Solutions Fluids Quiz 1

PROBLEM 1F

(10%) (a) Forces: gravity, surface tension, viscous stresses.

(10%) (b) Parameters: $D, g, v, t, \theta, h, \rho, \mu, \gamma \Rightarrow 9 \{N\}$

Units: $l, m, t \Rightarrow 3 \{k\}$

We have $N-k = 6$ Π groups. (No problem if you selected a subset of relevant parameters)

(40%) (c) By inspection:

$\Pi_1 = \theta$, contact angle

$\Pi_2 = \frac{D}{h}$, capillary diameter to column rise ratio

$\Pi_3 = \frac{t}{h/v}$, time to characteristic column rise time

$\Pi_4 = \frac{\gamma}{\mu v}$, surface tension stresses to viscous stresses.

$\Pi_5 = \frac{\rho v D}{\mu}$ or $\frac{\rho v h}{\mu}$, Inertia to viscous stress ratio (← Re #)

$\Pi_6 = \frac{gh}{v^2}$, hydrostatic pressure to dynamic pressure ratio.

Combinations of these factors give related non-dimensional groups, like:

$\Pi_5 \times \Pi_6 = \frac{\rho g h}{\mu v/D}$, hydrostatic to viscous stresses, ratio

$\frac{\Pi_4}{\Pi_5} = \frac{\gamma/D}{\rho v^2}$, surface tension pressure to dynamic pressure, ratio.

$\frac{\Pi_4}{\Pi_5 \times \Pi_6} = \frac{\gamma/D}{\rho g h}$, surface tension pressure to hydrostatic pressure, ratio.

$\Pi_4 \times \Pi_5 = \frac{\gamma D}{\mu^2/\rho}$, surface tension force to viscous friction, ratio.

among others ...

(30%) (d) if viscous stresses ~ surface tension, then $\Pi_4 \sim 1$

this means that $\frac{\gamma}{\mu v} \sim 1$ or $\boxed{v \sim \frac{\gamma}{\mu}}$ $v \sim \frac{0.048 \text{ N/m}}{0.021 \text{ Pa}\cdot\text{s}} \approx 2.3 \frac{\text{m}}{\text{s}}$

You could expect this happening when $\frac{D}{h} \sim 1$, otherwise gravity would become important in determining v.

(10%) (e) Inertial forces are much weaker than viscous stresses, since the Reynolds number (Π_5) is at most:

$$Re = \frac{\rho v D}{\mu} = \frac{1000 \frac{kg}{m^3} \cdot (2.3 \frac{m}{s}) \cdot (0.0005 m)}{0.021 \text{ Pa}\cdot\text{s}} = 55$$

which is a low value, and becomes smaller as $\Pi_2 << 1$

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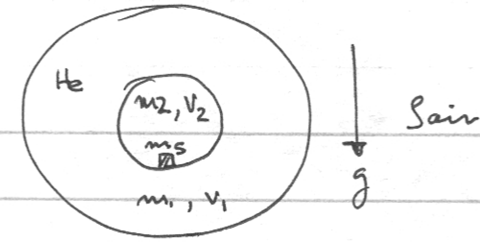
PROBLEM 2F



(30%) (a) Neutral buoyancy:

$$\text{big balloon} \Rightarrow (m_1 + m_2 + m_S) g = \rho_{air}\, g\, (V_1 + V_2) \quad (1)$$

$$\text{small balloon} \Rightarrow (m_2 + m_S) g = \rho_1 g V_2 \quad (2)$$

(30%) (b) if $m_1 = 4m_2$ and $m_2 = m_S$ $\Rightarrow$ $m_1 = 4m_S$

from (1) we find $6m_S = \rho_{air}(V_1 + V_2)$ or $V_1 + V_2 = \frac{6m_S}{\rho_{air}}$

from (2) we have, with $m_1 = \rho_1 V_1 \Rightarrow$ $2m_S = 4m_S \frac{V_2}{V_1}$ } $V_2 = \frac{V_1}{V_2}$

therefore: $V_1 + V_2 = V_1 + \frac{V_1}{2} = \frac{3}{2}V_1 = \frac{6m_S}{\rho_{air}}$

$$V_1 = \frac{4m_S}{\rho_{air}}$$

$$V_2 = \frac{2m_S}{\rho_{air}}$$

(20%) (c) Now, $V_2^{new} = \frac{3}{2}V_2 = \frac{3m_S}{\rho_{air}}$ but $V_1^{new} + V_2^{new} = \frac{6m_S}{\rho_{air}}$ (volume is fixed)

then $V_1^{new} = \frac{3m_S}{\rho_{air}} = V_2^{new}$ (volumes are equal)

The new buoyant force is: $F_B^{new} = \rho_1 g V_2^{new} = \frac{4m_S}{V_1^{new}} g V_2^{new} = 4m_S g$

So, there is a net upwards force $F_{NET} = 4m_S g - 2m_S g = 2m_S g$

The small balloon becomes positively buoyant.

(20%) (d) The buoyant force is still $F_B = \rho_{air}\, g\, (V_1^{new} + V_2^{new}) = F_B^{OLD} = 6m_S g$

The net upwards force is $F_{NET} = 6m_S g - 6m_S g = 0$

Remains neutrally buoyant.

Corollary: what remains fixed in this problem is the center of mass. While the small balloon climbs, the big one descends, until they hit each other and the 2-balloon system settles in buoyant equilibrium.