Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.001/16.002 Unified Engineering I, II
Fall 2008

Problem Set 7

Name: ________________________

Due Date: 10/24/2008

	Time Spent (min)
M10	
M11	
M12	
M13	
T14	
T15	
T16	
Study Time	

Announcements:

Unified Engineering --
Problem Set 7 **Lectures: M12, M13, M14** ***(w/look-ahead)***
Week 8 Fall, 2008 **Units: M2.1, M2.2** ***(w/look-ahead)***

M10 (M8.1) ***(5 points)*** Write out the following tensor equations in full.
Show all appropriate steps.

(**Note:** these equations do not necessarily have any real meaning)

(a) $E = 1/2\, \sigma_{\alpha\beta}\, \varepsilon_{\alpha\beta}$

(b) $D_{ab}\, (\partial h / \partial x_b) = g_a$

(c) $\sigma_{23} = \ell_{2i'}\, \ell_{3j'}\, \sigma'_{i'j'}$

(d) $C_n = d_{\sigma\gamma}\, S_{\sigma\gamma}\, p_n$

(e) $A_{rs} = Q_{rstv}\, \alpha_t\, y_v$ (for r = 3, s = 2)

M11 (M8.2) ***(5 points)*** Write out the succinct tensor equation that describes the following notation. Clearly explain all steps in arriving at the equation.

$$\begin{bmatrix} P_{111} & 2P_{112} & P_{122} \\ P_{211} & 2P_{212} & P_{222} \\ P_{311} & 2P_{312} & P_{322} \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{12} \\ S_{22} \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix}$$

M12 **(M8.3)** ***(10 points)*** A second axis system is defined by rotating the x_1- x_3 plane by an angle θ of +55° about the x_2 axis. A force vector, $\underline{F}$, is described in the original axis system as (NOTE: The vector has units of Newtons):

$$\underline{F} = -6N\,\underline{i}_1 + 3N\,\underline{i}_2 + 2N\,\underline{i}_3$$

(a) Determine the expression for the vector in the rotated coordinate system.

(b) Prove, by some means, that these two expressions are equivalent.

M13 **(M8.4)** ***(10 points)*** Consider the stress equations of equilibrium.

(a) Write these equations in engineering notation.

(b) Reduce the full three-dimensional equations to their plane stress form.

(Add a short summary of the concepts you are using to solve the problem)

Problem T14

Consider an ideal Brayton cycle in which the compressor inlet air conditions are 1 bar and 300K and the maximum cycle temperature is 1200K. You can assume air to be the working fluid with $\gamma = 1.4$ and $R = 287$ J/kg-K.

a) Sketch the p-V diagram for a number of Brayton cycles operating between the same inlet conditions and maximum cycle temperature. Indicate the cycle that maximizes the net cycle work.

b) Find the compressor outlet pressure that will result in maximum net cycle work.

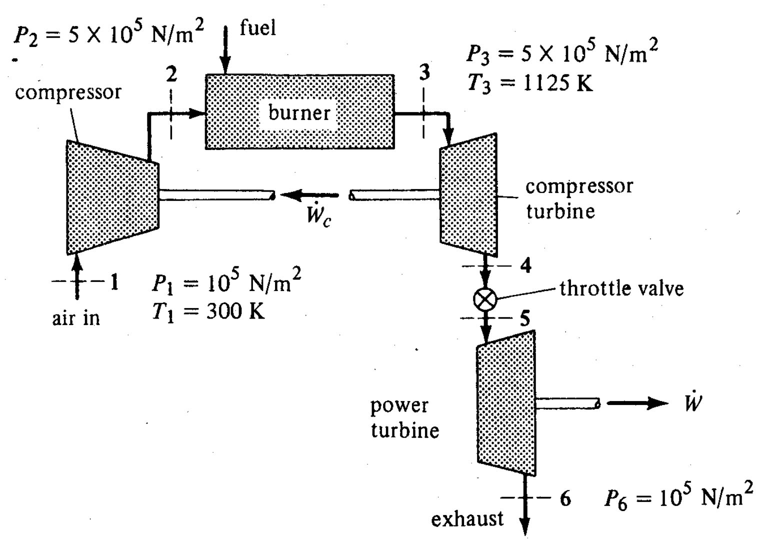
c) What is the thermal efficiency?

Unified Engineering Fall 2008
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T15 – T16

It has been proposed to use a gas turbine plant for automotive propulsion. The plan is to use a standard Brayton cycle with one modification: two turbines will be employed (making this a "split shaft plant"). One turbine will be used to drive the compressor while the other turbine will serve as the power turbine to drive the rear wheels of the vehicle. For purposes of control, the turbines will be connected with a throttle valve as shown in the figure below. Assume that the mass flow rate of fuel added in the burner is negligible compared to the mass flow rate of air. Model the burner as a constant pressure heat exchanger. Assume the air can be modeled as an ideal gas with $\gamma = 1.4$ and R = 287 J/kg-K.



Assume first that the power turbine operates at full load (throttle valve is fully open, such that $p_4 = p_5$).

a) Determine the pressures and temperatures of states 5 and 6.

b) Calculate the compressor work transfer per unit mass flow rate of air.

c) Calculate the power turbine work transfer per unit mass flow rate of air.

d) Calculate the heat transfer per unit mass flow rate of air in the burner.

e) If 250 kW output are required for a specific vehicle application, what is the necessary air flow rate?

(Continued on next page)

f) If the heating value of the fuel is 44 MJ/kg, calculate the fuel flow rate and the specific fuel consumption (kg/kWh).

g) Calculate the thermal efficiency of the plant.

The throttle valve may be partially closed, such that $p_5 = p_6$. For this case,

h) Determine the pressures and temperatures of states 5 and 6.

i) Calculate the power turbine work transfer per unit mass flow rate of air.