10:05 - 10: 55
34-101

Name SOLUTIONS

Unified Quiz FM5

December 3 2008

M - PORTION

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of Handouts "HO-M-8" and "HO-M-9" along with 2 sides of pages of handwritten material are allowed.***

EXAM SCORING

#1M (1/3)	
#2M (2/3)	
FINAL SCORE	

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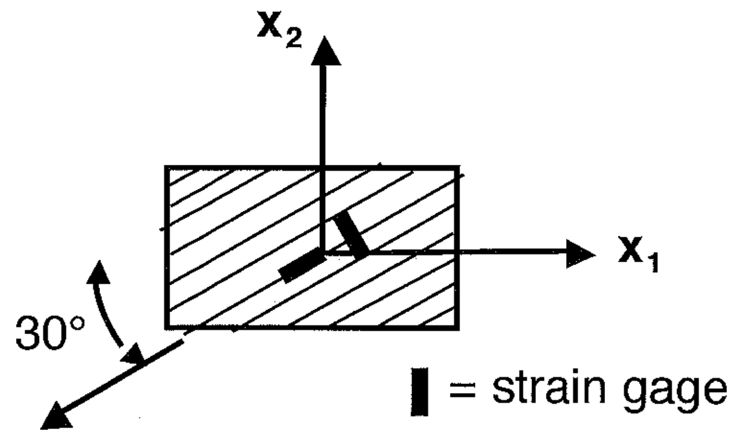
PROBLEM #1M (1/3)

A rectangular piece of unidirectional composite material is loaded in a state of plane stress with a pure shear of magnitude τ. The fibers of the material are aligned at a 30° angle with respect to the axis system in which the shear stress acts. There is one strain gage placed along the direction of the fibers and this reads A. A second strain gage is placed perpendicular to the direction of the fibers and reads B.



Is it possible to find a transformed axis system in the x_1 - x_2 plane such that the slab is loaded only by normal stresses, σ_{11} and σ_{22}? If not, describe why not. If yes, describe why, determine the angle to the transformed axis system, and determine the relative values of the normal stresses. **Clearly explain your reasoning.**

YES. These are the principal stresses (no shear stresses) and they and their associated axes can always be found.

→ For the case of a pure shear stress, the principal axes are 45° to the original axis system.

This can be shown via:

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}} \right)$$

with: $\sigma_{12} = \tau$, $\sigma_{11} = 0$, $\sigma_{22} = 0$

PROBLEM #1M (continued)

$$\Rightarrow \theta_p = \frac{1}{2} \tan^{-1}(\infty)$$
$$= \frac{1}{2}(90^\circ) \Rightarrow \boxed{\theta_p = 45^\circ}$$

→ And use this equation to determine the principal (normal) stresses:
(or place the value of θ_p in the general transformation equations):

τ: roots
$$\tau^2 - \tau(\sigma_{11} + \sigma_{22}) + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

Call applied $\tau = T$

$$\Rightarrow \tau^2 - T^2 = 0 \Rightarrow \tau = \pm T$$

so: principal stresses
= normal stresses $= (\tilde{\sigma}_{11}, \tilde{\sigma}_{22})$
$= +\tau, -\tau$

← original magnitude of shear stress

Check via invariant:
Σ extensional = constant = 0 ✓

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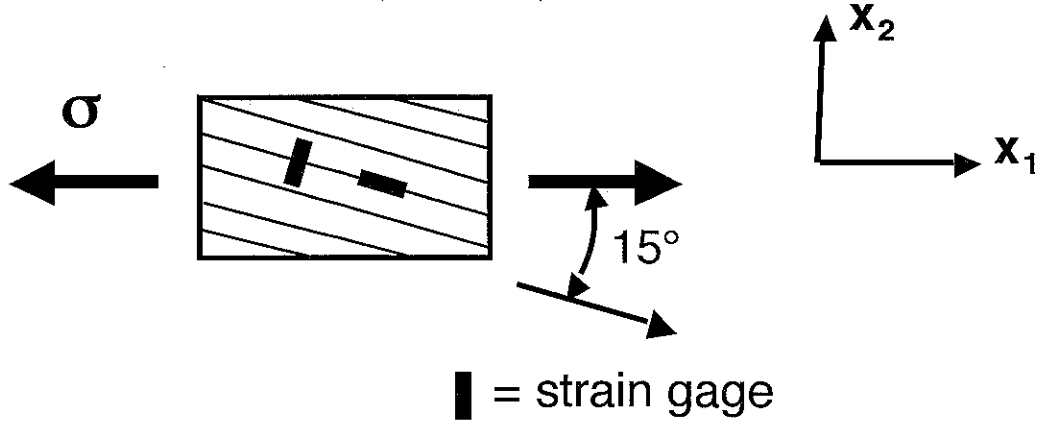
PROBLEM #2M (2/3)

A unidirectional composite ply has the following elastic constants:

E_1 = 24 Msi ν_{12} = 0.24 G_{12} = 6.0 Msi
E_2 = 3.0 Msi ν_{21} = 0.03

These are referenced with the x_1 direction along the fibers and the x_2 direction perpendicular to the fibers. The unidirectional ply is loaded by a stress of 20 ksi at an angle 15° off the fiber direction as pictured. Strain gages are placed on the surface of the ply parallel and perpendicular to the fiber direction (also shown).



Can the strain readings be determined? If so, do so. If not, clearly explain why not, using equations as appropriate.

→ In the axis system of the fibers, the material is an orthotropic material defined by the following in-plane stress-strain relations:

$$\epsilon_{11} = \frac{1}{E_1}\left[\sigma_{11} - \nu_{12}\sigma_{22}\right]$$

$$\epsilon_{22} = \frac{1}{E_2}\left[-\nu_{21}\sigma_{11} + \sigma_{22}\right]$$

$$2\epsilon_{12} = \frac{1}{G_{12}}\sigma_{12}$$

where ϵ_{11} is along the fibers and ϵ_{22} is perpendicular to the fibers.

→ It is necessary to transform the stress in the original x_1-x_2 axis system to the fiber axis system via:

PROBLEM #2M (continued)

$$\tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta\,\sigma_{12}$$
$$\tilde{\sigma}_{22} = \sin^2\theta\,\sigma_{11} + \cos^2\theta\,\sigma_{22} - 2\cos\theta\sin\theta\,\sigma_{12}$$
$$\tilde{\sigma}_{12} = -\sin\theta\cos\theta\,\sigma_{11} + \cos\theta\sin\theta\,\sigma_{22} + (\cos^2\theta - \sin^2\theta)\sigma_{12}$$

Here: $\theta = -15°$

$\tilde{\sigma}_{11} = \sigma$ (along fibers)

$\tilde{\sigma}_{22} = \sigma$ (perpendicular to fibers)

$\cos\theta = 0.966$ $\quad \cos^2\theta = 0.933$

$\sin\theta = -0.259$ $\quad \sin^2\theta = 0.067$

$\sin\theta\cos\theta = -0.250$

→ Putting this in the transformation equations:

σ (along fibers) $= (0.933)(20\text{ ksi}) = 18.7\text{ ksi}$

σ (perpendicular to fibers) $= (0.067)(20\text{ ksi}) = 1.34\text{ ksi}$

Check Σ extensional = constant = 20 ksi ✓

→ Use in stress-strain equations:

$$\epsilon_{11} = \epsilon\text{ (along fibers)} = \frac{1}{24\text{ Msi}}\left[18.7\text{ ksi} - (0.24)(1.34\text{ ksi})\right] = 766 \times 10^{-6}$$

$$\epsilon_{22} = \epsilon\text{ (perpendicular to fibers)} = \frac{1}{3.0\text{ Msi}}\left[-(0.03)(18.7\text{ ksi}) + 1.34\text{ ksi}\right] = 260 \times 10^{-6}$$

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PROBLEM #2M (continued)

Summarizing:

strain parallel to fibers = 766 μstrain

strain perpendicular to fibers = 260 μstrain