PAL
10/31/08

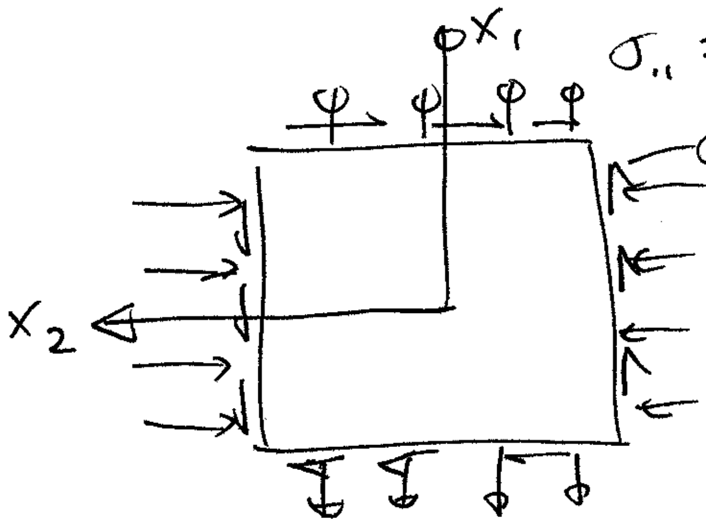
Unified Engineering

Problem Set 9 - week 10

Fall, 2008

SOLUTIONS

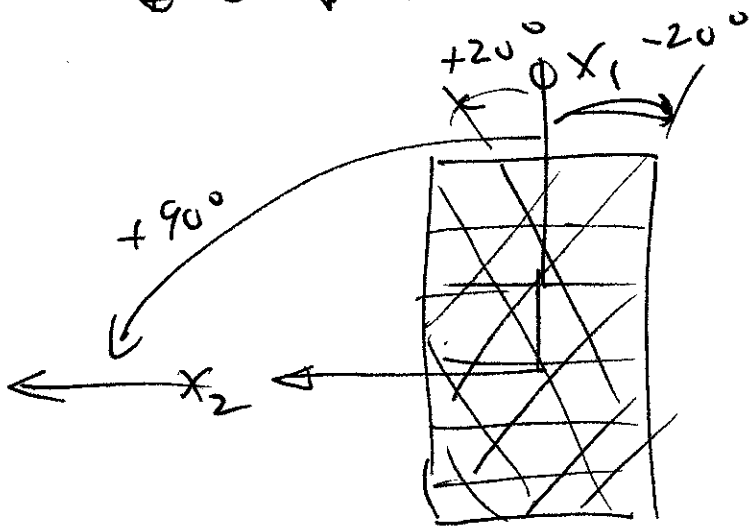
M17 (M10.1)



$\sigma_{11} = 20 MPa$

$\sigma_{12} = -10 MPa$ ← as drawn

$\sigma_{22} = -30 MPa$ ← as drawn



$\sigma_{11} = 20 MPa$

$\sigma_{22} = -30 MPa$

$\sigma_{12} = -10 MPa$

→ To find the stress state relative to the "fiber axes", align the transformed axis system with its $\tilde{x}_1$ axis along the fiber direction.

→ Then use the transformation equations (in 2-D):

$$\tilde{\sigma}_{11} = \cos^2\theta\, \sigma_{11} + \sin^2\theta\, \sigma_{22} + 2\cos\theta \sin\theta\, \sigma_{12}$$

$$\tilde{\sigma}_{22} = \sin^2\theta\, \sigma_{11} + \cos^2\theta\, \sigma_{22} - 2\cos\theta \sin\theta\, \sigma_{12}$$

$$\tilde{\sigma}_{12} = -\sin\theta \cos\theta\, \sigma_{11} + \sin\theta \cos\theta\, \sigma_{22} + (\cos^2\theta - \sin^2\theta)\, \sigma_{12}$$

→ Consider the three axis cases ($\theta = +20°$, $-20°$, $90°$) separately

For $\theta = +20^\circ$

$\sin\theta = 0.342 \Rightarrow \sin^2\theta = 0.117$

$\cos\theta = 0.940 \Rightarrow \cos^2\theta = 0.883$

and: $\sin\theta\cos\theta = 0.325$

Plugging into the transformation equations gives:

$$\tilde{\sigma}_{11} = (0.883)(20\,MPa) + (0.117)(-30\,MPa) + (0.650)(-10\,MPa) = 7.65\,MPa$$

$$\tilde{\sigma}_{22} = (0.117)(20\,MPa) + (0.883)(-30\,MPa) - (0.650)(-10\,MPa) = -17.6\,MPa$$

$$\tilde{\sigma}_{12} = -(0.325)(20\,MPa) + (0.325)(-30\,MPa) + (0.883 - 0.117)(-10\,MPa) = -23.9\,MPa$$

Summarizing:

for $\theta = +20^\circ$

$$\boxed{\begin{aligned}\tilde{\sigma}_{11} &= 7.65\,MPa\\ \tilde{\sigma}_{22} &= -17.6\,MPa\\ \tilde{\sigma}_{12} &= -23.9\,MPa\end{aligned}}$$

For $\theta = -20^\circ$

$\sin\theta = -0.342 \Rightarrow \sin^2\theta = 0.117$

$\cos\theta = 0.940 \Rightarrow \cos^2\theta = 0.883$

and: $\sin\theta\cos\theta = -0.325$

Plugging into the transformation equations gives:

$$\tilde{\sigma}_{11} = (0.883)(20\,MPa) + (0.117)(-30\,MPa) + (-0.650)(-10\,MPa) = 20.7\,MPa$$

$$\tilde{\sigma}_{22} = (0.117)(20\,MPa) + (0.883)(-30\,MPa) - (-0.650)(-10\,MPa) = -30.7\,MPa$$

$$\tilde{\sigma}_{12} = -(-0.325)(20\,MPa) + (-0.325)(-30\,MPa) + (0.883-0.117)(-10\,MPa) = 8.59\,MPa$$

Summarizing:

for $\theta = -20^\circ$

$$\boxed{\begin{aligned}\tilde{\sigma}_{11} &= 20.7\,MPa\\ \tilde{\sigma}_{22} &= -30.7\,MPa\\ \tilde{\sigma}_{12} &= 8.59\,MPa\end{aligned}}$$

<u>For $\theta = +90^\circ$</u>

$\sin\theta = 1 \Rightarrow \sin^2\theta = 1$

$\cos\theta = 0 \Rightarrow \cos^2\theta = 0$

And: $\sin\theta\cos\theta = 0$

Plugging into the transformation equation gives:

$$\tilde{\sigma}_{11} = (0)(20\,MPa) + (1)(-30\,MPa) + (0)(-10\,MPa) = -30\,MPa$$

$$\tilde{\sigma}_{22} = (1)(20\,MPa) + (0)(-30\,MPa) - (0)(-10\,MPa) = 20\,MPa$$

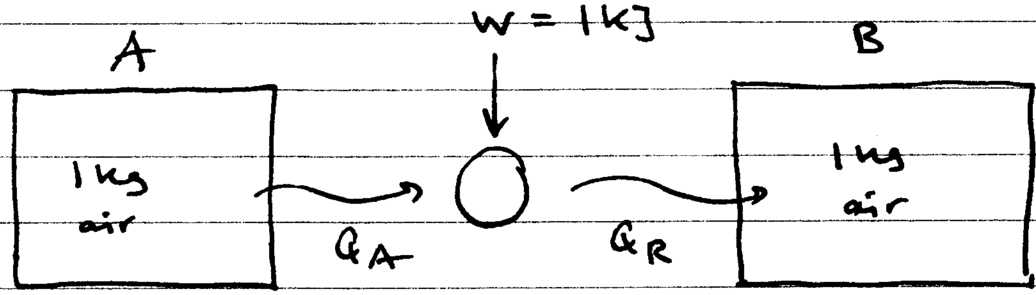
$$\tilde{\sigma}_{12} = -(0)(20\,MPa) + (0)(-30\,MPa) + (0-1)(-10\,MPa) = 10\,MPa$$

Summarizing:

for $\theta = +90^\circ$

$$\boxed{\begin{aligned}\tilde{\sigma}_{11} &= -30\,MPa\\ \tilde{\sigma}_{22} &= 20\,MPa\\ \tilde{\sigma}_{12} &= 10\,MPa\end{aligned}}$$

a)



Concepts:
- 1st law
- 2nd law
- rev. process

$\Delta T_A = -27°C$

b) Carnot heat pump: $0 = W + Q_A - Q_R$

$Q_A = m c_v (T_{A_i} - T_{Af})$ from 1st law $dU_A = -dQ_A \Big| \int_i^f$

$Q_R = m c_v (T_{Bf} - T_{Bi}) = m c_v \Delta T_B$ $\quad dU_B = +dQ_R \Big| \int_i^f$

$\Delta T_B = \dfrac{W + m c_v(-\Delta T_A)}{m c_v}$, $\underline{\Delta T_B = 28.4°C}$

c) $\Delta S_{total} = \Delta S_A + \Delta S_{HP} + \Delta S_B \equiv 0$ since Carnot HP

$\Delta S_A = \int_i^f \dfrac{dQ_A}{T} = m c_v \ln\left(\dfrac{T_{fA}}{T_{iA}}\right)$, $\Delta S_B = m c_v \ln\left(\dfrac{T_{fB}}{T_{iB}}\right)$

$\Delta S_{HP} = 0$ since cycle and $\underline{\Delta S_{total} = 0}$

d) from above: $\ln\left(\dfrac{T_{fA}}{T_{iA}}\right) + \ln\left(\dfrac{T_{fB}}{T_{iB}}\right) = 0$

$\rightarrow T_{fA} \cdot T_{fB} = T_{iA} T_{iB}$ and $T_{fA} = T_{iA} + \Delta T_A < T_{iA}$

$T_{fB} = T_{iB} + \Delta T_B > T_{iB}$

$\cancel{T_{iA} T_{iB}} + T_{iA}\Delta T_B + T_{iB}\Delta T_A + \Delta T_A \cdot \Delta T_B = \cancel{T_{iA} T_{iB}}$

also: $T_{iA} = T_{iB}$ so $\underline{T_{iA} = T_{iB}} = \dfrac{-\Delta T_A \Delta T_B}{\Delta T_A + \Delta T_B} = \underline{274.7°C}$

T20

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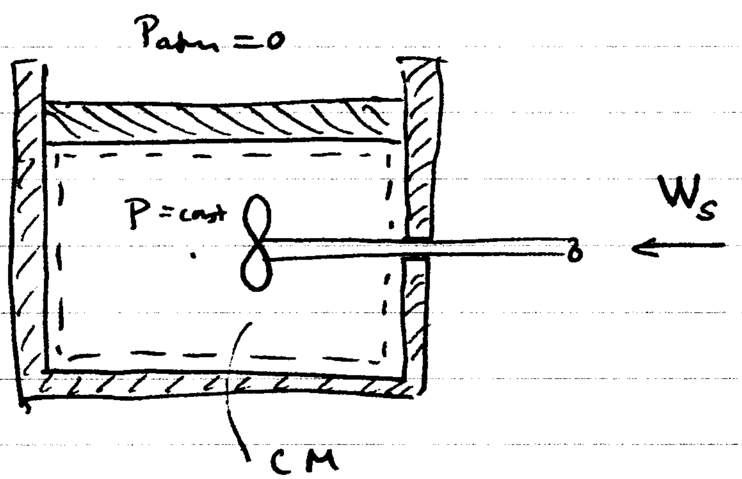
ZS



Given: $m = 0.01\,kg$
$T_i = 21°C$
$m_p = 100\,kg$
$\Delta z_p = 0.15\,m$

Assume: all adiabatic
air is ideal gas

Concepts: 1st law, 2nd law, entropy changes, lost work

a) $m = \rho V$ and $pV = mRT$ where $V = \underbrace{\frac{mR}{P}}_{constant} \cdot T$

$V_f = V_i + A\Delta z_p$ so $\frac{T_f}{T_i} = \frac{V_i + A\Delta z_p}{V_i}$; $Ap = m_p g$

$$T_f = T_i\left(1 + \frac{A\Delta z_p}{V_i}\right) = T_i\left(1 + \frac{A\Delta z_p}{mRT_i}P\right) = T_i\left(1 + \frac{m_p g}{mRT_i}\Delta z_p\right)$$

$\underline{T_f = 345.3\,K}$

b) 1st law CM : $U_f - U_i = -(W_p - W_s)$

$$W_p = \int_{V_i}^{V_f} p\,dV = \frac{m_p g}{A}(V_f - V_i) = m_p g \Delta z_p = \Delta E_{pot}$$

$W_s = m c_v (T_f - T_i) + m_p g \Delta z_p$ $\quad \underline{W_s = 515\,J/kg}$

$W_p = 147\,J/kg$

c) reversible or irrev.? $\Delta S_{total} = \Delta S_{gas} = ?$

Gibbs : $T\,ds = dh - v\,\cancel{dp}^{0}$ $\rightarrow \Delta S_{gas} = m c_p \int_{T_i}^{T_f} \frac{dT}{T} = m c_p \ln\left(\frac{T_f}{T_i}\right)$

$\underline{\Delta S_{gas} = 1.62\,J/K > 0}$ lost some work in irreversible process → manifested in entropy generated!

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Problem S2 (Signals and Systems)

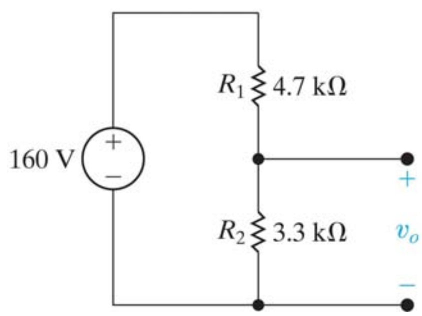

Figure 1

(a) Calculate the no-load voltage v_0 for the voltage divider circuit shown in Fig. 1.

(b) Calculate the power dissipated in R_1 and R_2.

(c) Assume that only 0.5 W resistors are available. The no-load voltage is to be the same as in (a). Specify the smallest ohmic values of R_1 and R_2.

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Solution for Problem S2 (Signals and Systems)

[a] $v_o = \dfrac{160(3300)}{(4700+3300)} = 66 \text{ V}$

[b] $i = 160/8000 = 20 \text{ mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88 \text{ W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32 \text{ W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160-66}{R_1}, \quad \text{Therefore,} \left(\frac{94}{R_1}\right)^2 R_1 \le 0.5$$

$$\text{Thus, } R_1 \ge \frac{94^2}{0.5} \quad \text{or} \quad R_1 \ge 17{,}672\,\Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$

Problem S3 (Signals and Systems)

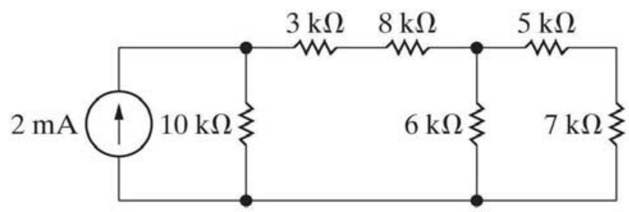


Figure 2

Look at the circuit in Fig. 2.

(a) Use current division to find the current flowing from top to bottom in the 10 kΩ resistor.

(b) Using your result in (a), find the voltage drop across the 10 kΩ resistor, positive at the top.

(c) Using your result from (b), use voltage division to find the voltage drop across the 6 kΩ resistor, positive at the top.

(d) Using your result from (c), use voltage division to find the voltage drop across the 5 kΩ resistor, positive at the left.

Solution for Problem S3 (Signals and Systems)

[a] The equivalent resistance to the right of the 10 kΩ resistor is $3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})] = 15\text{ k}\Omega$. Therefore,

$$i_{10\text{k}} = \frac{15\text{ k}\|10\text{ k}}{10\text{ k}}(0.002) = \frac{6\text{ k}}{10\text{ k}}(0.002) = 1.2\text{ mA}$$

[b] The voltage drop across the 10 kΩ resistor can be found using Ohm's law:

$$v_{10\text{k}} = (10,000)i_{10\text{k}} = (10,000)(0.0012) = 12\text{ V}$$

[c] The voltage $v_{10\text{k}}$ drops across the 3 kΩ resistor, the 8 kΩ resistor and the equivalent resistance of the 6 kΩ and the parallel branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{6\text{k}} = \frac{6\text{ k}\|(5\text{ k} + 7\text{ k})}{3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})]}(12) = \frac{4}{15}(12) = 3.2\text{ V}$$

[d] The voltage $v_{6\text{k}}$ drops across the branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{5\text{k}} = \frac{5\text{ k}}{5\text{ k} + 7\text{ k}}(3.2) = 1.33\text{ V}$$

Problem S4 (Signals and Systems)

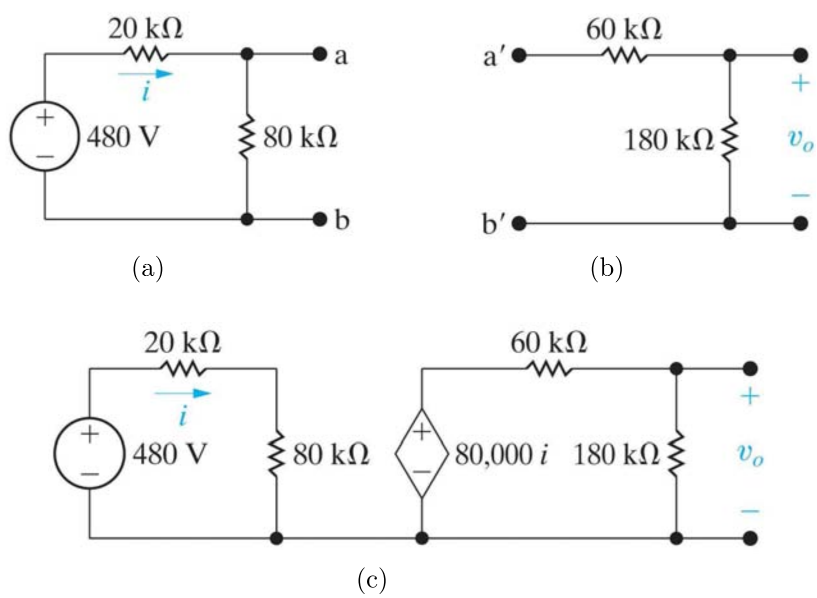


Figure 3

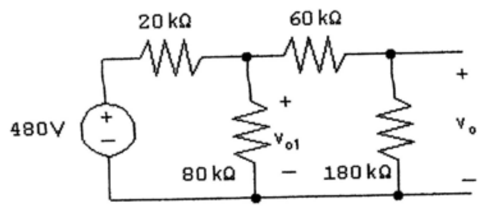
(a) The voltage divider in Fig. 3(a) is loaded with the voltage divider shown in Fig. 3(b): that is, a is connected to a′, and b is connected to b′. Find v_0.

(b) Now assume the voltage divider in Fig. 3(b) is connected to the voltage divider in Fig. 3(a) by means of current-controlled voltage source as shown in Fig. 3(c). Find v_0.

(c) What effect does adding the dependent-voltage source have on the operation of the voltage divider that is connected to the 480 V source?

Solution for Problem S4 (Signals and Systems)

[a]



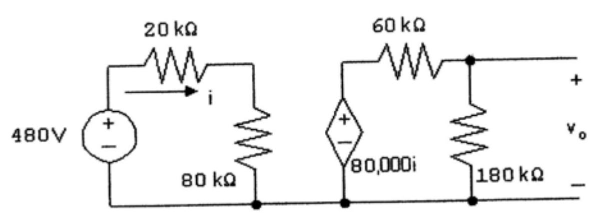
$$180 \text{ k}\Omega + 60 \text{ k}\Omega = 240 \text{ k}\Omega$$

$$80 \text{ k}\Omega \| 240 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$v_{o1} = \frac{60{,}000}{(20{,}000 + 60{,}000)}(480) = 360 \text{ V}$$

$$v_o = \frac{180{,}000}{(240{,}000)}(v_{o1}) = 270 \text{ V}$$

[b]



$$i = \frac{480}{100{,}000} = 4.8 \text{ mA}$$

$$80{,}000i = 384 \text{ V}$$

$$v_o = \frac{180{,}000}{240{,}000}(384) = 288 \text{ V}$$

[c] It removes loading effect of second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{80{,}000}{(100{,}000)}(480) = 384 \text{ V}$$

Now note this is the input voltage to the second voltage divider when the current controlled voltage source is used.