

Lect # 4

Note Title

10/26/2008

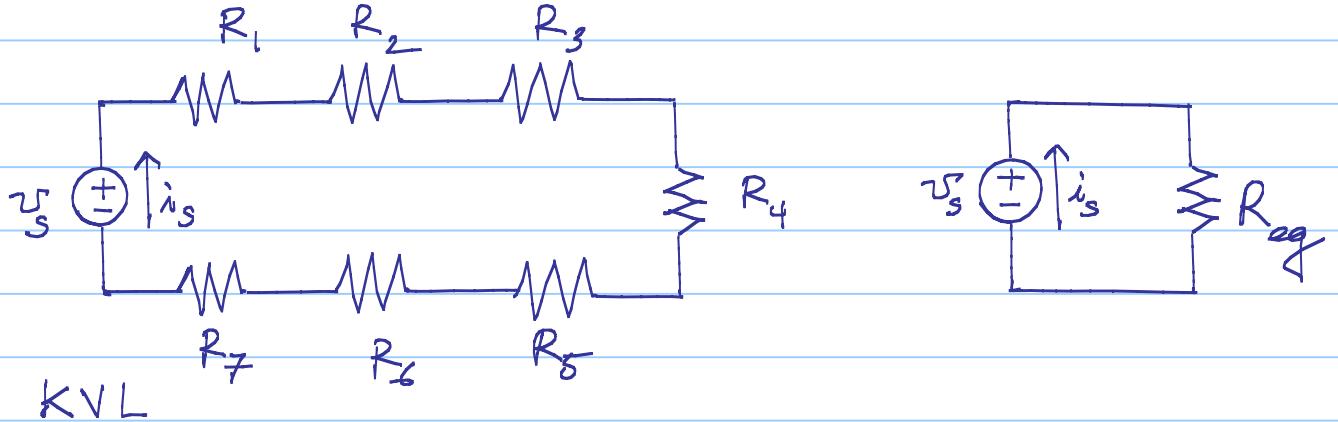
Next:

- Resistors in series
- Resistors in parallel
- Voltage Division
- Current Division
- Measuring Resistance - Wheatstone bridge

Objectives:

- Be able to recognize resistors connected in series and in parallel and use the rules for combining series-connected resistors and parallel-connected resistors to yield equivalent resistance
- Understand how a Wheatstone bridge is used to measure resistance

Resistors in Series.



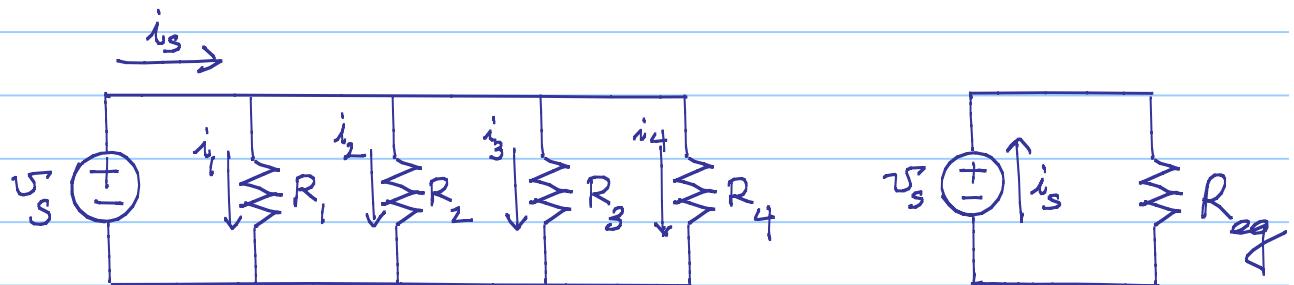
$$-U_s + i_s R_1 + i_s R_2 + \dots + i_s R_7 = 0$$

$$U_s = i_s \underbrace{(R_1 + R_2 + \dots + R_7)}_{R_{eq}}$$

$$R_{eq} = \sum_{i=1}^k R_i$$

Combining resistors in series

Resistors in Parallel



$$i_1 = \frac{U_s}{R_1} \quad i_2 = \frac{U_s}{R_2} \quad i_3 = \frac{U_s}{R_3} \quad i_4 = \frac{U_s}{R_4}$$

KCL

$$-i_s + i_1 + i_2 + i_3 + i_4 = 0$$

$$i_s = U_s \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

$$\frac{i_s}{U_s} = \frac{1}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \sum_{i=1}^k \frac{1}{R_i}$$

combining resistors in parallel

$$G_{eq} = \sum_{i=1}^k G_i \quad \text{in terms of conductance}$$

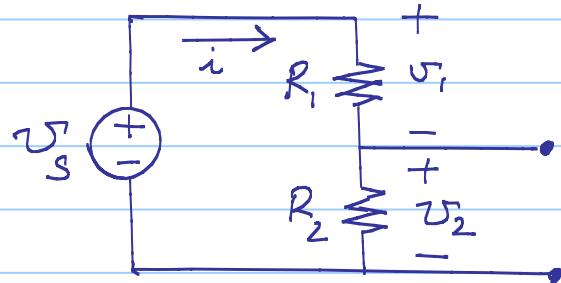
Many times only two resistors are connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

two resistors in parallel

Voltage Divider Circuit



$$V_s = iR_1 + iR_2 \Rightarrow i = \frac{V_s}{R_1 + R_2}$$

$$V_1 = iR_1 = V_s \frac{R_1}{R_1 + R_2}$$

$$V_2 = iR_2 = V_s \frac{R_2}{R_1 + R_2}$$

• V_1 & V_2 are fraction of voltage V_s with $V_1 + V_2 = V_s$

• each fraction is ratio of the resistance across which divided voltage is defined to the sum of the two resistances.

In general

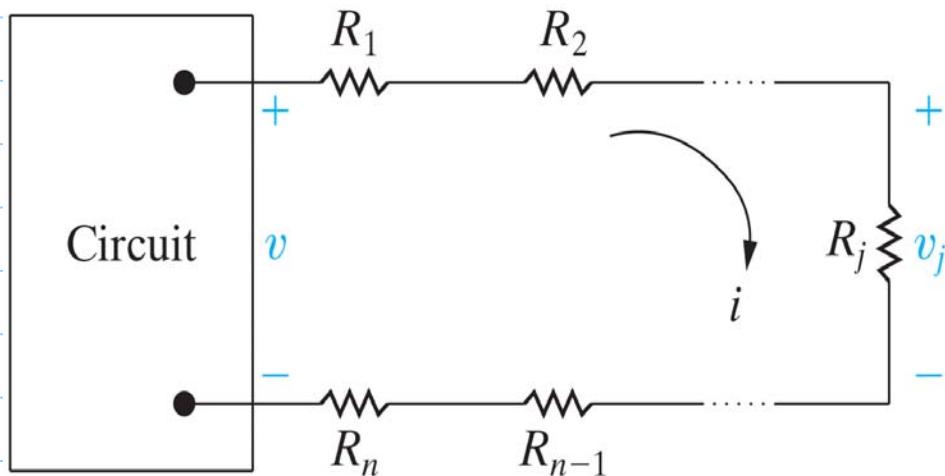


Figure: 03-18

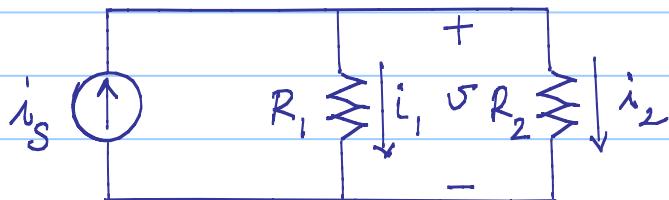
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$$v = i R_{\text{eq}} \quad \text{where} \quad R_{\text{eq}} = \sum_{i=1}^n R_i$$

$$v_j = i R_j = \frac{R_j}{R_{\text{eq}}} v$$

Voltage division eq.

Current Divider Circuit.



$$V = i_1 R_1 = i_2 R_2 = i_s R_{eq}$$

where $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

In general

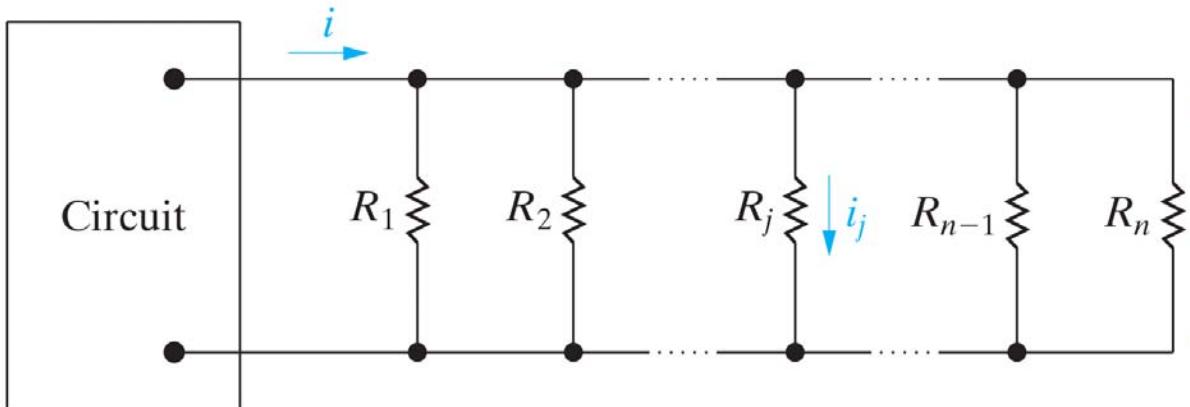


Figure: 03-19

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$$v = i R_{eq}$$

$$\text{where } \frac{1}{R_{eq}} = \sum_{i=1}^n \frac{1}{R_i}$$

$$v = i_j R_j$$

$$i_j = \frac{v}{R_j} = \frac{R_{eq} i}{R_j}$$

Current division eq

Measuring a Resistance - The Wheatstone Bridge

- Wheatstone bridge measure resistances of medium values (1Ω to $1M\Omega$)
- consists of four resistors, a dc voltage source and a detector. The resistance of one of the resistors can be varied
- To find the value of R_x , we adjust the value of R_4 no current in the detector (galvanometer). We then calculate the unknown resistor as

$$R_x = \frac{R_2}{R_1} R_3$$

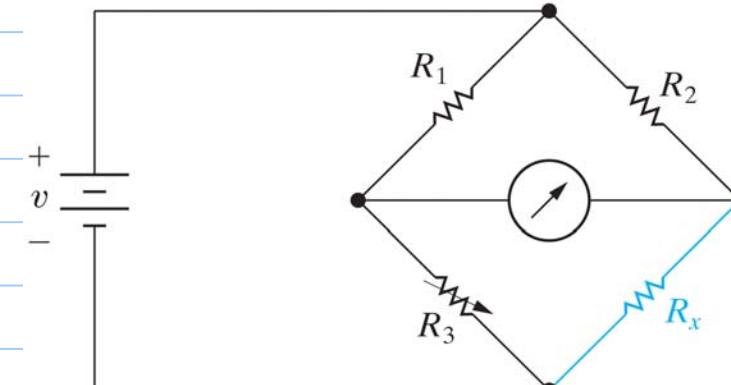


Figure: 03-26
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Proof: The derivation follows from KCL and KVL directly to the bridge circuit.

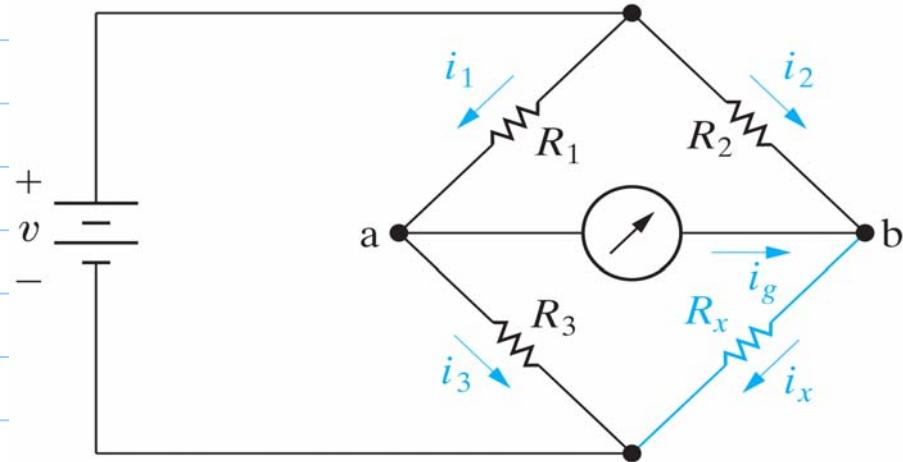


Figure: 03-27

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When $i_g = 0$ i.e., when the bridge is balanced

KCL \Rightarrow

$$i_1 = i_3 \quad (1)$$

$$i_2 = i_x \quad (2)$$

Because $i_g = 0$, there is no voltage drop across the detector and therefore points a and b are at the same potential.

Thus when the bridge is balanced,

$$\text{KVL} \Rightarrow i_3 R_3 = i_x R_x \quad (3)$$

$$i_1 R_1 = i_2 R_2 \quad (4)$$

Substituting (1) and (2) into (3) gives

$$i_1 R_3 = i_2 R_x \quad (5)$$

Dividing (5) by (4) gives

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

$$\Rightarrow R_x = \frac{R_2}{R_1} R_3 \quad \text{when } i_g = 0$$

Q.E.D



