10:20 - 10: 55
34-101

Name SOLUTIONS

Unified Quiz TMS4

November 19, 2008

M - PORTION

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of Handout "HO-M-7" along with 2 sides of pages of handwritten material are allowed.***

EXAM SCORING

#1M = FINAL SCORE	

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PROBLEM #1M

A large slab of a relatively compliant material is in the x_2- x_3 plane. This slab is outfitted with strain gages and the slab is loaded by stresses along multiple axes. While undergoing this multiaxial stress state, it is determined that the strain gages show that the strains are:

$$\varepsilon_{22} = -4000\,\mu\text{strain} \quad \varepsilon_{33} = +8000\,\mu\text{strain} \quad \gamma_{23} = +10{,}000\,\mu\text{strain}$$

where the shear strain is engineering shear strain. It is furthermore known that the strain does not vary through the thickness of the slab, i.e. with x_1, and any strains involving the x_1-direction are equal to zero.

(a) Can one draw a rectangle on the slab of material that will maintain its shape as a rectangle? If not, why not? If so, what is its orientation? How will its rectangular aspect ratio change in that case, if at all? **Clearly explain your reasoning.**

→ A rectangle will maintain its rectangular shape in a direction where there is no shear strain, and thus no angular deformation/change.

→ Thus it is the principal axis system of the strain where this will occur.

→ For an in-plane strain system, the principal strains are the roots of the equation:

$$\tau^2 - \tau(\epsilon_{22} + \epsilon_{33}) + (\epsilon_{22}\epsilon_{33} - \epsilon_{23}^2) = 0$$

→ Note in this case the shear strain is <u>engineering</u> shear strain, so this must be converted to <u>tensorial</u> shear strain: $\epsilon_{23} = \gamma_{23}/2$

So have: $\epsilon_{22} = -4000\,\mu s$

$\epsilon_{33} = +8000\,\mu s$ (μs = μstrain)

$\epsilon_{23} = +5000\,\mu s$

→ Proceed with the equation for the roots:

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$$\tau^2 - \tau(4\times10^3\mu s) + (-32\times10^6 - 25\times10^6)(\mu s)^2 = 0$$

giving: $\tau^2 - \tau(4\times10^3\mu s) - 57\times10^6(\mu s)^2 = 0$

→ solve via the quadratic formula: $\tau = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

here: $a = 1$
$b = -4\times10^3\mu s$
$c = -57\times10^6(\mu s)^2$

$$\Rightarrow \tau = \frac{4\times10^3 \pm\sqrt{(16\times10^6)-4(1)(-57\times10^6)}}{2}\mu s$$

$$= \frac{4\pm\sqrt{16+228}}{2}\times10^3\mu s$$

$$= \frac{4\pm\sqrt{244}}{2}\times10^3\mu s = \frac{4\pm15.6}{2}\times10^3\mu s$$

finally: $\tau = 9800\mu$strain, -5800μstrain

so: $\boxed{\text{Principal strains: } \epsilon_I, \epsilon_{II} = 9800\mu\text{strain}, -5800\mu\text{strain}}$

Note: Check via Σ extensional = constant
$= 4000\mu$strain ✓

→ Get the direction associated with this via:

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\epsilon_{23}}{\epsilon_{22}-\epsilon_{33}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{10\times10^3\mu\text{strain}}{-12\times10^3\mu\text{strain}}\right)$$

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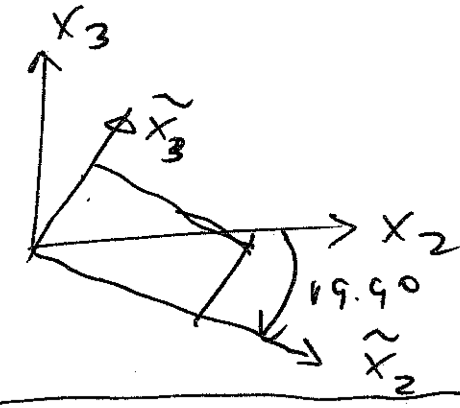
giving... $\theta_p = \frac{1}{2}\tan^{-1}\left(-\frac{5}{6}\right)$

$= \frac{1}{2}\left(-39.8^\circ\right)$

$\Rightarrow$ $\boxed{\text{Principal direction: } \theta_p = -19.9^\circ}$

from x_2-axis

So



(– clockwise)

$\boxed{\text{Rectangle in } \tilde{x}_2, \tilde{x}_3 \text{ system, } -19.9^\circ \text{ from } x_2, x_3 \text{ will stay a rectangle}}$

Its aspect ratio will change via:

$$\frac{\partial u_2}{\partial x_2} = \epsilon_{22} \; ; \quad \frac{\partial u_3}{\partial x_3} = \epsilon_{33}$$

for constant strain as is the case, the deformation change ratio is:

$$\left|\frac{\Delta u_3}{\Delta u_2}\right| = \left|\frac{\epsilon_{33}}{\epsilon_{22}}\right|$$

must look at this in axis system of rectangle $(\tilde{x}_2, \tilde{x}_3)$

$$\Rightarrow = \cancel{1-}\left|\frac{1+0.0098}{1-0.0058}\right| \Rightarrow \boxed{\text{change in aspect ratio} = 1.016} = 1.6\%$$

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(b) The stresses are altered along the various axes, but this does not result in a change in the strains measured by the strain gages, or any in the x_1-direction. Will this change the deformation of the slab? **Clearly explain your reasoning.**

If the strain does not change, the deformation will not change. The deformation is directly related to the strain and is not directly affected by the stresses. The deformation is affected by the stresses only if it were to affect the strains.

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(c) The loading is now changed so that the deformations are increased by a factor of thirty. How will this affect the values of the in-plane strains? **Clearly explain your reasoning.**

The key issue here is whether all remains "small and linear" such that the strains increase linearly by the same factor of 30 as the deformations. This would give $\epsilon_{22} = -120{,}000\ \mu strain$ and $\epsilon_{33} = +240{,}000\ \mu strain$. These are rather large, but it becomes clearer whether the assumption of small displacements and small angles is breaking down by looking at the shear strain and the associated angle change, i.e. γ_{23}. If this increases linearly by a factor of 30, this gives: $\gamma_{23} = +300{,}000\ \mu strain$. Recalling that the total angle change is equal to the engineering strain, this gives a change in radians of 0.30. This is 17.2° and gives $\cos\theta$ of 0.955. So this is 4.5% off $\cos\theta = 1$. Thus, there is coupling between extension and shear that needs to be taken into account.

Therefore, saying simply that the strains increase by a linear factor of 30 would be using an approximation that is "breaking down".