qquad (16)$$

and using (15) in (11) and (13):

$$V_c = -\frac{2MgkL^2}{k_T + 16kL^2}$$

$$V_A = +\frac{6MgkL^2}{k_T + 16kL^2}$$

Check: $V_A + V_B + V_c \stackrel{?}{=} Mg$

$$+\frac{6MgkL^2}{(k_T + 16kL^2)} + Mg - \frac{4MgkL^2}{(k_T + 16kL^2)}$$

$$-\frac{2MgkL^2}{(k_T + 16kL^2)} \stackrel{?}{=} Mg$$

✓ checks

summarizing.... (see next page):

$$V_A = \frac{6 M g k L^2}{(k_T + 16 k L^2)}$$

$$V_B = Mg - \frac{4 M g k L^2}{(k_T + 16 k L^2)}$$

$$V_C = - \frac{2 M g k L^2}{(k_T + 16 k L^2)}$$

spring 1: $\delta_3(x_1 = L) = - \frac{2 M g L^2}{(k_T + 16 k L^2)}$

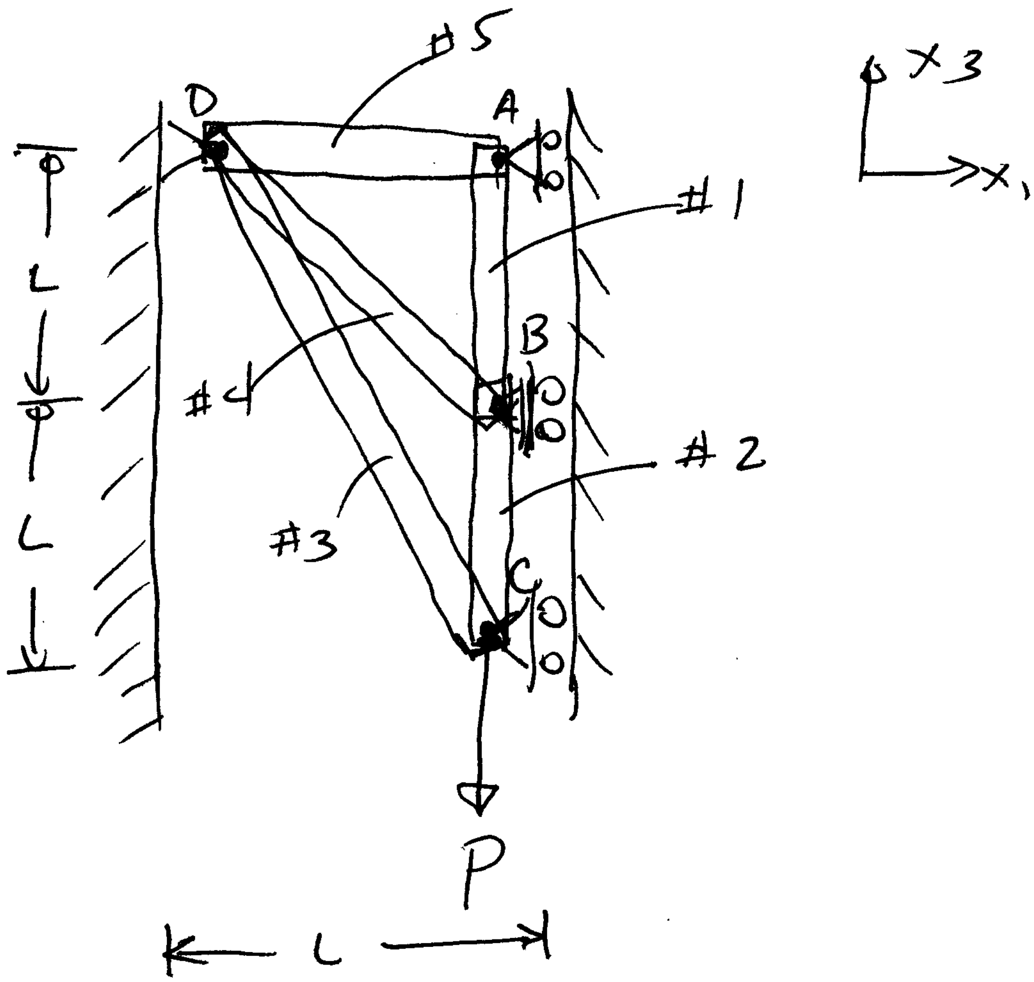
spring 2: $\theta = \frac{M g L}{(k_T + 16 k L^2)}$

spring 3: $\delta_3(x_1 = 5L) = + \frac{2 M g L^2}{(k_T + 16 k L^2)}$

overall bar displacement:

$$\delta_3 = \frac{M g L}{(k_T + 16 k L^2)} (x_1 - 3L)$$

M9 (M7.2)



(a) Manifestation of "compatibility of Displacement" in this case is $\boxed{\text{Compatibility at the joints}}$. All items connected at a joint must displace the same way at the joint.

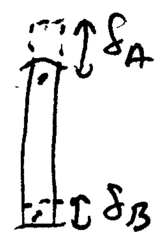
First define the displacement at each joint. Note that due to the supports, no displacement in x_1 can occur. All displacements considered are in x_3:

→ D is pinned, so $\delta_D = 0$

→ A, B, and C are rollers, so they do displace in x_3. Represent these displacements as $\delta_A, \delta_B, \delta_C$ (+ with respect to x_3)

Now consider the change in length (overall displacement) of each bar and define such as δ_{bar} (not just in x_3 but overall change). This can be equated to the displacements of the joints for each case:

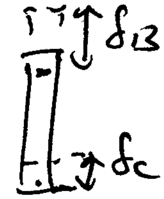
<u>Bar #1</u> moves along x_3:



Using the relative positive definition of the joint displacements:

$$\delta_{BAR_1} = \delta_A - \delta_B \quad (1)$$

<u>Bar #2</u> also moves along x_3:



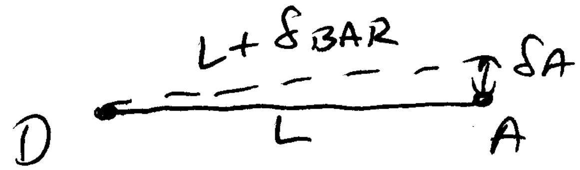
In a similar manner:

$$\delta_{BAR_2} = \delta_B - \delta_C \qquad (2)$$

In looking at the other three bars, one endpoint (D) does not move while the second endpoint does move (in x_3). Furthermore, the two endpoints are at different locations in x_1. Thus, the angle of the bar relative to the $x_1 - x_3$ system will change with displacement.

→ Consider each case . . . . .

<u>Bar #5</u> has movement only at one end and initially is at a 90° angle. So the geometry is:



Looking at this right triangle and squaring sides gives:

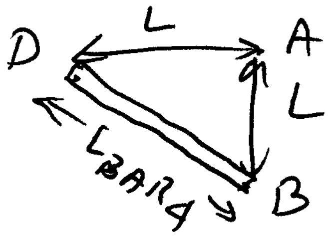
$$L^2 + \delta_A^2 = (L + \delta_{BAR_5})^2$$

working this yields:

$$L^2 + \delta_A^2 = L^2 + 2L\delta_{BAR_5} + \delta_{BAR_5}^2$$

$$\Rightarrow \delta_A^2 = \delta_{BAR_5}^2 + 2L\delta_{BAR_5} \qquad (3)$$

<u>Bar #4</u> is different as it starts at an angle. It is therefore necessary to determine its original length:

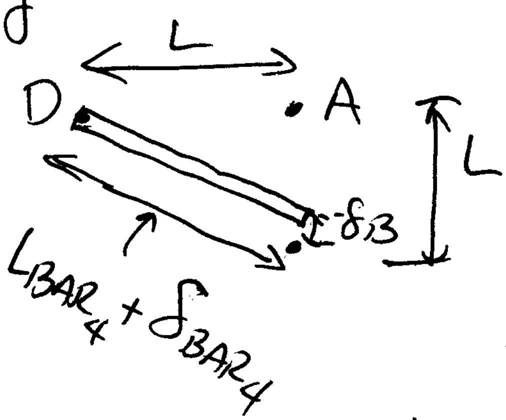


$$\Rightarrow L_{BAR_4}^2 = L^2 + L^2$$

giving:

$$L_{BAR_4} = 1.414L \quad (= \sqrt{2}L)$$

This tip of Bar 4 moves at B by δ_B. This geometry is now:



The side leg from A to the new point of B is $(L - \delta_B)$. Now square sides:

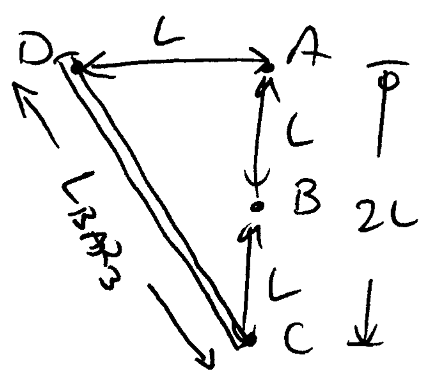
$$L^2 + (L - \delta_B)^2 = (L_{BAR_4} + \delta_{BAR_4})^2$$

proceeding and using the value for L_{BAR_4}:

$$L^2 + L^2 - 2L\delta_B + \delta_B^2 = 2L^2 + 2.828 L \delta_{BAR_4} + \delta_{BAR_4}^2$$

$$\Rightarrow \delta_B^2 - 2L\delta_B = \delta_{BAR_4}^2 + 2.828 L \delta_{BAR_4} \quad (4)$$

<u>Bar #3</u> is similar to consider as Bar #4. First to determine its original length:

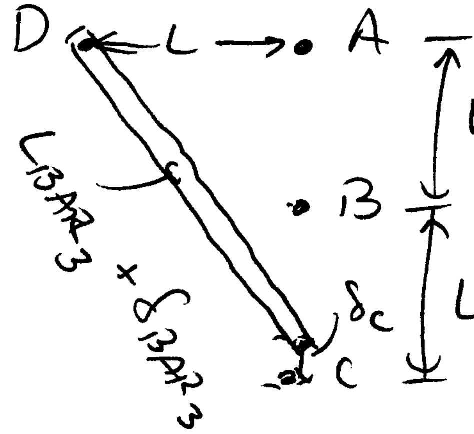


squaring sides gives:

$$L_{BAR_3}^2 = L^2 + (2L)^2 = 5L^2$$

$$\Rightarrow L_{BAR_3} = \sqrt{5}\,L = 2.237\,L$$

The tip of Bar 3 at C moves by δ_C. This geometry is similar to Bar 4 and becomes:



The side leg from A to the displaced point C is $(2L - \delta_c)$. Now square sides:

$$L^2 + (2L - \delta_c)^2 = (L_{BAR_3} + \delta_{BAR_3})^2$$

proceeding and using the value for L_{BAR_3}:

$$L^2 + 4L^2 - 4L\delta_c + \delta_c^2 = 5L^2 + 4.474L\delta_{BAR_3} + \delta_{BAR_3}^2$$

$$\Rightarrow \delta_c^2 - 4L\delta_c = \delta_{BAR_3}^2 + 4.474L\delta_{BAR_3} \quad (5)$$

Summarizing, the working equations of the Compatibility of Displacement are:

$$\boxed{\begin{array}{ll} \delta_{BAR_1} = \delta_A - \delta_B & (1) \\ \delta_{BAR_2} = \delta_B - \delta_C & (2) \\ \delta_{BAR_5}^2 + 2L\delta_{BAR_5} = \delta_A^2 & (3) \\ \delta_{BAR_4}^2 + 2.828L\delta_{BAR_4} = \delta_B^2 - 2L\delta_B & (4) \\ \delta_{BAR_3}^2 + 4.474L\delta_{BAR_3} = \delta_c^2 - 4L\delta_c & (5) \end{array}}$$

with all node displacements ($\delta_A, \delta_B, \delta_C$) defined as positive in the $+x_3$ direction and position bar deflections defined as elongation.

<u>NOTE</u>: Can check units are compatible in each equation. For (1) and (2) it is [L] ✓; for (3), (4) and (5) it is [L^2] ✓.

(b) If it is assumed that the deflections are small, any higher order terms (H.O.T.'s) in the deflections can be linearized by considering first order effects.

→ There are no changes for Bar #1 or Bar #2 in equations (1) and (2)

→ For Bar #5, one can equate the joint deflection with that of the bar:

$$\delta_{BAR_5} = \delta_A \qquad (3)'$$

(similar to ignoring $2L\delta_{BAR_5}$ in (3))

→ For Bar #4, a similar linearization is made through the equations and ignoring higher order terms in equation (4):

$$2.828\, L\delta_{BAR_4} = -2L\delta_B$$

$$\Rightarrow \quad \delta_{BAR_4} = -0.707\,\delta_B \qquad (4)'$$

→ Similarly for Bar #3, higher order terms are ignored in equation (5):

$$4.474L\,\delta_{BAR_3} = -4L\,\delta_c$$

$$\Rightarrow \quad \delta_{BAR} = -0.894\,\delta_c \qquad (5)'$$

Summarizing the linearized equations:

$$\boxed{\begin{aligned} \delta_{BAR_1} &= \delta_A - \delta_B && (1) \\ \delta_{BAR_2} &= \delta_B - \delta_c && (2) \\ \delta_{BAR_5} &= \delta_A && (3)' \quad * \\ \delta_{BAR_4} &= -0.707\,\delta_B && (4)' \\ \delta_{BAR_3} &= -0.894\,\delta_c && (5)' \end{aligned}}$$

* Notes: • One can also approximate $\delta_{BA5} \simeq 0$

• There are different ways to linearize the equations and this will result in different overall results. Only full inclusion results in the actual results...)

(c) "Small" deflections allow a linearization to take place. This then allows a linear solution of the full set of equations. If the deflections are "large" and the higher order (nonlinear) equations must be used, the overall set of governing equations are quite nonlinear and would need to be solved using numerical approximations or other techniques.

Solutions F7

Mass continuity:

$$\begin{cases} \int \rho \bar{V}\cdot\hat{n}\, dA + \frac{\partial}{\partial t}\int \rho\, d\mathcal{V} = 0 \\ \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\bar{V}) = 0 \quad (*) \\ \frac{D\rho}{Dt} + \rho\,\nabla\cdot\bar{V} = 0 \end{cases}$$

$$\rho = At \exp\left[\frac{-(x+y)}{L}\right]$$

(a) To find the velocity vector, we can use (*)

$$\frac{\partial \rho}{\partial t} = \frac{\rho}{t} \quad \text{and} \quad \nabla\cdot(\rho\bar{V}) = \rho\,\nabla\cdot\bar{V} + \bar{V}\cdot\nabla\rho$$

$$= \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] + \left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right]$$

All together:

$$\frac{\rho}{t} + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = 0$$

now $\frac{\partial u}{\partial x} = 0$ since $u = 2\frac{L}{t}$; $\quad \frac{\partial \rho}{\partial x} = -\frac{\rho}{L} = \frac{\partial \rho}{\partial y}$

then:

$$\frac{\rho}{t} + \rho\frac{\partial v}{\partial y} - \frac{2\rho}{t} - v\frac{\rho}{L} = 0$$

therefore:

$$\frac{dv}{dy} = \frac{v}{L} + \frac{1}{t} = \frac{1}{L}\left(v + \frac{L}{t}\right)$$

Integrate:

$$\int \frac{dv}{v + L/t} = \int \frac{dy}{L} \Rightarrow \ln\left(\frac{v + L/t}{v_0 + L/t_0}\right) = \frac{y - y_0}{L}$$

or

$$\boxed{v = \left(v_0 + \frac{L}{t_0}\right)e^{\frac{y-y_0}{L}} - \frac{L}{t}}$$

(v_0, t_0, y_0 are constants)

The velocity vector is then:

$$\boxed{\bar{V} = 2\frac{L}{t}\,\hat{\imath} + \left[\left(v_0 + \frac{L}{t_0}\right)e^{\frac{y-y_0}{L}} - \frac{L}{t}\right]\hat{\jmath}}$$

(b) Calculate $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \bar{V}\cdot\nabla\rho$ $\quad$ As before, $\frac{\partial \rho}{\partial t} = \frac{\rho}{t}$

and $\nabla\rho = \frac{\partial \rho}{\partial x}\hat{\imath} + \frac{\partial \rho}{\partial y}\hat{\jmath} = -\frac{\rho}{L}\hat{\imath} - \frac{\rho}{L}\hat{\jmath}$

(b) contd'.

$$\bar{v} \cdot \nabla \rho = -\frac{\rho}{L}\left(2\frac{L}{t}\right) - \frac{\rho}{L}\left(\left[v_0 + \frac{L}{t_0}\right] e^{\frac{y-y_0}{L}} - \frac{L}{t}\right)$$

then $$\frac{D\rho}{Dt} = \cancel{\frac{\rho}{t}} - \cancel{\frac{2\rho}{t}} + \cancel{\frac{\rho}{t}} - \boxed{\frac{\rho}{L}\left[\left(v_0 + \frac{L}{t_0}\right) e^{\frac{y-y_0}{L}}\right]}$$

(c) At a given location (x,y) the density variation with time is given by: $\frac{\partial \rho}{\partial t} = \frac{\rho}{t} = A\exp\left[\frac{-(x+y)}{L}\right]$

which for $A \neq 0$ and L finite is always $\neq 0$ (except at $x+y \to \infty$)

So in every point, there is a <u>local</u> density variation.

(d) If we follow a fluid element (Lagrangian view) its change of density with time is given by: $\frac{D\rho}{Dt}$

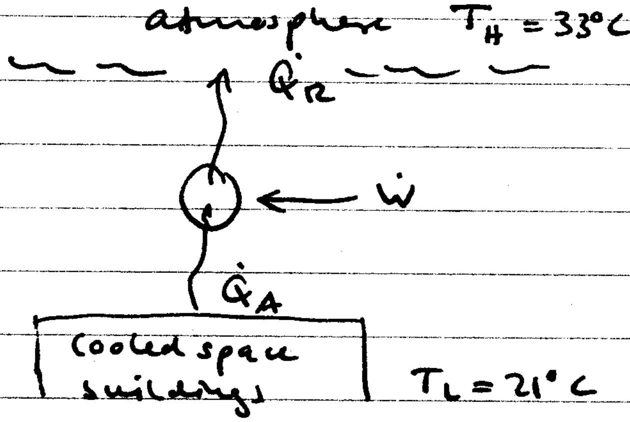
$$\frac{D\rho}{Dt} = -\frac{\rho}{L}\left[\left(v_0 + \frac{L}{t_0}\right) e^{\frac{y-y_0}{L}}\right]$$

(c) and (d) are not the same, in particular if we select intial conditions such that $v_0 = -\frac{L}{t_0}$, we could have $\frac{D\rho}{Dt} = 0$ <u>everywhere</u>

This is a very interesting result: even though the density at a point might be changing, it does not necessarily mean that the density of a fluid element needs to change as well.

T 11

16. Unified Fall 08



Assume: - Carnot refrig. cycle
- daily power for air conditioning $\dot{W} = 2 \times 10^9$ W

a) Carnot Refrig.: $COP_R^C = \dfrac{\dot{Q}_A}{\dot{W}}$ 1st law: $0 = \dot{Q}_A - \dot{Q}_R + \dot{W}$

$COP_R^C = \dfrac{\dot{Q}_A}{\dot{Q}_R - \dot{Q}_A}$; carnot: $\dfrac{\dot{Q}_A}{T_L} = \dfrac{\dot{Q}_R}{T_H}$

find $COP_R^C = \dfrac{T_L}{T_H - T_L}$ so $\dot{Q}_R = \dot{Q}_A + \dot{W}$, $\dot{Q}_A = \dot{W}\dfrac{T_L}{T_H - T_L}$

$\dot{Q}_R = \dot{W}\left(1 + \dfrac{T_L}{T_H - T_L}\right)$ $\underline{\dot{Q}_R = 5.1 \times 10^{10} \text{ W}}$ daily

b) COP_R^C is an upper bound (best one could do since using Carnot cycle)

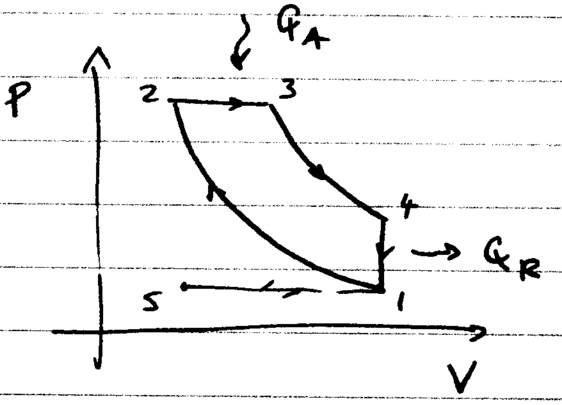
$COP_R^{real} < COP_R^C$ $COP_R = \dfrac{\dot{Q}_A}{\dot{W}} = \dfrac{\dot{Q}_R - \dot{W}}{\dot{W}} = \dfrac{\dot{Q}_R}{\dot{W}} - 1$

so $\dfrac{\dot{Q}_R^{real}}{\dot{W}} < \dfrac{\dot{Q}_R^{C}}{\dot{W}}$

The estimated pollution is an <u>upper</u> bound

T12 16. Unified Fall 08

a)



Assume: - ideal diesel cycle
- air is working fluid
- cut-off 6% of displacement

$T_1 = 333\,K$, $r = 14$

$p_1 = 1\,bar$, $V_1 = 0.01\,m^3$

b) $1 \to 2$ ad. rev process: $\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}$, $\underline{T_2} = r^{\gamma-1} T_1 = \underline{957\,K}$

$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$, $\underline{p_2} = p_1 (r)^{\gamma} = \underline{40.2\,bar}$, $\underline{V_2} = \frac{V_1}{r} = \underline{7.14 \cdot 10^{-4}\,m^3}$

$V_3 - V_2 = 0.06(V_1 - V_2)$ so $V_3 = V_2 + 0.06(V_1 - V_2)$ $\underline{V_3 = 1.27 \cdot 10^{-3}\,m^3}$

$m = V_1 \frac{p_1}{RT_1} = V_3 \frac{p_2}{RT_3}$ $T_3 = \frac{V_3}{V_1} \frac{p_2}{p_1} T_1$ $\underline{T_3 = 1701\,K}$

$\frac{T_4}{T_3} = \left(\frac{V_3}{V_1}\right)^{\gamma-1}$ so $T_4 = T_3 \cdot \left(\frac{V_3}{V_1}\right)^{\gamma-1} \longrightarrow \underline{T_4 = 745\,K}$

and $p_4 = p_3 \left(\frac{V_3}{V_1}\right)^{\gamma} \longrightarrow \underline{p_4 = 2.23\,bar}$

c) $m = V_1 \frac{p_1}{RT_1} = 0.0105\,kg$ $\underline{Q_A} = m c_p (T_3 - T_2) = \underline{7.82\,kJ}$

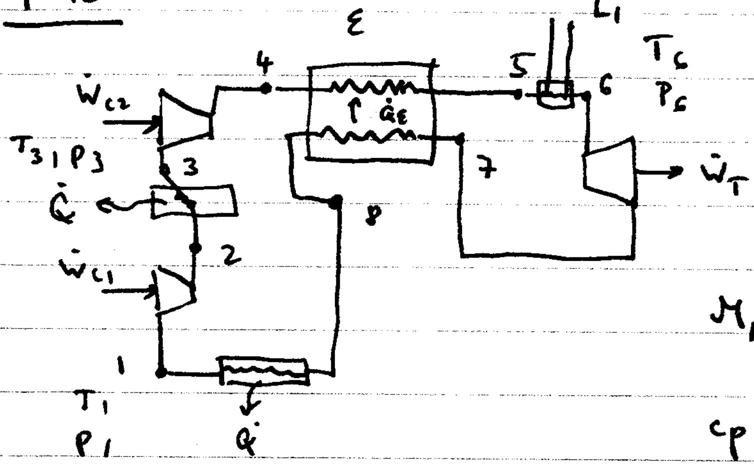
$c_p = \frac{\gamma}{\gamma-1} R = 1004.5\,J/kg\text{-}K$ $\underline{Q_R} = m c_v (T_4 - T_1) = \underline{3.09\,kJ}$

$c_v = \frac{1}{\gamma-1} R = 717.5\,J/kg\text{-}K$

d) $W = Q_A - Q_R \longrightarrow \underline{W = 4.73\,kJ}$

$\eta_{th}^{Diesel} = \frac{W}{Q_A} \longrightarrow \underline{\eta_{th}^{Diesel} = 0.6}$

T 13   16. Unified Fall 08

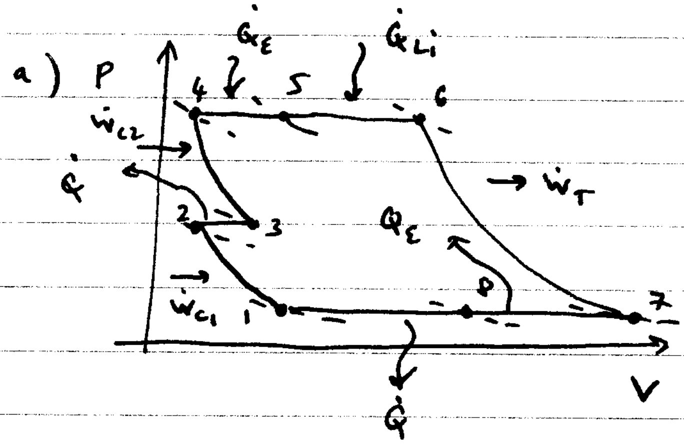


Assume: - ideal cycle
- He working fluid
- heat exchange @ p=const

$\mathcal{M}_{He} = 4$ g/mol $R = \frac{\mathcal{R}}{\mathcal{M}} = 2078 \ \mathrm{J/kg\text{-}K}$

$c_p = \frac{5}{2} R = 5195 \ \mathrm{\frac{J}{kg\text{-}K}}, \ \gamma = \frac{5}{3} = 1.667$

a)



b) find $q = \frac{\dot Q}{\dot m} = c_p(T_8 - T_1)$
$= c_p(T_2 - T_3)$

→ find T_8! or T_2!

$T_4 = T_3 \left(\frac{P_6}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$, $\underline{T_4 = 421 \text{ K}}$

$T_7 = T_6 \left(\frac{P_1}{P_6}\right)^{\frac{\gamma-1}{\gamma}}$, $\underline{T_7 = 771 \text{ K}}$

$T_2 = T_1 \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$, $\underline{T_2 = 396 \text{ K}}$ so $q = c_p(T_2 - T_3)$ $\underline{q = 379.2 \text{ kJ/kg}}$

c) $\eta_{th} = \frac{w_T - w_{c1} - w_{c2}}{q_{Li}} = \frac{T_6 - T_7 - (T_2 - T_1) - (T_4 - T_3)}{T_6 - T_5}$

find T_5: $\varepsilon = \frac{T_5 - T_4}{T_7 - T_4}$ $\underline{T_5 = 736 \text{ K}}$

find $\underline{\eta_{th} = 0.598}$

d) $\dot m_{He} = \frac{\dot W_{net}}{w_T - w_{c1} - w_{c2}} = \frac{10^6}{c_p(T_6 - T_7 - T_2 + T_1 - T_4 + T_3)}$

$\underline{\dot m_{He} = 0.599 \text{ kg/s}}$ per MW net power