10:00 - 11:00
34-101

Name SOLUTIONS

Unified Quiz TM3

October 29, 2008

M - PORTION

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of Handout "HO-M-5" along with 2 sides of pages of handwritten material are allowed.***

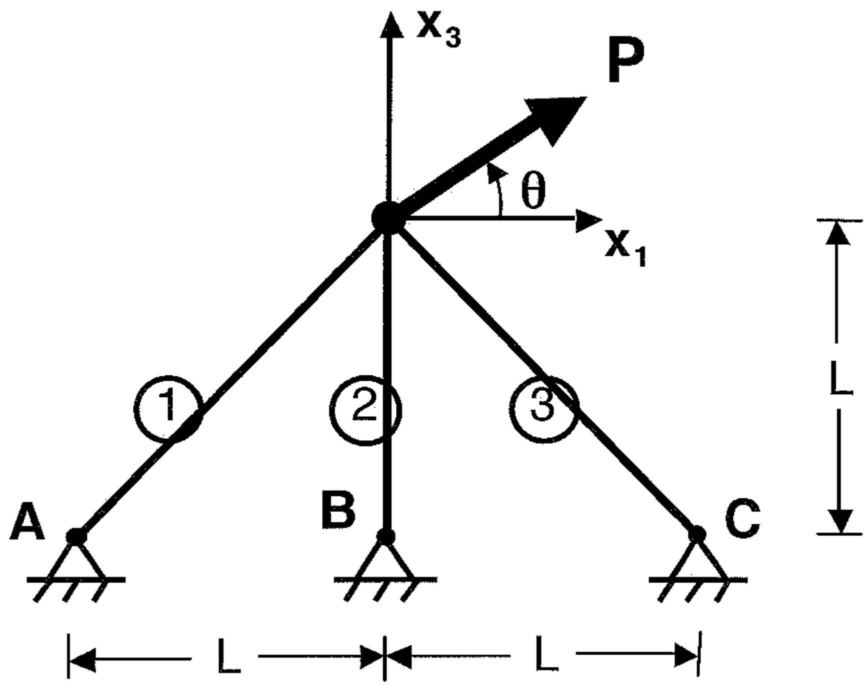
EXAM SCORING

#1M = FINAL SCORE	

Unified Quiz TM3 - Materials and Structures Portion
October 29, 2008 **Name** SOLUTIONS

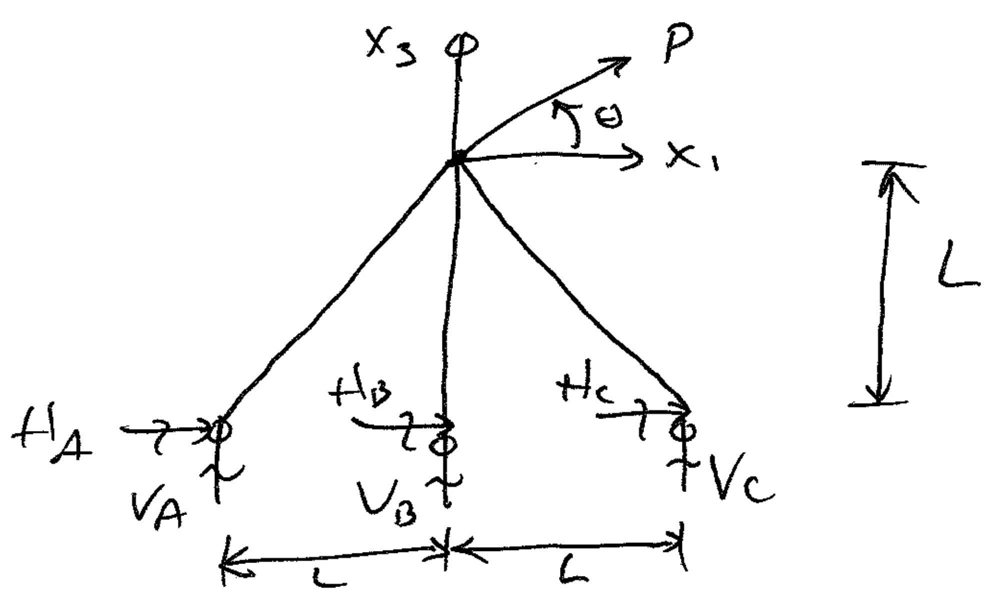
PROBLEM #1M

A three-bar truss is pinned at the junction of the three bars. Each bar is supported at its base via a pin support. The overall structure, prior to being loaded, has a height of L and length of 2L, with the pin support, B, of Bar 2 at the mid-point. Bar 2 is initially vertical. A load of magnitude P is applied at the junction of the bars at an angle of θ (measured positive counterclockwise) to the x_1- x_3 coordinate system. The bars have force-displacement relations of: $\delta_{bar} = PL_{bar} / (EA)_{bar}$ and the EA of all bars is the same.



(a) What is the "class/category" of this structural configuration (Dynamic, Statically Determinate, Statically Indeterminate)? **Clearly** explain your reasoning.

Free Body Diagram shows:



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PROBLEM #1M (continued)

So: 6 reactions but only 3 d.o.f. giving only 3 equations of equilibrium

$\Rightarrow$ $\boxed{\text{Statically Indeterminate}}$

(b) Set up the equations available to determine the bar loads and reactions. **Clearly explain** the approach needed and the steps taken. Indicate whether there are sufficient equations to determine the loads or indicate the additional information needed for such. **Do not solve the equations.**

As the system is statically indeterminate, it is necessary to obtain equations using each of the 3 principles:

- equilibrium
- constitutive relations
- Compatibility of Displacement

The junction of the three bars can move.

→ Define the movement of this points as Δx_1 and Δx_3.

→ Each of the bars can change angle from its initial setting depending upon the extent of displacement. Define the angle that each bar makes with the x_1 direction as α_1, α_2, α_3.

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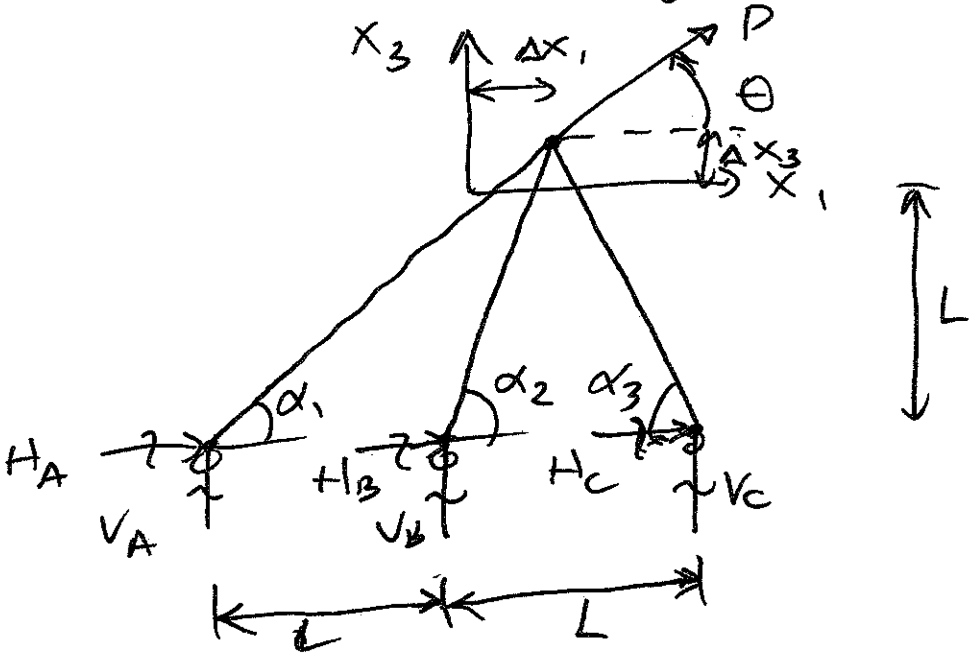
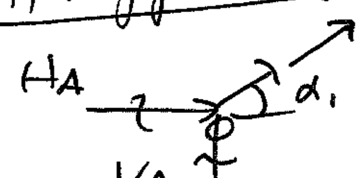
PROBLEM #1M (continued)

→ Draw the resulting configuration:



Consider each of the subsystems

→ At support A:



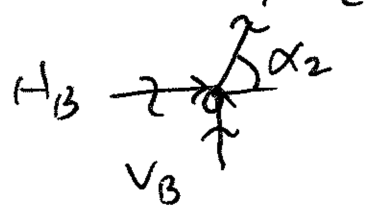
P_1 = load in bar 1

$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow H_A + P_1 \cos\alpha_1 = 0$ (1)

$\Sigma F_3 = 0 \uparrow + \Rightarrow V_A + P_1 \sin\alpha_1 = 0$ (2)

→ At support B:

P_2 = load in bar 2



$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow H_B + P_2 \cos\alpha_2 = 0$ (3)

$\Sigma F_3 = 0 \uparrow + \Rightarrow V_B + P_2 \sin\alpha_2 = 0$ (4)

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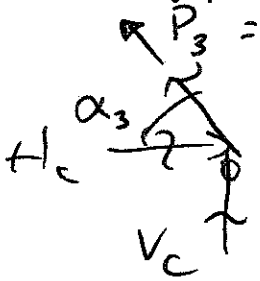
PROBLEM #1M (continued)

→ <u>At support C</u>

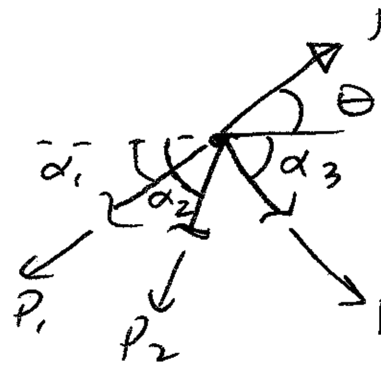
P_3 = load in bar 3



$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow H_c - P_3 \cos\alpha_3 = 0 \quad (5)$$

$$\sum F_3 = 0 \uparrow + \Rightarrow V_c + P_3 \sin\alpha_3 = 0 \quad (6)$$

→ <u>At junction of bars</u>



$$\sum F_1 = 0 \xrightarrow{+}$$

$$\Rightarrow P\cos\theta - P_1\cos\alpha_1 - P_2\cos\alpha_2 + P_3\cos\alpha_3 = 0 \quad (7)$$

$$\sum F_3 = 0 \uparrow +$$

$$\Rightarrow P\sin\theta - P_1\sin\alpha_1 - P_2\cos\alpha_2 - P_3\sin\alpha_3 = 0 \quad (8)$$

• <u>Current</u>:

8 equations

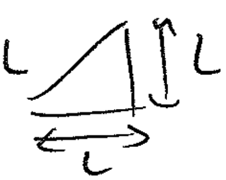
12 unknowns (6 reactions: $H_A, H_B, H_C, V_A, V_B, V_C$
3 bar loads: P_1, P_2, P_3
3 bar angles: α_1, α_2, α_3)

→ Add the <u>constitutive relation</u> for each bar:

$$\delta_{bar} = P L_{bar} / (EA)_{bar}$$

EA = constant (same for each bar)

$L_{bar_1} = L_{bar_3} = \sqrt{2}\, L$



$L_{bar_2} = L$

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PROBLEM #1M (continued)

$$\Rightarrow \delta_1 = \delta_{bar_1} = P_1 \sqrt{2} L / EA \qquad (9)$$

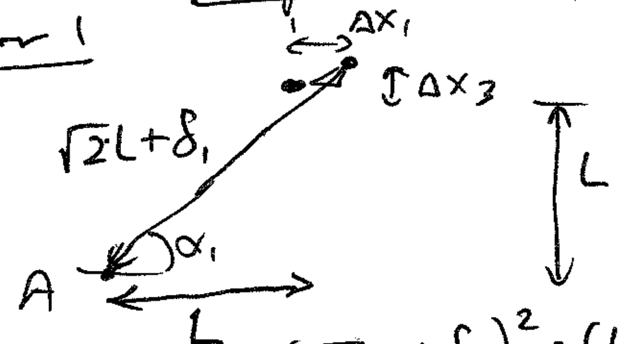
$$\delta_2 = \delta_{bar_2} = P_2 L / EA \qquad (10)$$

$$\delta_3 = \delta_{bar_3} = P_3 \sqrt{2} L / EA \qquad (11)$$

- Current: 11 equations
  15 unknowns (3 bar displacements added)

→ Use Compatibility of Displacement

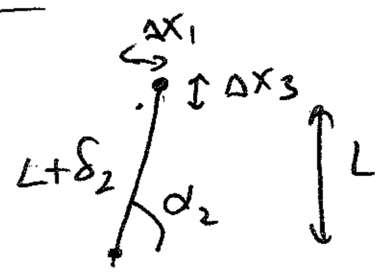
Bar 1



$$(\sqrt{2} L + \delta_1)^2 = (L + \Delta x_1)^2 + (L + \Delta x_3)^2 \qquad (12)$$

$$\alpha_1 = \tan^{-1} \frac{L + \Delta x_3}{L + \Delta x_1} \qquad (13)$$

Bar 2

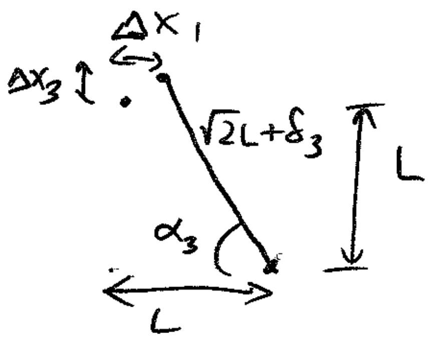


$$(L + \delta_2)^2 = (L + \Delta x_3)^2 + (\Delta x_1)^2 \qquad (14)$$

$$\alpha_2 = \tan^{-1} \frac{L + \Delta x_3}{\Delta x_1} \qquad (15)$$

PROBLEM #1M (continued)

Bar 3



$$(\sqrt{2}L+\delta_3)^2 = (L-\Delta x_1)^2 + (L+\Delta x_3)^2 \qquad (16)$$

$$\alpha_3 = \tan^{-1}\frac{L+\Delta x_3}{L-\Delta x_1} \qquad (17)$$

- Current Status:

17 equations
17 unknowns (Δx_1, Δx_3 added)

$\Rightarrow$ $\boxed{\text{sufficient equations}}$ to determine loads and reactions

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PROBLEM #1M (continued)

(c) Indicate if the pertinent equations can be simplified if it can be assumed that the deflections of the junction of the bars are small and that any changes in the angles of the bars can be ignored. If the equations can be simplified, indicate how.

$\boxed{\text{YES}}$ Equations (12), (14), and (16) can be simplified if it can be assumed that the deflections of the junction of the bars are small.

→ these equations can be written out and "higher order terms" (deflections squared: Δx_1^2, Δx_3^2, δ_1^2, δ_2^2, δ_3^2) ignored and the equations linearized.

If it can be assumed that any changes in angles can be ignored, then

→ equations (13), (15), and (17) and α_1, α_2, and α_3 can be ignored

→ In equations (1) - (8), can set:
$\alpha_1 = 45°$, $\alpha_2 = 90°$, $\alpha_3 = 45°$

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PROBLEM #1M (continued)

(d) If equations are simplified via assumptions of small deflections and relative angle changes, indicate (generically, not specifically with numbers for these equations) how such assumptions would be checked once the solutions of the simplified equations are obtained.

Once the results are determined for Δx_1 and Δx_3, these values can be compared to the length of the bars (L and $\sqrt{2}L$) to assess the percentage change.

In addition, the angular change induced can be assessed via equations (13), (15) and (17) to determine the new angles for those displacements.