10:00 - 11:00
34-101

Name SOLUTIONS

PAL
10/12/08

Unified Quiz FM2

October 15, 2008

M - PORTION

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of Handout "HO-M-4" along with 2 sides of pages of handwritten material are allowed.***

EXAM SCORING

#1M (1/3)	
#2M (1/3)	
#3M (1/3)	
FINAL SCORE	

Unified Quiz FM2 - Materials and Structures Portion
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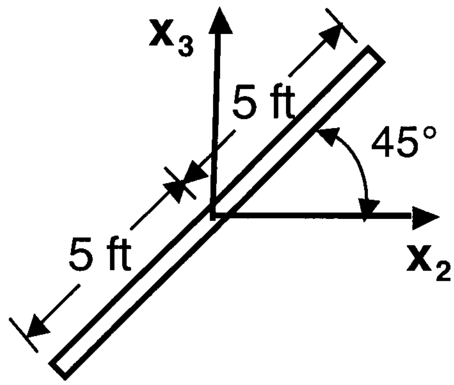
PROBLEM #1M (1/3)

A set of four forces acts in the x_2-x_3 plane on a 10-foot long bar that passes through and is centered on the origin, and that makes a 45° angle with the axis system as shown. The force vectors and the distance along the bar from the origin at which they act are as follows:

$\underline{F}_A = (3 \text{ lbs})\, \underline{i}_2 + (6 \text{ lbs})\, \underline{i}_3$	acts at +4 feet
$\underline{F}_B = (-2 \text{ lbs})\, \underline{i}_2 + (-2 \text{ lbs})\, \underline{i}_3$	acts at 0 feet
$\underline{F}_C = (4 \text{ lbs})\, \underline{i}_2 + (-1 \text{ lbs})\, \underline{i}_3$	acts at -1 feet
$\underline{F}_D = (6 \text{ lbs})\, \underline{i}_2$	acts at -3 feet



(a) Determine the force system acting at the origin that is equipollent to this force system.

The total force and moment with respect to the origin must be determined in finding the equipollent force system acting at the origin.

<u>Force</u>: Must have same magnitude and direction as effect of these four forces.

Find: $\sum \underline{F}_i = \underline{F}_A + \underline{F}_B + \underline{F}_C + \underline{F}_D$

x_3 + , x_2 +

$$= \{(3\text{ lbs}) + (-2\text{ lbs}) + (4\text{ lbs}) + (6\text{ lbs})\}\, \underline{i}_2 + \{(6\text{ lbs}) + (-2\text{ lbs}) + (-1\text{ lb})\}\, \underline{i}_3$$

$$\Rightarrow \boxed{\underline{F}_{origin} = (11\text{ lbs})\, \underline{i}_2 + (3\text{ lbs})\, \underline{i}_3}$$

<u>Moment</u>: Must include pure moment equal to sum of moments caused by four forces acting about origin:

$$\sum M_{origin} = \sum M_i = \sum (\underline{r}_i \times \underline{F}_i)$$

Cross product accounts for relative angle.
Can also take x_2-distance (for x_3-force) and

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PROBLEM #1M (continued)

x_3 - distance (for x_2 - force) along with their "sense")

x_2 - distance = (distance along bar) $\cos 45°$

x_3 - distance = (distance along bar) $\sin 45°$

$\sin 45° = \cos 45° = 0.707$

$$\overset{+}{\circlearrowleft}\ \Sigma M_{origin} = (0.707)\{-(3\,lbs)(4\,ft) + (6\,lbs)(4\,ft) + (4\,lbs)(1\,ft) + (1\,lb)(1\,ft) + (6\,lbs)(3\,ft)\}$$

$$= (0.707)\{-12 + 24 + 4 + 1 + 18\}\ ft \cdot lbs$$

$$= 35(0.707)\ ft \cdot lbs \Rightarrow \boxed{M_{origin} = 24.8\ ft \cdot lb}\ \overset{+}{\circlearrowleft}$$

Summarizing:

$$\boxed{\begin{array}{l} \underline{F}_{origin} = (11\,lbs)\,\underline{i}_2 + (3\,lbs)\,\underline{i}_3 \\ M_{origin} = 24.8\ ft \cdot lb\ \overset{+}{\circlearrowleft} \end{array}}$$

(b) Can this system be put in equilibrium by applying one force at any point along the bar? If so, what is that force and what is the location? If not, clearly explain why not.

One can have the same magnitude and opposite direction for equilibrium. For the force, this is:

$$\underline{F} = (-11\,lbs)\,\underline{i}_2 + (-3\,lbs)\,\underline{i}_3$$

This force must be applied to cause a moment with magnitude about the origin of

$$M_{origin} = -24.8\ ft \cdot lb \quad \overset{+}{\circlearrowleft}$$

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PROBLEM #1M (continued)

The position is such that $x_2 = x_3$, so this can occur if:

$$(11\ lbs)(x_3) + (-3\ lbs)(x_2) = -24.8\ ft\cdot lb$$

$$x_2 = x_3 = -\frac{24.8\ ft\cdot lb}{8\ lb} = -3.1\ ft$$

along the bar:

$$x_2 = x_3 = A(0.707)$$

$$\Rightarrow -3.1\ ft = A(0.707)$$

$$\Rightarrow \quad A = \text{distance along bar} =$$

$\boxed{\text{YES}}$ this equilibrium can be achieved

with:

$$\boxed{\begin{array}{c}\text{Force} = (-11\ lbs)\,\underline{i}_2 + (-3\ lbs)\,\underline{i}_3 \\ \text{at} -4.38\ ft \text{ along bar}\end{array}}$$

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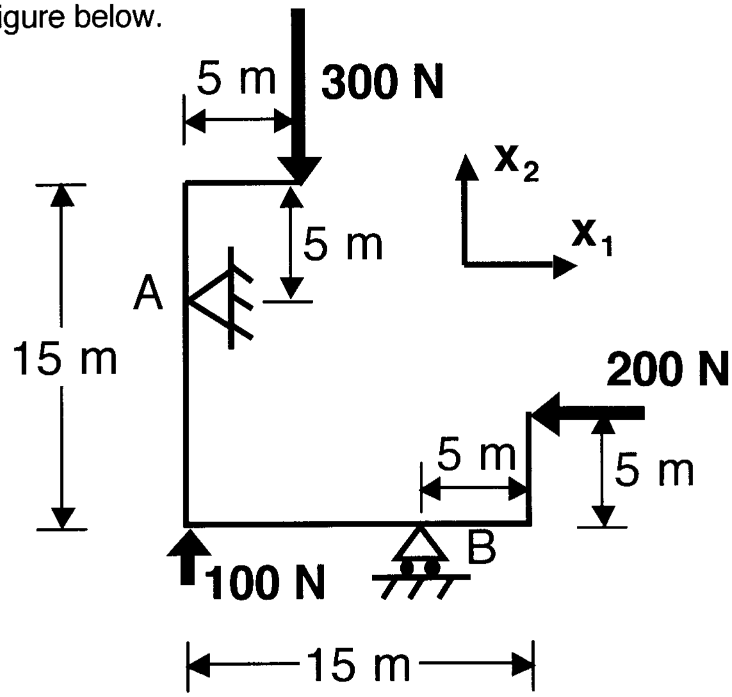
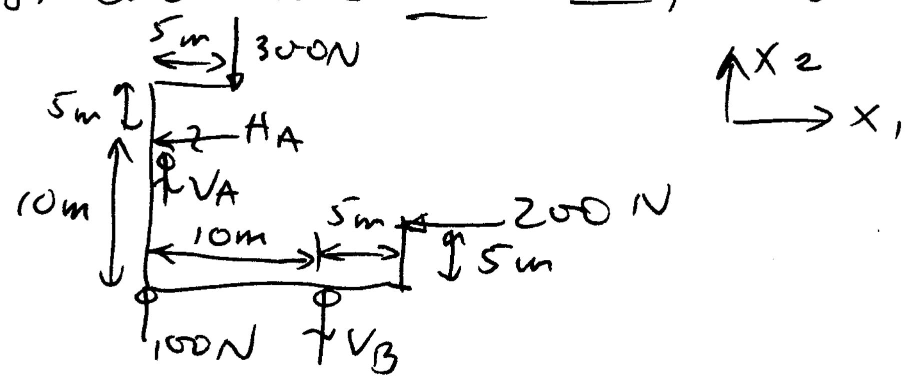
PROBLEM #2M (1/3)

A structural configuration has overall dimensions of 15 meters by 15 meters. The structure is supported at two different heights with a pin at the far left 10 meters above the bottom of the structure where a roller support is located. Loads are applied at three points: a 300-Newton vertical load at the upper tip, a 200-Newton horizontal load at the right tip, and a 100-Newton vertical load at the lower left corner of the structure. This overall configuration is shown in the figure below.



(a) Determine the reaction forces at the two support points A and B *or* indicate all information available to determine the reaction forces and the additional information that is needed to fully determine these forces.

First draw the Free Body Diagram



There are 3 reactions and 3 degrees of freedom (for 2-D system). So this is statically determinate and the reactions can be determined.

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PROBLEM #2M (continued)

Use equations of equilibrium.

$\sum F_1 = 0 \overset{+}{\rightarrow} \Rightarrow -H_A - 200\,N = 0 \Rightarrow \boxed{H_A = -200\,N}$

$\sum F_2 = 0 \uparrow + \Rightarrow V_A + V_B + 100\,N - 300\,N = 0$

giving: $V_A + V_B = 200\,N$

$\sum M_A = 0 \circlearrowleft + \Rightarrow -(300\,N)(5\,m) + V_B(10\,m) - (200\,N)(5\,m) = 0$

giving: $2V_B = 500\,N \Rightarrow \boxed{V_B = 250\,N}$

using $\sum F_2 = 0 \Rightarrow V_A = 200\,N - 250\,N$

giving: $\boxed{V_A = -50\,N}$

Summarizing:

$$\boxed{\begin{aligned} V_A &= -50\,N \\ V_B &= 250\,N \\ H_A &= -200\,N \end{aligned}}$$

(b) How are these answers affected by the material from which the structure is made? Explain clearly.

The system is statically determinate and thus the reactions can be determined solely from the consideration of equilibrium. Thus, the material **does not affect** the results.

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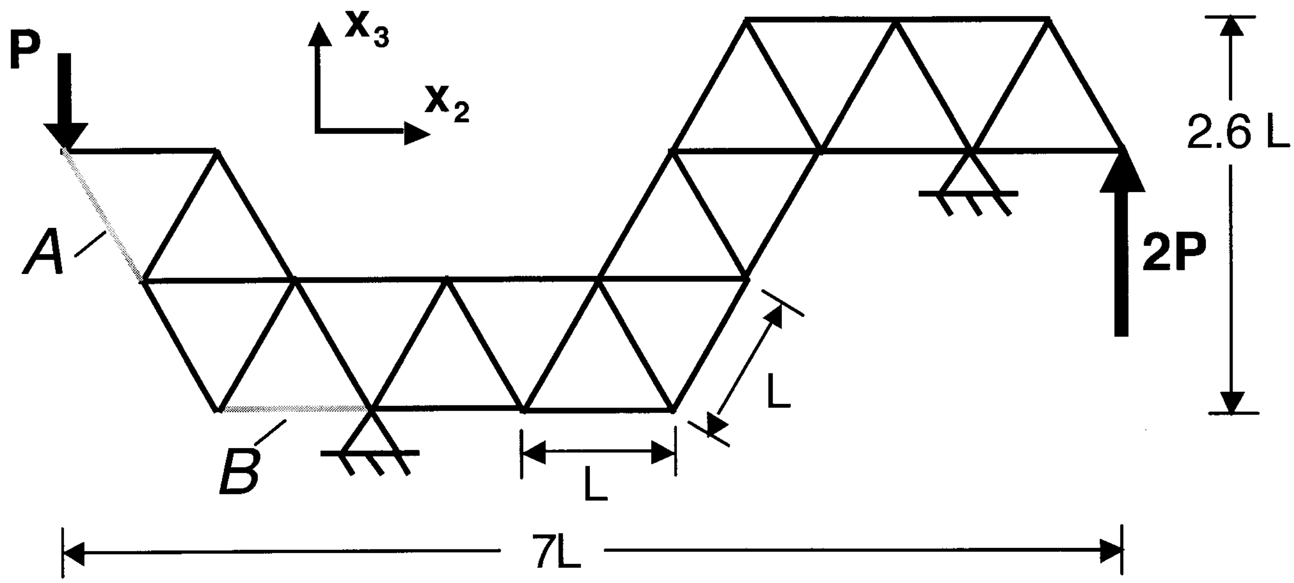
PROBLEM #3M (1/3)

The following truss has various triangular bays made of bars all of length L. The truss is loaded by two tip loads, P and 2P, has overall dimensions of 7L in the x_2-direction and 2.60 L in the x_3-direction, and is pinned at two points, as shown in the figure below.



(a) What is the "class/category" of this structural configuration (Dynamic, Statically Determinate, Statically Indeterminate)? Clearly explain your reasoning.

There are two supports and each is a pin. A pin has two reactions -- along each axis (x_2 and x_3)

# of reactions = 4

This is a 2-D system and thus:

# of degrees of freedom = 3

# of reactions > # of degrees of freedom

$\Rightarrow$ $\boxed{\text{Statically Indeterminate}}$

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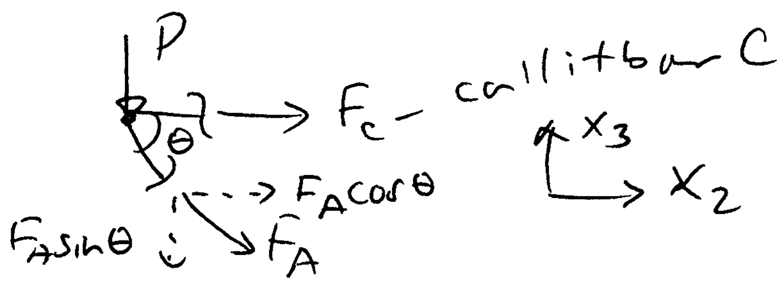
PROBLEM #3M (continued)

(b) Determine the load in the diagonal bar of the truss labeled "A" (highlighted in the figure) ***or*** indicate the additional information needed in order to determine this load.

Use the Method of Joints:



Each bay is an equilateral triangle $\Rightarrow \theta = 60^\circ$

so: $F_A \sin\theta = 0.866\, F_A$

use equilibrium:

$$\sum F_3 = 0 \;\; \uparrow + \;\; \Rightarrow -P - F_A \sin\theta = 0$$

$$\text{so:} \quad F_A = -\frac{P}{0.866}$$

$$\Rightarrow \boxed{F_A = -1.15\,P}$$

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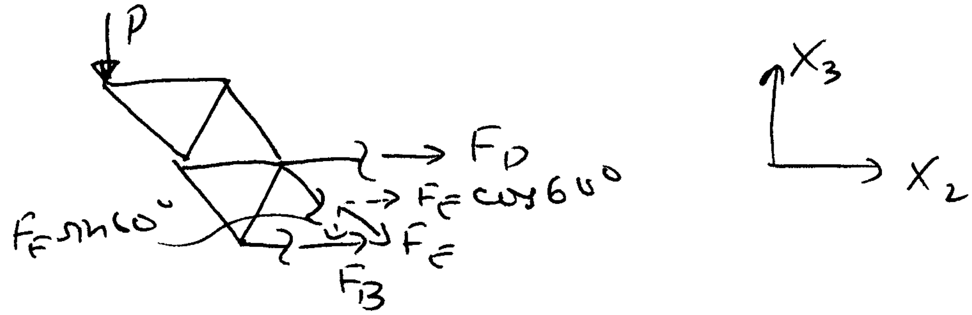
PROBLEM #3M (continued)

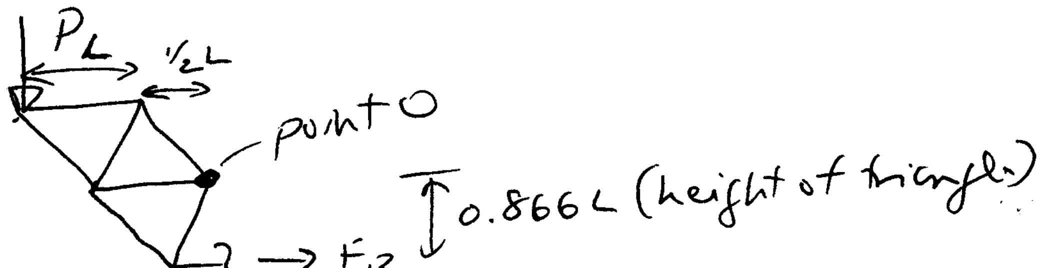
(c) Determine the load in the diagonal bar of the truss labeled "B" (highlighted in the figure) ***or*** indicate the additional information needed in order to determine this load.

Use the Method of Sections:



Take moments about the point where bars "E" and "D" touch. This isolates F_{13} as the other two bars act through this point.

Determine distances:



$$\sum M_O = 0 \; \circlearrowleft + \Rightarrow P(1.5L) + (0.866L)F_{13} = 0$$

$$\Rightarrow F_{13} = -P\left(\frac{1.5}{0.866}\right)$$

giving: $\boxed{F_{13} = -1.73P}$