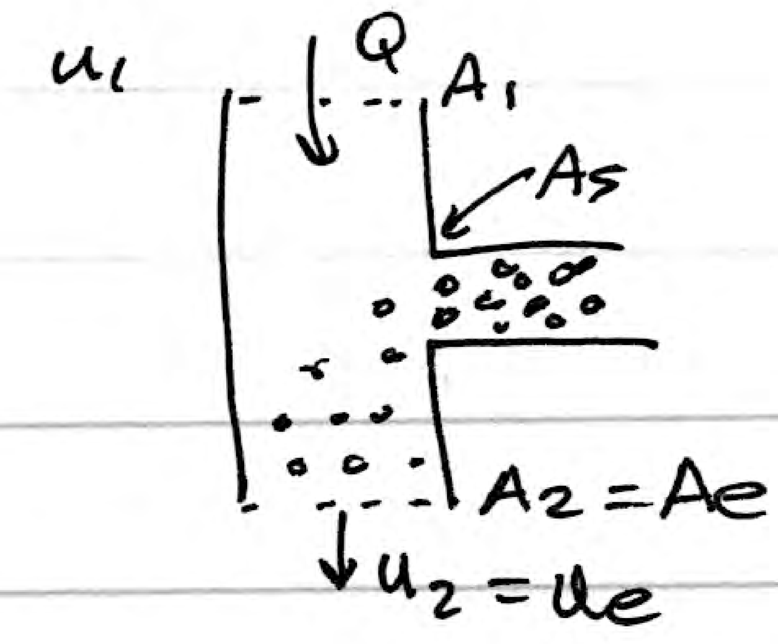
Solutions FQ2



(a) find the flux of particles in A_s:

the mass flow is simply: $\dot{m}_s = \dot{N} m_s$ ← mass of particles.

$m_s = \rho_s V_s = \rho_s \frac{4}{3}\pi R^3$

$\dot{m}_s = \int T_m dA = T_m A_s \Rightarrow \boxed{T_m = \frac{\dot{m}_s}{A_s} = \frac{4\rho_s \dot{N} \pi R^3}{3A_s}}$ verify units: $\left[\frac{kg}{s \cdot m^2}\right]$

(b) Continuity $\int \rho \vec{v}\cdot\hat{n}\, dA + \cancel{\frac{\partial}{\partial t}\int \rho\, dV} = 0$ ← 0 in steady state.

this means that $\dot{m}_{in} = \dot{m}_{out}$ for both particles and water:

⇒ for particles $\dot{N} m_s = \rho_s u_2 A_2^s$

⇒ for water $\rho_w u_1 A_1 = \rho_w u_2 A_2^w$

A_2^s and A_2^w are the areas on which particles and water are respectively crossing.

We also have $\boxed{A_2^s + A_2^w = A_2}$

then: $A_2^s = \frac{\dot{N} m_s}{\rho_s u_2}$ and $A_2^w = \frac{u_1 A_1}{u_2}$

add them ⇒ $\frac{\dot{N} m_s}{\rho_s u_2} + \frac{u_1 A_1}{u_2} = A_2$

solve for u_2 ⇒ $u_2 = \frac{1}{A_2}\left(\frac{\dot{N} m_s}{\rho_s} + u_1 A_1\right)$

or since $Q = u_1 A_1$, $A_2 = A_e$, $u_2 = u_e$ and $m_s = \rho_s \frac{4}{3}\pi R^3$

$$\boxed{u_e = \frac{4\pi R^3 \dot{N} + 3Q}{3A_e}}$$

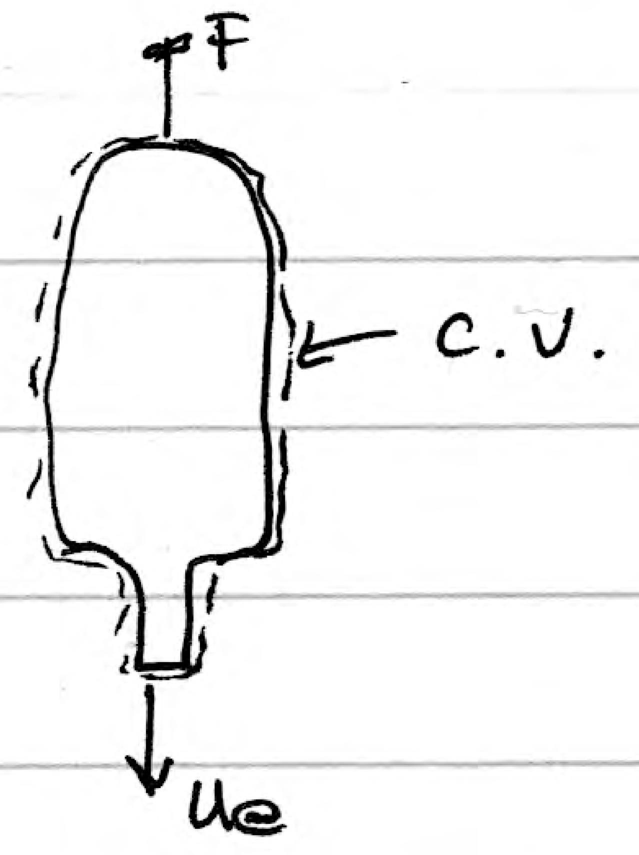
(c) Overall continuity: $\rho_w u_1 A_1 + \dot{N} m_s = \rho u_2 A_2 = \dot{m}_{NET}$

then $\rho = \frac{\rho_w Q + \dot{N} m_s}{u_2 A_2}$ or $\rho = \frac{\rho_w Q + \dot{N} \rho_s \frac{4}{3}\pi R^3}{\cancel{A_2}\left(\frac{4\pi R^3 \dot{N} + 3Q}{3\cancel{A_2}}\right)}$

or

$$\boxed{\rho = \frac{\rho_w Q + \rho_s \dot{N} \frac{4}{3}\pi R^3}{Q + \dot{N}\frac{4}{3}\pi R^3}}$$

if $\rho_s = \rho_w \Rightarrow \boxed{\rho = \rho_w}$ limit case

(d) 

$$\int \rho u(\bar{v}\cdot n)\,dA + \int p\,dA\,\hat{n} = -\cancel{F} + \cancel{F_4}^{\,0}\ (\text{outside})$$

$+R$

p is p_{atm} on C.V. $\Rightarrow$ $\boxed{\int p\,dA = 0}$

$$-F = \int \rho u(\bar{v}\cdot\hat{n})\,dA = -(\overbrace{m_s\dot{N} + \rho_w Q}^{\rho_e u_e A_e})\,u_e$$

$$\boxed{F = \underbrace{\left[\tfrac{4}{3}\pi R^3 \rho_s \dot{N} + \rho_w Q\right]}_{\dot{m}_{NET}} \underbrace{\left(\frac{4\pi R^3 \dot{N} + 3Q}{3A_e}\right)}_{u_e}}$$

About signs: F is (+) as shown.

This is a static problem (the rocket doesn't move)

so, $\bar{R} = -\bar{F}$

↑ reaction force should be equal in magnitude but opposite to F.